SOME NEW b*ĝ-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract: The determination of this paper is to introduce some new functions namely strongly b*ĝ-continuous, slightly b*ĝ-continuous, perfectly b*ĝ-continuous and totally b*ĝ-continuous functions using b*ĝ-closed sets and we investigate some of its properties. Additionally, we relate and compare these functions with some other known existing functions in topological spaces.

Keywords: b*ĝ-closed sets, b*ĝ-continuous, contra b*ĝ-continuous, strongly b*ĝ-continuous, slightly b*ĝ-continuous, perfectly b*ĝ-continuous and totally b*ĝ-continuous.

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1. INTRODUCTION

In 1960, N.Levine [5] introduced the strong continuity in topological spaces. RC Jain [4] established the concept of totally continuous function and slightly continuous functions in topological spaces. In 2016, K.Bala Deepa Arasi and G.Subasini [1] introduced the b*ĝ-closed sets and studied their properties in topological spaces. A subset A is called so if b*Cl(A) ⊆U whenever A⊆U and U is ĝ-open in X. Later, in 2017, we [2] defined a new version of maps namely b*ĝ-continuous, b*ĝ-irresolute functions; b*ĝ-open map, b*ĝ-closed map and contra b*ĝ-continuous function in topological spaces. Also, we proved some properties of these functions and studied their relationships with the other existing functions.

By continuing this work, we introduce a new functions namely strongly b*ĝ-continuous, slightly b*ĝ-continuous, perfectly b*ĝ-continuous and totally b*ĝ-continuous functions in topological spaces and we investigate some of its properties. Additionally, we relate and compare these functions with some other known existing functions in topological spaces.

2. PRELIMINARIES

Throughout this paper (X,τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1

1) A subset A of a topological space $(X,)$ is said to be a **b*ĝ-closed set** [1] if b*Cl(A) ⊆ U whenever A ⊆ U and U is ĝ-open in X. The complement of a b*ĝ-closed set is called **b*ĝ-open set**.
2) A space (X,τ) is called a $\mathbf{T_{b^*\hat{g}}}$**-space** [1], if every b*ĝ-closed set in X is closed.
3) A space (X,τ) is called a **locally indiscrete space**, if every open set of X is closed in X.

Definition 2.2: A function $f:(X,\tau)\to(Y,\sigma)$ is called a

1) **continuous** [2] if $f^{-1}(V)$ is closed in (X,τ) for every closed set V in (Y,σ).
2) **b*ĝ-continuous** [2] if $f^{-1}(V)$ is b*ĝ-closed in (X,τ) for every closed set V in (Y,σ).
3) **contra continuous** [3] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).
4) **contra b*ĝ-continuous** [3] if $f^{-1}(V)$ is b*ĝ-closed in (X,τ) for every open set V in (Y,σ).
5) **strongly continuous** [5] if $f^{-1}(V)$ is clopen in (X,τ) for every subset V in (Y,σ).
6) **totally continuous** [6] if $f^{-1}(V)$ is clopen in (X,τ) for every open set V in (Y,σ).
7) **slightly continuous** [4] if $f^{-1}(V)$ is open in (X,τ) for every clopen set V in (Y,σ).
8) **perfectly continuous** [6] if $f^{-1}(V)$ is both open and closed in (X,τ) for every open set V in (Y,σ).

3. STRONGLY b*ĝ-CONTINUOUS FUNCTION

Definition 3.1: The function $f:(X,\tau)\to(Y,\sigma)$ is said to be **strongly b*ĝ-continuous** if the inverse image of every b*ĝ-closed set in Y is closed in X.

That is, $f^{-1}(V)$ is closed of (X,τ) for every b*ĝ-closed set V of (Y,σ).

Example 3.2: Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X,\phi, \{a\},\{b\},\{a,b\},\{a,c\}\}$ and $\sigma = \{Y,\phi,\{a\},\{b,c\}\}$; b*ĝC(Y) = $\{Y,\phi, \{a\},\{b,c\}\}$; C(X) = $\{X,\phi,\{b\},\{c\},\{b,c\}, \{a,c\}\}$. Define a map $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a, f(c) = c$. Here, f is strongly b*ĝ-continuous, since the inverse image of b*ĝC(Y) {b,c} and {a} are {a,c} and {b} respectively which are C(X).

Theorem 3.3: Every strongly continuous function is strongly b*ĝ-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be strongly continuous function and V be any b*ĝ-closed set in Y. Since f is strongly continuous, $f^{-1}(V)$ is closed in X. Hence f is strongly b*ĝ-continuous.

Remark 3.4: The converse of the above theorem need not be true as can be seen from the following example.

Example 3.5: Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X,\phi,\{b\},\{c\},\{b,c\}\}$ and $\sigma = \{Y,\phi,\{a,c\}\}$; b*ĝC(Y) = $\{Y,\phi,\{b\},\{a,b\}, \{b,c\}\}$; C(X) = $\{X,\phi,\{a\},\{a,b\}, \{a,c\}\}$; C(Y) = $\{Y,\phi,\{b\}\}$. Define a map $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a, f(c) = c$. Here, the inverse image of b*ĝC(Y) {b},{a,b} and {b,c} are {a},{a,b} and {a,c} respectively which are C(X) so f is strongly b*ĝ-continuous. But the inverse image of C(Y) {b} is {a} which is closed but not open in X so f is not strongly continuous.

Theorem 3.6: Every strongly b*ĝ-continuous function is b*ĝ-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be strongly b*ĝ-continuous function and V be any closed set in Y. By proposition 3.4 in [1], V is b*ĝ-closed set in Y. Since f is strongly b*ĝ-continuous, $f^{-1}(V)$ is closed in X. Again by proposition 3.4 in [1], $f^{-1}(V)$ is b*ĝ-closed in X. Hence f is b*ĝ-continuous.

Remark 3.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 3.8: Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X,\phi, \{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a\},\{b,c\}\}$; b*ĝC(Y) = $\{Y, \phi,\{a\},\{b,c\}\}$; C(X) = $\{X,\phi,\{c\},\{b,c\},\{a,c\}\}$; b*ĝC(X) = $\{X,\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,c\}\}$. Define a map $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Here, the inverse image of C(Y) {a} and {b,c} are {b} and {a,c} respectively which are b*ĝC(X) so f is b*ĝ-continuous. But the inverse image of b*ĝC(Y) {a} is {b} which is not C(X) so f is not strongly b*ĝ-continuous.

Theorem 3.9: If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\delta)$ are strongly b*ĝ-continuous functions, then $g \circ f:(X,\tau) \to (Z,\delta)$ is strongly b*ĝ-continuous functions.

Proof: Let V be any b*ĝ-closed set in Z. Since g is strongly b*ĝ-continuous, $g^{-1}(V)$ is closed in Y. By proposition 3.4 in [1], $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is strongly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ is closed in X. By proposition 3.4 in [1], $f^{-1}(g^{-1}(V))$ is b*ĝ-closed in X. Hence, $g \circ f$ is strongly b*ĝ-continuous.

Theorem 3.10: If $f:(X,\tau) \to (Y,\sigma)$ is strongly b*ĝ-continuous and $g:(Y,\sigma) \to (Z,\delta)$ is contra b*ĝ-continuous, then $g \circ f:(X,\tau) \to (Z,\delta)$ is contra continuous.

Proof: Let V be any open set in Z. Since g is contra b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is strongly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ is closed in X. Hence $g \circ f$ is contra continuous.

Theorem 3.11: If $f:(X,\tau) \to (Y,\sigma)$ is strongly b*ĝ-continuous and $g:(Y,\sigma) \to (Z,\delta)$ is contra b*ĝ-continuous, then $g \circ f:(X,\tau) \to (Z,\delta)$ is contra b*ĝ-continuous.

Proof: Let V be any open set in Z. Since g is contra b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is strongly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ is closed in X. By proposition 3.4 in [1], $f^{-1}(g^{-1}(V))$ is b*ĝ-closed in X. Hence $g \circ f$ is contra b*ĝ-continuous.

Theorem 3.12: If $f:(X,\tau) \to (Y,\sigma)$ is continuous and $g:(Y,\sigma) \to (Z,\delta)$ is strongly b*ĝ-continuous, then $g \circ f$ is strongly b*ĝ-continuous.

Proof: Let V be any b*ĝ-closed set in Z. Since g is strongly b*ĝ-continuous, $g^{-1}(V)$ is closed in Y. Since f is continuous, $f^{-1}(g^{-1}(V))$ is closed in X. Hence $g \circ f$ is strongly b*ĝ-continuous.

Theorem 3.13: If $f:(X,\tau) \to (Y,\sigma)$ is b*ĝ-continuous and $g:(Y,\sigma) \to (Z,\delta)$ is strongly b*ĝ-continuous, then $g \circ f$ is b*ĝ-continuous.

Proof: Let V be any closed set in Z. By proposition 3.4 in [1], V is b*ĝ-closed set in Z. Since g is strongly b*ĝ-continuous, $g^{-1}(V)$ is closed in Y. Since f is b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ is b*ĝ-closed in X. Hence $g \circ f$ is b*ĝ-continuous.

Theorem 3.14: Let X be any topological spaces and Y be a $T_{b*\hat{g}}$-space and $f:(X,\tau) \to (Y,\sigma)$ be a map. Then the following are equivalent:

(i) f is strongly b*ĝ-continuous
(ii) f is continuous
(iii) f is b*ĝ-continuous

Proof:

(i)⟹(ii) Let V be any closed set in Y. By proposition 3.4 in [1], V is b*ĝ-closed set in Y. Then by (i), $f^{-1}(V)$ is closed in X. Hence f is continuous.

(ii)⟹(i) Let V be any b*ĝ-closed set in Y. Since Y is a $T_{b*\hat{g}}$-space, V is closed set in Y. Then by (ii), $f^{-1}(V)$ is closed in X. Hence f is strongly b*ĝ-continuous.

(i)⟹(iii) The proof follows from theorem 3.6

(iii)⟹(i) Let V be any b*ĝ-closed set in Y. Since Y is a $T_{b*\hat{g}}$-space, V is closed set in Y. Then by (iii), $f^{-1}(V)$ is b*ĝ-closed in X. Again since Y is a $T_{b*\hat{g}}$-space, $f^{-1}(V)$ is closed in X. Hence f is strongly b*ĝ-continuous.

4. SLIGHTLY b*ĝ-CONTINUOUS FUNCTION

Definition 4.1: The function $f:(X,\tau)\to(Y,\sigma)$ is said to be **slightly b*ĝ-continuous** if the inverse image of every clopen set in Y is b*ĝ-closed in X. That is, $f^{-1}(V)$ is b*ĝ-closed of (X,τ) for every clopen set V of (Y,σ).

Example 4.2 Let $X = Y$ = {a,b,c} with topologies τ = {X,ϕ,{a},{b},{a,b},{a,c}} and σ = {Y,ϕ,{a,c}}; b*ĝC(X) = {X,ϕ,{b}, {c},{b,c},{a,c}}; C(Y) = {Y,ϕ,{b}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c$, $f(b)=b$, $f(c)=a$. Here, f is slightly b*ĝ-continuous, since the inverse image of clopen in Y {a,c} and {b} are {a,c} and {b} respectively which are b*ĝC(X).

Theorem 4.3: Every slightly continuous function is slightly b*ĝ-continuous.

Proof: Let f be slightly continuous function and V be a clopen set in Y. Since f is slightly continuous, $f^{-1}(V)$ is closed in X. By proposition 3.4 in [1], $f^{-1}(V)$ is b*ĝ-closed. Hence f is slightly b*ĝ-continuous.

Remark 4.4: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.5: Let $X = Y$ = {a,b,c} with topologies τ = {X,ϕ,{a},{a,b}} and σ = {Y,ϕ,{a,c}}; σ^c = {Y,ϕ,{b}}. b*ĝC(X) = {X, ϕ,{b},{c},{b,c},{a,c}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c$, $f(b)=b$, $f(c)=a$. Here, the inverse image of clopen in Y {b} and {a,c} are {b} and {a,c} respectively which are b*ĝC(X) but not closed in X. Hence f is slightly b*ĝ-continuous but not slightly continuous.

Theorem 4.6: If $f:(X,\tau)\to(Y,\sigma)$ is slightly b*ĝ-continuous and (X,τ) is $T_{b*\hat{g}}$-space, then f is slightly continuous.

Proof: Let V be a clopen set in Y. Since f is slightly b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed in X. Since X is $T_{b*\hat{g}}$-space, $f^{-1}(V)$ is closed in X. Hence f is slightly continuous.

Theorem 4.7: Every b*ĝ-continuous function is slightly b*ĝ-continuous.

Proof: Let f be b*ĝ-continuous function and V be a clopen set in Y. Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed in X. Hence f is slightly b*ĝ-continuous.

Theorem 4.8: If $f:(X,\tau)\to(Y,\sigma)$ is slightly b*ĝ-continuous and (Y,σ) is a locally indiscrete space, then f is b*ĝ-continuous.

Proof: Let V be any open subset in Y. Since Y is locally indiscrete space, V is closed set in Y. Since f is slightly b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed in X. Hence f is b*ĝ-continuous.

Theorem 4.9: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ be the two functions. Then the following holds:

(i) If f is b*ĝ-irresolute and g is slightly b*ĝ-continuous, then $g\circ f$ is slightly b*ĝ-continuous.

(ii) If f is b*ĝ-irresolute and g is b*ĝ-continuous, then $g\circ f$ is slightly b*ĝ-continuous.

(iii) If f is b*ĝ-continuous and g is slightly continuous, then $g\circ f$ is slightly b*ĝ-continuous.

(iv) If f is strongly b*ĝ-continuous and g is slightly b*ĝ-continuous, then $g\circ f$ is slightly continuous.

Proof:

(i) Let V be a clopen set in Z. Since g is slightly b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is b*ĝ-irresolute, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is b*ĝ-closed in X. Hence $g\circ f$ is slightly b*ĝ-continuous.

(ii) Let V be a clopen set in Z. Since g is b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is b*ĝ-irresolute, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is b*ĝ-closed in X. Hence $g\circ f$ is slightly b*ĝ-continuous.

(iii) Let V be a clopen set in Z. Since g is slightly continuous, $g^{-1}(V)$ is closed in Y. Since f is b*ĝ-continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is b*ĝ-closed in X. Hence $g\circ f$ is slightly b*ĝ-continuous.

(iv) Let V be a clopen set in Z. Since g is slightly b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is strongly b*ĝ-continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is closed in X. Hence $g\circ f$ is slightly continuous.

5. PERFECTLY b*ĝ-CONTINUOUS FUNCTION

Definition 5.1: The function $f:(X,\tau)\to(Y,\sigma)$ is said to be **perfectly b*ĝ-continuous** if the inverse image of every b*ĝ-closed set in Y is both open and closed in X. That is, $f^{-1}(V)$ is clopen of (X,τ) for every b*ĝ-closed set V of (Y,σ).

Example 5.2: Let $X = Y$ = {a,b,c} with topologies τ = {X,ϕ, {a},{b},{a,b},{a,c}} and σ = {Y,ϕ,{c},{a,b}}; b*ĝC(Y) = {Y,ϕ, {a,b}, {c}}; C(X) = {X,ϕ,{b},{c},{b,c},{a,c}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=c$, $f(c)=a$. Here, f is perfectly b*ĝ-continuous, since the inverse image of b*ĝC(Y) {c} and {a,b} are {b} and {a,c} respectively which are both closed and open.

Theorem 5.3: Every perfectly b*ĝ-continuous function is perfectly continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be perfectly b*ĝ-continuous and V be any closed set in Y. By proposition 3.4 in [1] V is b*ĝ-closed in Y. Since f is perfectly b*ĝ-continuous, $f^{-1}(V)$ is both open and closed in X. Hence f is perfectly continuous.

Remark 5.4: The converse of the above theorem need not be true as can be seen from the following example.

Example 5.5: Let $X = Y$ = {a,b,c} with topologies τ = {X,ϕ,{a},{c},{a,c},{b,c}} and σ = {Y,ϕ,{a,c}}; b*ĝC(Y) = {Y,ϕ,{b}, {a,b}, {b,c}}; C(X) = {X,ϕ,{a },{b},{a,b},{b,c}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=c, f(c)=a$. Here, the inverse image of O(Y) {a,c} is {b,c} respectively which is both open and closed in X but the inverse image of b*ĝC(Y) {a,b} is {a,c} which is open but not closed in X. Hence f is perfectly continuous but not perfectly b*ĝ-continuous.

Theorem 5.6: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ be the two functions. Then the following holds:

(i) If f is perfectly b*ĝ-continuous and g is perfectly b*ĝ-continuous, then $g\circ f$ is perfectly b*ĝ-continuous.

(ii) If f is continuous and g is perfectly b*ĝ-continuous, then $g\circ f$ is perfectly b*ĝ-continuous.

(iii) If f is perfectly b*ĝ-continuous and g is b*ĝ-irresolute, then $g\circ f$ is perfectly b*ĝ-continuous.

(iv) If f is perfectly b*ĝ-continuous and g is b*ĝ-continuous, then $g\circ f$ is perfectly continuous.

(v) If f is perfectly continuous and g is strongly b*ĝ-continuous, then $g\circ f$ is perfectly b*ĝ-continuous.

Proof:

(i) Let V be a b*ĝ-closed set in Z. Since g is perfectly b*ĝ-continuous, $g^{-1}(V)$ is both open and closed in Y. By proposition 3.4 in [1], $g^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in Y. Since f is perfectly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is both open and closed in X. Hence $g\circ f$ is perfectly b*ĝ-continuous.

(ii) Let V be a b*ĝ-closed set in Z. Since g is perfectly b*ĝ-continuous, $g^{-1}(V)$ is both open and closed in Y. Since f is continuous, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is both open and closed in X. Hence $g\circ f$ is perfectly b*ĝ-continuous.

(iii) Let V be a b*ĝ-closed set in Z. Since g is b*ĝ-irresolute, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is perfectly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is both open and closed in X. Hence $g\circ f$ is perfectly b*ĝ-continuous.

(iv) Let V be a closed set in Z. Since g is b*ĝ-continuous, $g^{-1}(V)$ is b*ĝ-closed in Y. Since f is perfectly b*ĝ-continuous, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is both open and closed in X. Hence $g\circ f$ is perfectly continuous.

(v) Let V be a b*ĝ-closed set in Z. Since g is strongly b*ĝ-continuous, $g^{-1}(V)$ is closed in Y. Since f is perfectly continuous, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is both open and closed in X. Hence $g\circ f$ is perfectly b*ĝ-continuous.

6. TOTALLY b*ĝ-CONTINUOUS FUNCTION

Definition 6.1: The function $f:(X,\tau)\to(Y,\sigma)$ is said to be **totally b*ĝ-continuous** if the inverse image of every closed set in Y is b*ĝ-clopen in X. That is, $f^{-1}(V)$ is b*ĝ-clopen of (X,τ) for every closed set V of (Y,σ).

Example 6.2: Let $X = Y$ = {a,b,c} with topologies τ = {X,ϕ, {a},{b},{a,b}} and σ = {Y,ϕ,{c},{a,b}}; b*ĝC(X) = {X,ϕ,{a}, {b}, {c},{b,c},{a,c}}; b*ĝO(X) = {X,ϕ, {a},{b}, {a,b},{b,c},{a,c}}; C(Y) = {Y,ϕ,{c},{a, b}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=c, f(c)=a$. Here, f is totally b*ĝ-continuous, since the inverse image of C(Y) {c} and {a,b} are {b} and {a,c} respectively which are both b*ĝC(X) and b*ĝO(X).

Theorem 6.3: Every perfectly b*ĝ-continuous function is totally b*ĝ-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be perfectly b*ĝ-continuous and V be any closed set in Y. By proposition 3.4 in [1], V is b*ĝ-closed in Y. Since f is perfectly b*ĝ-continuous, $f^{-1}(V)$ is both open and closed in X. Again by proposition 3.4 in [1], $f^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in X. Hence f is totally b*ĝ-continuous.

Remark 6.4: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.5: Let $X = Y$ = {a,b,c} with topologies τ = {X, ϕ,{a},{c},{a,c},{b,c}} and σ = {Y,ϕ,{a,c}}; σ^c = {Y,ϕ,{b}}; b*ĝC(Y) = {Y,ϕ,{b},{a,b},{b,c}}; C(X) = {X,ϕ,{a}, {b}, {a,b},{b,c}}; b*ĝC(X) = {X,ϕ,{a}, {b},{a,b}, {b,c}}; b*ĝO(X) = {X,ϕ,{a},{c},{a,c}, {b,c}}. Define a map $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=c, f(c)=a$. Here, the inverse image of closed set in Y {b} is {a} respectively which is both b*ĝ-open and b*ĝ-closed in X but the inverse image of b*ĝC(Y) {a,b} is {a,c} which is open but not closed in X. Hence f is totally b*ĝ-continuous but not perfectly b*ĝ-continuous.

Theorem 6.6: Every totally b*ĝ-continuous function is b*ĝ-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be totally b*ĝ-continuous and V be any closed set in Y. Since f is totally b*ĝ-continuous, $f^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in X. Hence f is b*ĝ-continuous.

Theorem 6.7: Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\delta)$ be the two functions. Then the following holds:

(i) If f is b*ĝ-irresolute and g is totally b*ĝ-continuous, then $g \circ f$ is totally b*ĝ-continuous.

(ii) If f is totally b*ĝ-continuous and g is continuous, then $g \circ f$ is totally b*ĝ-continuous.

Proof:

(i) Let V be any closed set in Z. Since g is totally b*ĝ-continuous, $g^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in Y. Since f is b*ĝ-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in X. Hence $g \circ f$ is totally b*ĝ-continuous.

(ii) Let V be any closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is totally b*ĝ-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is both b*ĝ-open and b*ĝ-closed in X. Hence $g \circ f$ is totally b*ĝ-continuous.

7. References

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