# Interval Valued Fuzzy Ideals In Boolean Like Semi-Rings

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## Abstract :

In this paper, we introduce the notion of Interval valued fuzzy ideals in Boolean like semi- ring R and also characterize some of their properties and illustrate with examples of interval valued fuzzy ideals in Boolean like semi- ring. Let $\tilde{\mu}$ be an interval valued fuzzy subset of R. Then $\tilde{\mu}$ is said to be an interval valued fuzzy ideal of R, if

i) $\tilde{\mu}(x-y) \geq \min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$

ii) $\tilde{\mu}(ra) \geq \tilde{\mu}(a)$

iii) $\tilde{\mu}((r+a)s+rs) \geq \tilde{\mu}(a)$

## Keywords :

Boolean like semi-ring, interval valued fuzzy subset, interval valued fuzzy ideal.

## 1.Introduction :

The notion of fuzzy sets and fuzzy logic was introduced by Lotfi A.Zadeh in 1965. Fuzziness occurs when the knowledge is not precise. A Fuzzy set can be defined by assigning to each individual of the universe under consideration, a value of membership. Fuzzy theory is associated with information theory and uncertainty. Fuzzy ideals of rings were introduced by Ziu, and it has been studied by several authors.

Boolean like semi-rings were introduced in roll by K.Venkatesawarlu, B.V.N. Murthy and N.Amaranth during 2011. A Boolean like ring is a commutative ring with unity and is of characteristic 2. It is clear that every Boolean ring is a Boolean like ring but not conversely. In this paper, we introduce the concept of interval valued fuzzy ideals in Boolean like semi rings.

## 2. Preliminaries :

### Definition : 2.1

A non empty set R with two binary operations '+' and '.' is called a **near-ring** if

i) (R,+) is a group

ii) (R,·) is a semigroup

iii) x.(y+z) = x.y + x.z, ∀ x,y,z ∈ R

### Definition : 2.2

A system (R,+, ·) a **Boolean semi ring** if and only if the following properties hold

i) (R,+) is an additive(abelian)group(whose 'zero' will be denoted by'0')

ii) (R,·) is a semigroup of idempotents in the sense aa=a, ∀ a ∈ R

iii) a(b+c) = ab+ac &

iv) abc =bac, ∀ a,b,c ϵ R

## Definition : 2.3

A non-empty set R together with two binary operations + and · satisfying the following conditions is called a **Boolean like semi-ring**.

i) (R,+) is an abelian group

ii) (R,·) is a semigroup

iii) a.(b+c) = a.b+a.c, ∀ a,b,c ϵ R

iv) a+a = 0, ∀ a ϵ R

v) ab(a+b+ab) = ab, ∀ a,b ϵ R

## Definition : 2.4

A non-empty subset I of R is said to be an **ideal** if,

i) (I,+) is a subgroup of (R,+), (i,e) for a,b ϵ R ⇒ a+b ϵ R

ii) ra ϵ R for all a ϵ I, r ϵ R, (i,e), RI⊆I

iii) (r+a)s+rs ϵ I for all r,s ϵ R, a ϵ I.

## Definition : 2.5

Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy ideal** of R if

i) $\mu(x-y) \geq \min\{\mu(x),\mu(y)\}$,for all x,yϵR

ii) $\mu(ra) \geq \mu(a)$,for all r,aϵR

iii) $\mu((r+a)s+rs) \geq \mu(a)$, for all r,a,sϵR

## Definition : 2.6

A fuzzy set μ in a Boolean like semi-ring R is called an **anti-fuzzy ideal** of M, if

i) $\tilde{\mu}(x-y) \leq \max\{\tilde{\mu}(x),\tilde{\mu}(y)\}$,∀ x,yϵR

ii) $\tilde{\mu}(ra) \leq \tilde{\mu}(a)$, ∀ r,aϵR

iii) $\tilde{\mu}((r+a)s+rs) \leq \tilde{\mu}(a)$, ∀ r,a,sϵR

## Definition : 2.7

Let R & S be Boolean like semi-rings. A map f:R→S is called a Boolean like semi-ring homomorphism if f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y) for all x,yϵR.

## Definition : 2.8

Let X be any set. A mapping $\tilde{\eta}$: X→D[0,1] is called an interval valued fuzzy subset of X, where D[0,1] denotes the family of closed subintervals of [0,1] and $\tilde{\eta}(x) = [\eta^-(x), \eta^+(x)]$ for all x ϵ X, where $\eta^-$ and $\eta^+$ are fuzzy subsets of X such that $\eta^-(x) \leq \eta^+(x)$ for all x ϵ X.

## Definition : 2.9

An interval number ã, we mean an interval $[a^-,a^+]$ such that $0\leq a^-\leq a^+\leq 1$ and where $a^-$ and $a^+$ are the lower and upper limits of ã respectively. We also identify the interval [a,a] by the number aϵ[0,1]. For any interval numbers $\tilde{a}_j = [a_j^-,a_j^+]$, $\tilde{b}_j = [b_j^-,b_j^+]$ ϵ D[0,1], j ϵ Ω. (where Ω is index set) We define $\max^i\{\tilde{a}_j,\tilde{b}_j\} = [\max\{a_j^-,b_j^-\}, \max\{a_j^+,b_j^+\}]$, $\min^i\{\tilde{a}_j,\tilde{b}_j\} = [\min\{a_j^-,b_j^-\}, \min\{a_j^+,b_j^+\}]$, $\inf^i\tilde{a}_j = [\cap_{j\epsilon\Omega} a_j^-, \cap_{j\epsilon\Omega} a_j^+]$, $\sup^i\tilde{a}_j = [\cup_{j\epsilon\Omega} a_j^-, \cup_{j\epsilon\Omega} a_j^+]$ and let

i) $\tilde{a} \leq \tilde{b}$ iff $a^- \leq b^-$ and $a^+ \leq b^+$

ii) $\tilde{a} = \tilde{b}$ iff $a^- = b^-$ and $a^+ = b^+$

iii) $\tilde{a} < \tilde{b}$ iff $\tilde{a} \leq \tilde{b}$ and $\tilde{a} \neq \tilde{b}$

iv) $k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$

# 3. Interval Valued Fuzzy Ideals :

## Definition : 3.1

An interval valued fuzzy subset $\tilde{\mu}$

in a Boolean like semi ring R is called an **interval valued(i.v) fuzzy ideal** of R if

i) $\tilde{\mu}(x-y) \geq \min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$

ii) $\tilde{\mu}(ra) \geq \tilde{\mu}(a)$

iii) $\tilde{\mu}((r+a)s+rs) \geq \tilde{\mu}(a)$

## Example :3.2

Consider the Boolean like semi-ring (R,+,·), where '+' and '·' are defined as follows,

| + | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | a | b | c |
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | a | b | a | 0 |

| . | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | 0 | a | a |
| b | 0 | 0 | b | b |
| c | 0 | a | b | c |

Let $\tilde{\mu}$ be an interval valued fuzzy ideal defined on R by $\tilde{\mu}(0) = [0.5, 0.6]$, $\tilde{\mu}(a) = [0.7,0.8]$, $\tilde{\mu}(b) = [0.7,0.8]$, $\tilde{\mu}(c) = [0.8,0.9]$. Then $\tilde{\mu}$ is an interval valued fuzzy ideal in Boolean like semi ring R.

## Theorem : 3.3

Let $\{\tilde{\mu}_i / i\epsilon\Omega\}$ be family of i.v fuzzy ideals of a Boolean like semi ring R, then $\bigcap_{i\epsilon\Omega} \tilde{\mu}_i$ is also an i.v fuzzy ideal of R, where $\Omega$ is any index set.

## Proof :

Let$\{\tilde{\mu}_i / i\epsilon\Omega\}$ be family of i.v fuzzy ideals of R. Let x,y$\epsilon$R & $\tilde{\mu} = \bigcap_{i\epsilon\Omega} \tilde{\mu}_i$

Then, $\tilde{\mu}(x) = \bigcap_{i\epsilon\Omega} \tilde{\mu}_i(x)$

$= (\inf^i{}_{i\epsilon\Omega}\tilde{\mu}_i)(x)$

$= \inf^i{}_{i\epsilon\Omega}\tilde{\mu}_i(x)$

Now, let x,y $\epsilon$ R

$\tilde{\mu}(x-y) = \inf^i{}_{i\epsilon\Omega}\tilde{\mu}_i(x-y)$

$\geq \inf^i{}_{i\epsilon\Omega} \min^i\{\tilde{\mu}_i(x),\tilde{\mu}_i(y)\}$

$= \min^i\{\inf^i{}_{i\epsilon\Omega}\tilde{\mu}_i(x), \inf^i{}_{i\epsilon\Omega}\tilde{\mu}_i(y)\}$

$= \min^i\{\bigcap_{i\epsilon\Omega} \tilde{\mu}_i(x),\bigcap_{i\epsilon\Omega} \tilde{\mu}_i(y)\}$

$= \min^i\{\tilde{\mu}_i(x),\tilde{\mu}_i(y)\}$

Let r,a $\epsilon$ R, then

$\bigcap_{i\epsilon\Omega} \tilde{\mu}_i(ra) = \inf^i\{\tilde{\mu}_i(ra) / i\epsilon\Omega\}$

$\geq \inf^i\{\tilde{\mu}_i(a) / i\epsilon\Omega\}$

$= \{\bigcap_{i\epsilon\Omega} \tilde{\mu}_i(a)\}$

Let r,a,s $\epsilon$ R, then

$\bigcap_{i\epsilon\Omega} \tilde{\mu}_i((r+a)s+rs) = \inf^i\{\tilde{\mu}_i((r+a)s+rs) / i\epsilon\Omega\}$

$\geq \inf^i\{\tilde{\mu}_i(a) / i\epsilon\Omega\}$

$= \{\bigcap_{i\epsilon\Omega} \tilde{\mu}_i(a)\}$

Therefore, $\bigcap_{i\epsilon\Omega} \tilde{\mu}_i$ is an i.v fuzzy ideal of R.

## Theorem : 3.4

Let $\{\tilde{\mu}_i / i\epsilon\Omega\}$ be family of i.v fuzzy ideals of a Boolean like semi ring R, then $\bigcup_{i\epsilon\Omega} \tilde{\mu}_i$ is also an i.v fuzzy ideal of R, where $\Omega$ is any index set.

## Proof :

Let$\{\tilde{\mu}_i / i\epsilon\Omega\}$ be family of i.v fuzzy ideals of R. Let x,y$\epsilon$R & $\tilde{\mu} = \bigcup_{i\epsilon\Omega} \tilde{\mu}_i$

Then, $\tilde{\mu}(x) = \cup_{i\in\Omega} \tilde{\mu}_i(x)$

$= (\sup^i{}_{i\in\Omega}\tilde{\mu}_i)(x)$

$= \sup^i{}_{i\in\Omega}\tilde{\mu}_i(x)$

Now, let x,y ϵ R

$\tilde{\mu}(x-y) = \sup^i{}_{i\in\Omega}\tilde{\mu}_i(x-y)$

$\geq \sup^i{}_{i\in\Omega} \max^i\{\tilde{\mu}_i(x),\tilde{\mu}_i(y)\}$

$= \max^i\{\sup^i{}_{i\in\Omega}\tilde{\mu}_i(x), \sup^i{}_{i\in\Omega}\tilde{\mu}_i(y)\}$

$= \max^i\{\cup_{i\in\Omega} \tilde{\mu}_i(x),\cup_{i\in\Omega} \tilde{\mu}_i(y)\}$

$= \max^i\{\tilde{\mu}_i(x),\tilde{\mu}_i(y)\}$

Let r,a ϵ R, then

$\cup_{i\in\Omega} \tilde{\mu}_i(ra) = \sup^i\{\tilde{\mu}_i(ra) / i\in\Omega\}$

$\geq \sup^i\{\tilde{\mu}_i(a) / i\in\Omega\}$

$= \{\cup_{i\in\Omega} \tilde{\mu}_i(a)\}$

Let r,a,s ϵ R, then

$\cup_{i\in\Omega} \tilde{\mu}_i((r+a)s+rs) = \sup^i\{\tilde{\mu}_i((r+a)s+rs) / i\in\Omega\}$

$\geq \sup^i\{\tilde{\mu}_i(a) / i\in\Omega\}$

$= \{\cup_{i\in\Omega} \tilde{\mu}_i(a)\}$

Therefore, $\cup_{i\in\Omega} \tilde{\mu}_i$ is an i.v fuzzy ideal of R.

## Theorem : 3.5

Let f: $R_1 \rightarrow R_2$ be a homomorphism between Boolean like semi ring $R_1$& $R_2$. If $\bar{\gamma}$ is an i.v fuzzy ideal of $R_2$, then $f^{-1}(\bar{\gamma})$ is an i.v fuzzy ideal of $R_1$.

**Proof :**

Let $\bar{\gamma}$ is an i.v fuzzy ideal of $R_2$. Let x,y ϵ $R_1$. Then,

$f^{-1}(\bar{\gamma})(x-y) = \bar{\gamma} (f(x-y))$

$= \bar{\gamma} (f(x)-f(y))$

$\geq \min^i\{\bar{\gamma} (f(x)),\bar{\gamma} (f(y))\}$

$= \min^i\{ f^{-1}(\bar{\gamma}(x)), f^{-1}(\bar{\gamma}(y))\}$

Let r,a ϵ $R_1$, $f^{-1}(\bar{\gamma})(ra) = \bar{\gamma} (f(ra))$

$\geq \bar{\gamma} (f(a))$

$= f^{-1}(\bar{\gamma}(a))$

Let r,s,a ϵ $R_1$,

$f^{-1}(\bar{\gamma})((r+a)s+rs) = \bar{\gamma} (f((r+a)s+rs))$

$= \bar{\gamma} (f(rs)+f(as)+f(rs))$

$\geq \bar{\gamma} (f(a))$

$= f^{-1}(\bar{\gamma}(a))$

Therefore, $f^{-1}(\bar{\gamma})$ is an i.v fuzzy ideal of $R_1$.

## Theorem : 3.6

Let Rbe a Boolean like semi ring and$\tilde{\mu}$ be an i.v fuzzy ideal of R. Then the set $R_{\tilde{\mu}} = \{ x \in R / \tilde{\mu}(x) = \tilde{\mu}(0) \}$ is an i.v fuzzy ideal of R.

**Proof :**

Let $\tilde{\mu}$ be an i.v fuzzy ideal

Let x,y ϵ $R_{\tilde{\mu}}$ implies $\tilde{\mu}(x) =\tilde{\mu} (0)$ and $\tilde{\mu}(y) =\tilde{\mu} (0)$

Then, $\tilde{\mu} (x-y) \geq \min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}$

$= \min^i\{\tilde{\mu}(0),\tilde{\mu}(0)\}$

$=\tilde{\mu}(0)$

Hence, x-y ϵ $R_{\tilde{\mu}}$

Now ,for every r,s ϵ R, a ϵ $R_{\tilde{\mu}}$

$\tilde{\mu}((r + a)s + rs) \geq \tilde{\mu}(a)$

$=\tilde{\mu}(0)$

Hence, (r + a)s + rs ϵ $R_{\tilde{\mu}}$

$\tilde{\mu}(ra) \geq \tilde{\mu}(a) =\tilde{\mu}(0)$

Hence, ra ϵ $R_{\tilde{\mu}}$

## Theorem : 3.7

Let R be a Boolean like semi ring. Then a fuzzy set $\tilde{\mu}$ is an i.v fuzzy ideal of Riff $\tilde{\mu}^c$ is an i.v fuzzy ideal of R.

**Proof :**

Let x,y ϵ Rand $\tilde{\mu}$ be an i.v fuzzy ideal of Rthen we have,

$\tilde{\mu}^c(x - y) = 1- \tilde{\mu}(x - y)$

$\leq 1- \min^i\{\tilde{\mu}(x),\tilde{\mu} (y)\}$

$= \max^i\{1-\tilde{\mu}(x),1-\tilde{\mu} (y)\}$

$= \max^i\{\tilde{\mu}^c(x),\tilde{\mu}^c(y)\}$

Let r,a,s ϵ R. Then,

$\tilde{\mu}^c(ra) = 1-\tilde{\mu}(ra)$

$\leq 1- \tilde{\mu}(a)$

$= \tilde{\mu}^c(a)$

Let r,a,s ϵ R. Then,

$\tilde{\mu}^c((r + a)s + rs ) = 1- \tilde{\mu}((r + a)s + rs )$

$\leq 1-\tilde{\mu}(a)$

$= \tilde{\mu}^c(a)$

Hence, $\tilde{\mu}^c$ is a i.v fuzzy ideal of R similarly the converse follows.

## Theorem : 3.8

Let $\tilde{\mu}$ be an i.v fuzzy ideal of a Boolean like semi ring R and $\tilde{\mu}^+$ be a fuzzy set in R given by $\tilde{\mu}^+(x) = \tilde{\mu}(x) + 1 - \tilde{\mu}(1)$ for all x ϵ R. Then $\tilde{\mu}^+$ is an i.v fuzzy ideal of R.

**Proof :**

Let $\tilde{\mu}$ be an i.v fuzzy ideal of a Boolean like semi ring R for all x,y,r,a,s ϵR. Then,

$\tilde{\mu}^+(x - y) = \tilde{\mu}(x-y)+1-\tilde{\mu}(1)$

$\geq \min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\}+1-\tilde{\mu}(1)$

$= \min^i\{\tilde{\mu}(x)+1-\tilde{\mu}(1),\tilde{\mu}(y)+1-\tilde{\mu}(1)\}$

$= \min^i\{\tilde{\mu}^+(x),\tilde{\mu}^+ y)\}$

$\tilde{\mu}^+(ra) = \tilde{\mu}(ra)+1-\tilde{\mu}(1)$

$\geq \tilde{\mu}(a)+1-\tilde{\mu}(1)$

$= \tilde{\mu}^+(a)$

$\tilde{\mu}^+((r + a)s+rs ) = \tilde{\mu}((r + a)s+rs ) +1-\tilde{\mu}(1)$

$\geq \tilde{\mu}(a)+1-\tilde{\mu}(1)$

$= \tilde{\mu}^+(a)$

Hence $\tilde{\mu}^+$ is an i.v fuzzy ideal of a Boolean like semi ring R.

## Theorem : 3.9

Let $\tilde{\mu}$ be an i.v fuzzy ideal of a Boolean like semi ring R then $(\tilde{\mu}^+)^+ = \tilde{\mu}^+$

**Proof :**

For any x ϵ R, we have

$(\tilde{\mu}^+)^+(x) = \tilde{\mu}^+(x) + 1-\tilde{\mu}(1)$

$= \tilde{\mu}(x)+1-\tilde{\mu}(1)$ [by Theorem:3.8]

$= \tilde{\mu}^+(x)$

Hence $(\tilde{\mu}^+)^+ = \tilde{\mu}^+$

## Theorem : 3.10

Let $\tilde{\mu}$ be an i.v fuzzy ideal of a Boolean like semi ring R &

$\pi:[0,\tilde{\mu}(0)]\to[0,1]$ be an increasing function. Let $\tilde{\mu}_\pi$ be a fuzzy set in R defined by $\tilde{\mu}_\pi(x) = \pi(\tilde{\mu}(x))$ for all x ϵ R. Then $\tilde{\mu}_\pi$ is an i.v fuzzy ideal of R.

**Proof :**

Let x,y,r,a,s ϵ R. Then

$\tilde{\mu}_\pi(x - y) = \pi(\tilde{\mu}(x-y))$

$\geq \pi(\min^i\{\tilde{\mu}(x),\tilde{\mu}(y)\})$

$= \min^i\{\pi(\tilde{\mu}(x)),\pi(\tilde{\mu}(y))\}$

$= \min^i\{\tilde{\mu}_\pi(x),\tilde{\mu}_\pi(y)\}$

$\tilde{\mu}_\pi(ra) = \pi(\tilde{\mu}(ra))$

$\geq \pi(\tilde{\mu}(a))$

$=\tilde{\mu}_\pi(a)$

$\tilde{\mu}_\pi((r + a)s + rs) = \pi\tilde{\mu}((r + a)s + rs)$

$\geq \pi(\tilde{\mu}(a))$

$=\tilde{\mu}_\pi(a)$

Hence $\tilde{\mu}_\pi$ is an i.v fuzzy ideal of R.

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