On intuitionistic fuzzy k-bi-ideals of Boolean like semi rings

[1]M.Priyanka,[2]R.Rajeswari and [3]N.Meenakumari
[1]PG Student, [1]A.P.C.Mahalaxmi College for Women, Thoothukudi,Tamilnadu,India
[1]rajeshmanik1996@gmail.com
[2,3]PG& Research Department of Mathematics, A.P.C.Mahalaxmi College for Women, Thoothukudi,Tamilnadu,India
[2]rajeswarir30@yahoo.com,[3]meenakumari.n123@gmail.com

Abstract

Boolean like semi rings were introduced by K.Venkatesawarlu, B.V.N.Murthy and N.Amaranath during 2011. The idea of intuitionistic fuzzy set was introduced by Atanassov. In this paper we introduce the concept of intuitionistic fuzzy k – bi – ideals in Boolean like semi rings. An intuitionistic fuzzy bi – ideal A = (μ_A, γ_A) of R is called an intuitionistic fuzzy k – bi_ ideal of R if for x, y ∈R,

i)μ_A(x)≤ $\wedge \{\vee \{\mu_A(x+y), \mu_A(y+x)\},\ \mu_A(y)\}$

ii)γ_A(x)≤$\vee \{\wedge \{\gamma_A(x+y), \gamma_A(y+x)\},\ \gamma_A(y)\}$.

We investigate some of its properties

Keywords Boolean like semi ring, IFS, intuitionistic fuzzy bi - ideal, intuitionistic fuzzy k - bi - ideal.

1.Introduction:

Boolean like semi rings were introduced in roll by K.Venkatesawarlu, B.V.N. Murthiand N. Amaranath during 2011. Boolean like rings of A.L. Foster arise naturally from general ring dulity considerations and preserve many of the formal properties of Boolean ring. It is clear that every Boolean ring a Boolean like semi ring but not conversely. Fuzziness occurs when the knowledge is not precise. Fuzzy sets introduce vagueness by eliminating sharp boundary between the members of the class and nonmembers of the class whereas crisp sets dichotomize the individuals to members and nonmembers. A fuzzy set can be defined by assigning to each individual of the universe under consideration, a value of membership. Fuzzy theory is associated with information theory and uncertainly. The concept of the fuzzy set was first introduced by Zadeh in [6] . Since then , fuzzy set theory developed by Zadeh and others has evoked great interest among researches working in different branches of mathematics. Many notions of mathematics are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In this paper by N we mean a near-ring and by M we mean Γ-near-ring. In this paper we introduce the concept of intuitionistic fuzzy k – bi – ideals in Boolean like semi rings.

2.Preliminaries

Definition2.1: A system (R,+,·) a Boolean semi ring if and only if the following properties hold

i) (R,+) is an additive (abelian) group (whose ‘zero’ will be denoted by ‘0’)
ii) (R,·) is a semigroup of idempotents in the sense $aa = a$, for all a ∈R
iii) $a(b + c) = ab + ac$ and
iv) $abc = bac$, for all a, b, c ∈R.

Example2.2: Let (G,+) be any abelian group defined $ab = b$ for all a, b ∈G. Then (G,+,·) is a Boolean semiring.

Definition2.3: A nonempty set R together with two binary operations + and · satisfying the following conditions is called Boolean like semi ring.

i) (R,+) is an abelian group
ii) (R,·) is a semi group
iii) $a \cdot (b + c) = a \cdot b + a \cdot c$ for all $a, b, c \in$ R
iv) $a + a = 0$ for all a in R
v) $ab(ab + ab) = ab$ for all a, b ∈R

Definition2.4: Let μ be a fuzzy set defined on R then μ is said to be a fuzzy ideal of R if,

i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, for all x, y ∈R
ii) $\mu(ra) \geq \mu(a)$, for all $r, a \in R$
iii) $\mu\big((r + a)s + ra\big) \geq \mu(a)$, for all r, a, s ∈R

Definition2.5: Let μ be a fuzzy set defined on R. Then μ is said to be a fuzzy bi-ideal of R if,

i) $\mu(x - y) \geq min\{\mu(x), \mu(y)\}, for\ all\ x, y$ ∈R

ii) $\mu(xyz) \geq min\{\mu(x), \mu(z)\}$, $forall$

$x, y, z \epsilon$R

Definition2.6: A non-empty set R with two binary operations '+' and '·' is called a nearing if

i) (R,+) is a group (not necessarily abelian)
ii) (R,·) is a semigroup
iii) $x.(y + z) = x.y + x.z$ for all x, y, z ∈R

Definition2.7: A subgroup B of (N,+) is said to be a bi-ideal of N if BNB∩(BN)∗B ⊆B. In the case of zero symmetric near- ring subgroup B of (N,+) is a bi-ideal BNB⊆B.

Definition 2.8: A fuzzy set in a set M is a function $\mu: M \to [0,1]$.

Definition2.9: A complement of a fuzzy set μ, denoted by $\mu' = 1 - \mu(x)$ for all $x \in$ M.

Definition2.10: By μ_t we denote a level subset of μ, for $\{x \in M / \mu(x) \geq t\}$ where t∈[0,1].

Definition2.11: If μ is a fuzzy set M and f is a function defined on M, then the fuzz set ϑ in f(M) defined by, ϑ(y)=$Sup_{x \in f^{-1}(y)}\mu$(x) for all y∈f(M) is called the image of μ under f.

Definition2.12: If ϑ is a fuzzy set f(M), then the fuzzy set $\mu = \vartheta \circ f$ in M i.e., the fuzzy set defined by $\mu(x) = \vartheta(f(x))$ for all x in M is called the pre image of ϑ under f.

Definition2.13: A fuzzy set μ in M is said to have the sup property if for any subset T of M there exists t_0∈T such that $\mu(t_0) = \underset{t \in T}{Sup}\mu(t)$.

Definition 2.14: Let A and B be sets such that A⊆B. Define χ_A: B→ [0,1] by $\chi_A(x) = 1$ if x∈A, $\chi_A(x)=0$ if x∉A. Then χ_A is called as characteristic function of A.

Definition 2.15: μ is said to be fuzzy normal divisor with respect to the addition it μ satisfies

i) $\mu(x - y) \geq min\{\mu(x), \mu(y)\}$
ii) $\mu(y + x - y) \geq \mu(x)$, forall$x, y$ ∈M

3 Intuitionistic Fuzzy k – bi – ideal of Boolean like semi rings:

Definition 3.1: An IFS A= $(\mu_A; \gamma_A)$ in R called an intuitionistic fuzzy subgroup of R if

i) $\mu_A(x - y) \geq \wedge \{\mu_A(x), \mu_A(y)\}$
ii) $\gamma_A(x - y) \leq \vee \{\gamma_A(x), \gamma_A(y)\}$, for all x, y ∈R

Definition 3.2: An intuitionistic fuzzy subgroup A= $(\mu_A; \gamma_A)$ of R is called an intuitionistic fuzzy k-bi-ideal of R if,

i) $\mu_A(xyz) \geq \wedge \{\mu_A(x), \mu_A(z)\}$

ii) $\gamma_A(xyz) \leq \vee\{\gamma_A(x), \gamma_A(z)\}$ for all $x, y, z \in$R

Definition 3.3: An intuitionistic fuzzy bi-ideals A= $(\mu_A; \gamma_A)$ of R is called an intuitionistic fuzzy k-bi-ideal of R, if for all $x, y \in$ R

i) $\mu_A(x) \geq \wedge\{\vee\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$
ii) $\gamma_A(x) \leq \vee\{\wedge\{\gamma_A(x+y), \gamma_A(y+x)\}, \gamma_A(y)\}$

Theorem 3.4 : An IFS A= (μ_A, γ_A)in R is an intuitionistic fuzzy k – bi – ideal of R. Then the fuzzy sets μ_A and $\gamma_A^{'}$ are fuzzy k – bi – ideals of R.

Proof : Let A=(μ_A, γ_A)be an intuitionistic fuzzy k – bi – ideal of R. Then clearly $\mu_A$is a fuzzy k-bi-ideal of R. We claim that $\gamma_A^{'}$ is a fuzzy k – bi – ideal of R. Let $x, y \in$ R.

Then,$\gamma_A(x-y) = 1 - \gamma_A(x-y)$

$\geq 1 - \vee\{\gamma_A(x), \gamma_A(y)\}$

$= \wedge\{1-\gamma_A(x), 1-\gamma_A(y)\}$

$= \wedge\{\gamma_A^{'}(x), \gamma_A^{'}(y)\}$.

Let $x, y, z \in$R. Then,

$\gamma_A^{'}(xyz) = 1 - \gamma_A(xyz)$

$\geq 1 - \vee\{\gamma_A(x), \gamma_A(z)\}$

$= \wedge\{1-\gamma_A(x), 1-\gamma_A(z)\}$

$= \wedge\{\gamma_A^{'}(x), \gamma_A^{'}(y)\}$.

Now,$\gamma_A^{'}(x) = 1 - \gamma_A(x)$

$\geq 1 - \vee\{\wedge\{\gamma_A(x+y), \gamma_A(y+x)\}, \gamma_A(y)\}$

$= \wedge\{1 - \wedge\{\gamma_A(x+y), \gamma_A(y+x)\}, 1 - \gamma_A(y)\}$

$= \wedge\{\vee\{1-\gamma_A(x+y), 1-\gamma_A(y+x)\}, 1-\gamma_A(y)\}$

$= \wedge\{\vee\{\gamma_A^{'}(x+y), \gamma_A^{'}(y+x)\}, \gamma_A^{'}(y)\}$.

Hence, $\gamma_A^{'}$ is a fuzzy k-bi- ideal of R.

Theorem 3.5: Let f:R→R$'$ be a homomorphism of Boolean like semi rings. If B= (μ_B,γ_B) is an intuitionistic fuzzy k-bi-ideal of R$'$ then the Pre image f^{-1}(B)= $(f^{-1}((\mu_B); f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy k – bi – ideal of R.

Proof: Assume that B = (μ_B, γ_B) is an intuitionistic fuzzy k – bi – ideal of R$'$.

Let $x, y \in$R. Then,

$f^{-1}(\mu_B)(x-y) = \mu_B(f(x-y))$

$= \mu_B(f(x) - f(y))$

$\geq \wedge\{\mu_B(f(x), \mu_B(f(y)\}$

$= \wedge\{f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)\}$

$f^{-1}(\gamma_B)(x-y) = \gamma_B(f(x-y))$

$= \gamma_B(f(x) - f(y))$

$\leq \vee\{\gamma_B(f(x), \gamma_B f(y))\}$

$= \vee\{f^{-1}(\gamma_B)(x), f^{-1}(\gamma_B)(y)\}$

$\Rightarrow f^{-1}(B) = (f^{-1}(\mu_B); f^{-1}(\gamma_B))$ is an intuitionistic fuzzy subgroup of R.

Let $x, y, z \in R$,then,

$f^{-1}(\mu_B)(xyz) = \mu_B(f(xyz))$

$= \mu_B(f(x)f(y)f(z))$

$\geq \wedge\{\mu_B(f(x), \mu_B(f(z)\}$

$= \wedge\{f^{-1}\mu_B(x), f^{-1}\mu_B(z)\}$

$f^{-1}(\gamma_B)(xyz) = \gamma_B(f(xyz))$

$= \gamma_B(f(x)f(y)f(z))$

$\geq \vee\{\gamma_B(f(x), \gamma_B(f(z)\}$

$= \wedge\{f^{-1}\gamma_B(x), f^{-1}\gamma_B(z)\}$

Therefore,$f^{-1}(B)$ is intuitionistic fuzzy bi-ideal of R. Now, for $x, y \in$ R we have

$f^{-1}(\mu_B)(x) = \mu_B(f(x))$

$= \wedge\{\vee\{\mu_B(f(x)+f(y)), \mu_B(f(y)+f(x))\}, \mu_B(f(y))$

$= \wedge\{\vee\{\mu_B(f(x+y), \mu_B(f(y+x)\}, \mu_B(f(y))$

$=\wedge\{\vee\{f^{-1}(\mu_B)(x+y), f^{-1}(\mu_B)(y+x))\}, f^{-1}(\mu_B)(y)\}$

$f^{-1}(\gamma_B)(x) = \gamma_B(f(x))$

$=\wedge\{\vee\{\gamma_B(f(x)+f(y)),\ \gamma_B(f(y)+f(x))\}, \gamma_B(f(y))\}$

$=\wedge\{\vee\{\gamma_B(f(x+y),\ \gamma_B(f(y+x))\}, \gamma_B(f(y))\}$

$=\vee\{\wedge\{f^{-1}(\gamma_B)(x+y), f^{-1}(\gamma_B)\ (y+x))\}, f^{-1}(\gamma_B)(y)\}$

Hence, $f^{-1}(B)=(f^{-1}(\mu_B); f^{-1}(\gamma_B))$ is an intuitionistic fuzzy k - bi-ideal of R.

Definition3.6: Let A=$(\mu_A;\gamma_A)$ be an intuitionistic fuzzy set in R. Then the intuitionistic level subset of A is an object having the form $(\mu_A^t, \gamma_A^{t'})$ where

$$\mu_A^t = \{x \in R/\mu_A(x) \geq t\}$$

$$\gamma_A^{t'} = \{x \in R/\gamma_A(x) \leq t'\}$$

Theorem3.7: An IFS A=$(\mu_A;\gamma_A)$ in R is an intuionistic fuzzy k – bi – ideal of R if and only if the components $\mu_A^t \neq 0$ for any t∈ $Im\gamma_A$ and $\gamma_A^{t'} \neq 0$ for any $t' \in Im\gamma_A$ of the intuitionistic level subset are k-bi-ideal of R.

Proof: Assume that A = (μ_A, γ_A) in R, is an intuitionistic fuzzy k – bi-ideal of R.

Fix, t ∈ Im μ_A such that $\mu_A^t \neq 0$.

$t' \in$ Im γ_A such that $\gamma_A^t \neq 0$.

Suppose that,

$y \in \mu_A^t$.Then, $\mu_A(x) \geq t$, $\mu_A(y) \geq t$

$\Rightarrow\wedge\{\mu_A(x), \mu_A(y)\} \geq t$.

Weget, $\mu_A(x-y) \geq\wedge\{\mu_A(x), \mu_A(y)\} \geq t$

$\Rightarrow x-y \in \mu_A^t$.

Let $x, z \in \mu_A^t$, $y \in$R.

Then, $\mu_A(x) \geq t$, $\mu_A(z) \geq t$.

Thus,$\mu_A(xyz) \geq\wedge\{\mu_A(x), \mu_A(z)\} \geq$ t

$\Rightarrow xyz \in \mu_A^t$. Let $y \in \mu_A^t$

Also, Let $x+y \in \mu_A^t$ (or) $y+x \in \mu_A^t$ Then, $\{\mu_A(y)\} \geq t$ and $\{\mu_A(x+y)\} \geq t$ (or) $\mu_A(x+y) \geq t$.

Now,$\mu_A(x) \geq\wedge\{\vee\{\mu_A(x+y), \mu_A(y+x)\},\ \mu_A(y)\} \geq$t

$\Rightarrow x \in \mu_A^t$, Therefore μ_A^t is a k-bi-ideal of R.

Similarly, let $y \in \gamma_A^{t'}$.Then,$\gamma_A(x) \leq t', \gamma_A(y) \leq t', \Rightarrow\vee\{\gamma_A(x), \gamma_A(y)\} \leq t'$.

Thus we have ,

$\gamma_A(x-y) \leq \vee\{\gamma_A(x), \gamma_A(z)\} \leq t'$

$\Rightarrow x-y \in \gamma_A^{t'}$. Let $x, z \in \gamma_A^{t'}; y \in R$.

Then, $\gamma_A(x) \leq t'$; $\gamma_A(z) \leq t'$;

$\gamma_A(xyz) \leq \vee\{\gamma_A(x), \gamma_A(z)\} \leq t'$

$\Rightarrow xyz \in \gamma_A^{t'}$Let $y \in \gamma_A^{t'}$.

Also Let $x+y \in \gamma_A^{t'}$ (or) $y+x \in \gamma_A^{t'}$.

Then $\gamma_A(y) \leq t'$ and $\gamma_A(x+y) \leq t'$ (or) $\gamma_A(y+x) \leq t'$.

Now, $\gamma_A(x) \leq \vee\{\gamma_A(x+y), \gamma_A(y+x)\gamma_A(y),\} \leq t'$;$\Rightarrow x, \in \gamma_A^{t'}$

Hence, $\gamma_A^{t'}$ is also a k-bi-ideal of R.

Conversly, Assume that, the components μ_A^t and $\gamma_A^{t'}$ of the intuitionistic level subset are k-bi-ideal of R.Fix any $x, y \in$R and Let $\mu_A(x) = t_1, \mu_A(y) = t_2$. Let $t = \wedge\{t_1, t_2\}$,

Then, $\mu_A(x) \geq t$ and $\mu_A(y) \geq t \Rightarrow x, y \in \mu_A^t \Rightarrow x-y \in \mu_A^t$. Therefore,$\mu_A(x-y) \geq t$

$= \wedge\{t_1, t_2\}$,$= \wedge\{\mu_A(x), \mu_A(y)\}$.

Fix any $x, y \in$ R. Let $\gamma_A(x) = t_3$ and $\gamma_A(y) = t_4$.Let $t' = \vee\{t_3, t_4\}$.

Then $\gamma_A(x) \leq t'\gamma_A(y) \leq t' \Rightarrow x, y \in \gamma_A^{t'} \Rightarrow x-y \in \gamma_A^{t'}$.

Therefore, $\gamma_A(x-y)\gamma_A(x)= \vee\{t_3,t_4\}$ $=\vee\{\gamma_A(x),\gamma_A(y)\}$. Let $\mu_A(x)=t_1$ and $\mu_A(z)=t_2$. Let $t=\wedge\{t_1,t_2\}$,

Then, $\mu_A(x)\geq t$ and $\mu_A(z)\geq t \Rightarrow x,y\in \mu_A^t \Rightarrow xyz\in\mu_A^t$. Then, $\mu_A(xyz)\geq t = \wedge\{t_1,t_2\} = \wedge\{\mu_A(x),\mu_A(z)\}$. let $\gamma_A(x)=t_3$ and $\gamma_A(y)=t_4$. Let $t' = \vee\{t_3,t_4\}$.

$\gamma_A(x)\leq t'$ and $\gamma_A(z)\leq t' \Rightarrow x,y\in\gamma_A^{t'}$

$\Rightarrow xyz\in\gamma_A^{t'}$. Therefore $\gamma_A(xyz)\leq t'$ $=\vee\{t_3,t_4\} = \vee\{\gamma_A(x),\gamma_A(z)\}$

Thus, $A=(\mu_A,\gamma_A)$ in R is an intuitionistic fuzzy bi-ideal of R.

For any $x,y\in$R. Let $\mu_A(y)=t_1$, $\mu_A(x+y)= t_2$, $\mu_A(y+x)= t_3$, $(t_i\in Im\mu_A)$.

If we let $t=\wedge\{\vee\{t_2,t_3\},t_1\}$. Then $\mu_A(y)\geq t$ and $\mu_A(x+y)\geq t \mu_A(y+x)\geq t. \Rightarrow y\in \mu_A^t$ and $x+y\in\mu_A^t$ (or) $y+x\in\mu_A^t$. Since μ_A^t is a k – bi – ideal of R. We get $x\in\mu_A^t$

ie) $\mu_A(x)\geq t=\wedge\{\vee\{t_2,t_3\},t_1\}$.

$\mu_A(x)=\wedge\{\vee\{\mu_A(x+y),\mu_A(y+x)\}, \mu_A(y)\}$.

For any, $x,y\in$R,

Let, $\gamma_A(y)=t_4$, $\gamma_A(x+y)= t_5$, $\gamma_A(y+x)= t_6$, $(t_i\in Im\gamma_A)$.

If we get, $t'=\vee\{\wedge\{t_5,t_6\},t_4\}$., then $\gamma_A(y)\leq t'$ and $\gamma_A(x+y)\leq t'$, $\gamma_A(y+x)\leq t' \Rightarrow y\in\gamma_A^{t'}$ and $(x+y)\in\gamma_A^{t'}$ (or) $(y+x)\in\gamma_A^{t'} \Rightarrow x\in\gamma_A^{t'}$. Since $\gamma_A^{t'}$ is a k – bi – ideal of R. Therefore $\gamma_A(x)\leq t'$

$$=\vee\{\wedge\{t_5,t_6\},t_4\}$$

$$=\vee\{\wedge\{\gamma_A(x+y),\gamma_A(y+x)\}, \gamma_A(y)\}$$

Hence, A=$(\mu_A;\gamma_A)$ in R is an intuitionistic fuzzy k – bi- ideal of R.

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