NEAR MEAN CORDIAL LABELING OF CYCLE RELATED GRAPHS

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ABSTRACT :

Let $G = (V,E)$ be a simple graph. A ***Near Mean Cordial Labeling*** of G is a function in $f : V(G) \rightarrow \{1, 2, 3, . . . , p-1, p+1\}$ such that the induced map f^* defined by $f^*(uv) = \begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a ***Near Mean Cordial Graph*** if it admits a near mean cordial labeling.

In this paper, It is to be proved that Double Fan (DF_n) , Triangular snake(TS_n)*(When* $n \equiv 0,1,3\ (mod\ 4)$*)*and Jelly fish(J(m,n)) and $K_{1,n}$ @P_n@$K_{1,m}$are **Near Mean Cordial** graphs. And also Triangular snake(TS_n)(*When* $n \equiv 2\ (mod\ 4)$*) is* ***not Near Mean Cordial Graph.***

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Keywords and Phrases: *Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.*

I. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology ,we referred Harary [4].For labeling of graphs, we referred Gallian[1]. A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels of u and v.

A graph G is said to labeled if the n vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots\ldots.. v_n$. In a labeling of a particular type, the vertices are assigned distinct values from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2].In this paper, It is to be proved that Double Fan (DF_n) , Triangular snake(TS_n)*(When* $n \equiv 0,1,3\ (mod\ 4)$*)*and Jelly fish(J(m,n)) and $K_{1,n}$ @P_n@$K_{1,m}$are **Near Mean Cordial** graphs. And also Triangular snake(TS_n)(*When* $n \equiv 2 (mod\ 4)$*) is* ***not Near Mean Cordial Graph.***

II.PRELIMINARIES

Definition 2.1 : Let G= (V,E) be a simple graph. Let f:V(G) → {0,1} and the induced edge label,assigning $|f(u) - f(v)|$ is called a **Cordial Labeling** if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1differ by atmost 1 .A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2: Let G = (V,E) be a simple graph. G is said to be a **Mean Cordial Graph** if f:V(G)→{0,1,2} such that for each edge uv the induced map f*defined by $f^*(uv) = \left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where [x] denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \leq 1$ where e_f (0) is the number of edges with zero label. e_f (1) is the number of edges with one label.

Definition 2.3: Let $G = (V,E)$ be a simple graph. A ***Near Mean Cordial Labeling*** of G is a function in $f : V(G) \rightarrow \{1, 2, 3, . . . , p-1, p+1\}$ such that the induced map f^* defined by

$$f^*(uv) = \begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a ***Near Mean Cordial Graph*** if it admits a near mean cordial labeling.

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Definition 2.4: The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 U G_2$ and all lines joining V_1 with V_2 as vertex set V ($G_1 U\ G_2$) = V (G_1) U V(G_2) and edge set E(G_1 U G_2) =E (G_1) U E(G_2) U [uv : u ϵ V(G_1) and v ϵ V(G_2)] **.** The graph $P_n + K_1$ is called a Fan and $P_n + 2K_1$ is called the Double fan (DF_n)

Definition 2.5: A Triangular Snake is obtained from the path ($v_1, v_2, \ldots.., v_n$) by replacing every edge by a triangle C_3

Definition 2.6: For integers m,n ≥ 0. We consider the graph J(m,n) with vertex and edge set V(J(m,n))={u,v,x,y}U{$x_1, x_2, \ldots$, x_m}U{$y_1, y_2, \ldots, y_n$}and E(J(m,n))={(uv),(ux),(uy),(vx),(vy)}U{($x_i x$):1≤i≤m}U {($y_i y$) : 1 ≤ i ≤ n}. J(m,n) is called a jelly fish.

Definition 2.7 : $K_{1,n}$ @P_n@$K_{1,m}$ is a graph which is obtained by joining the root of the star $K_{1,n}$ at one end of the path P_n and joining the another root of the star $K_{1,m}$ at the other end of the path P_n.

III. MAIN RESULTS

Theorem 3.1: Double Fan (DF_n) is a Near Mean Cordial Graph.

Proof: Let V(G) = {u, u_i: $1 \le i \le n$, v_i : $1\le i \le n$ }

Let E(G) = {(uu_i) : $1\le i \le n$} U {(uv_i) :$1\le i \le n$} U{($v_i v_{i+1}$) : $1 \le i \le n-1$} U {($u_i u_{i+1}$): $1 \le i \le n-1$}

Define f : V(G) → {1,2,3, …., 2n, 2n+2} by

<u>Case (i)</u>:When <u>$n \equiv 0 \pmod 4$:</u>

Let f(u)= n+1

$f(u_{2i-1}) = i$, $\quad 1\le i \le \frac{n}{2}$

$f(u_{2i}) = \frac{n}{2} + i$, $\quad 1\le i \le \frac{n}{2}$

$f(v_1) = 2n + 2$

$f(v_{2i+1}) = 2n-(i-1)$, $\quad 1 \le i \le \frac{n}{2} - 1$

$f(v_{2i}) = n+i+1$, $\quad 1 \le i \le \frac{n}{2}$

<u>Case (ii)</u>:When <u>$n \equiv 1 \pmod 4$:</u>

Let f(u) = 2n +2

$f(u_{2i-1}) = i$, $\quad 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = \frac{3(n+1)}{2} + (i-1)$, $\quad 1\le i \le \frac{n-1}{2}$

$f(v_{2i-1}) = \frac{n+1}{2} + i$, $\quad 1\le i \le \frac{n+1}{2}$

$f(v_{2i}) = \frac{3n+1}{2} - (i-1)$, $\quad 1\le i \le \frac{n-1}{2}$

<u>Case (iii)</u>:When <u>$n \equiv 2 \pmod 4$:</u>

Let f(u) = $\frac{n+2}{2}$

$f(u_{2i-1}) = i$, $\quad 1\le i \le \frac{n}{2}$

$f(u_2) = 2n + 2$

$f(u_{2i}) = 2n - (i-2)$, $\quad 2\le i \le \frac{n}{2}$

$f(v_{2i}) = \frac{n+2}{2} + i$, $\quad 1\le i \le \frac{n}{2}$

$f(v_{2i-1}) = \frac{3n+2}{2} - (i-1)$, $\quad 1\le i \le \frac{n}{2}$

<u>Case (iv)</u>:When <u>$n \equiv 3 \pmod 4$:</u>

Let f(u) = $\frac{3(n+1)}{2}$

$f(u_{2i-1}) = i$, $\quad 1\le i \le \frac{n+1}{2}$

$f(u_2) = 2n+2$

$f(u_{2i}) = 2n - (i-2)$, $\quad 2\le i \le \frac{n-1}{2}$

$f(v_{2i-1}) = \frac{n+1}{2} + i$, $\quad 1\le i \le \frac{n+1}{2}$

$f(v_{2i}) = \frac{3n+1}{2} - (i-1)$, $\quad 1\le i \le \frac{n-1}{2}$

The induced edge labelings are,

$f^*(uu_i)=\begin{cases} 1 & \text{if } f(u) + f(u_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n$

$f^*(uv_i)=\begin{cases} 1 & \text{if } f(u) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n$

$f^*(u_i u_{i+1})=\begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n-1$

$f^*(v_i v_{i+1})=\begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1\le i \le n-1$

Edge condition:-

Here, $e_f(0) = e_f(1) = 2n-1$

So, in all the cases, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, DF_n or (P_n+2K_1) is a Near Mean Cordial graph.

For example, the Near Mean cordial labeling of DF_8, DF_9, DF_{10}& $DF_7$are shown in Figures 3.1.1-3.1.4.

When <u>$n \equiv 0 \pmod 4$:</u>

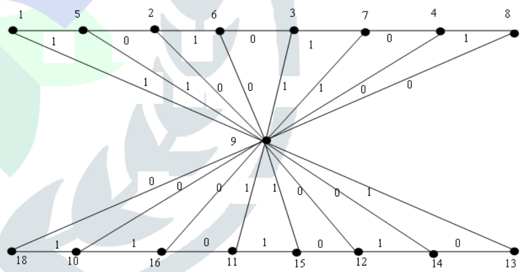

Figure 3.1.1

When <u>$n \equiv 1 \pmod 4$:</u>

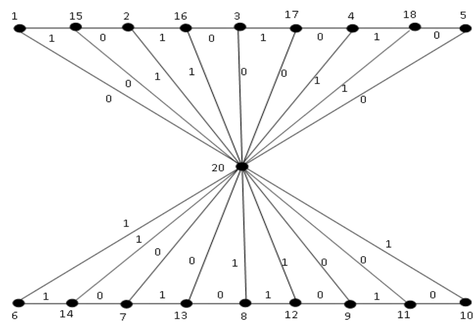

Figure 3.1.2

When <u>$n \equiv 2 \pmod 4$:</u>

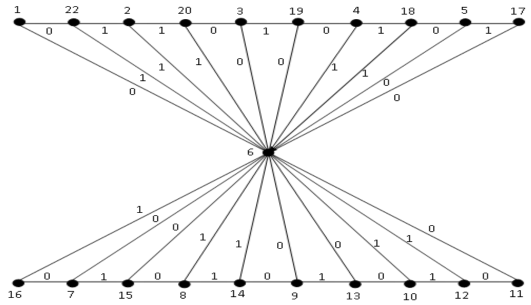

Figure 3.1.3

When $n \equiv 3 \pmod 4$:

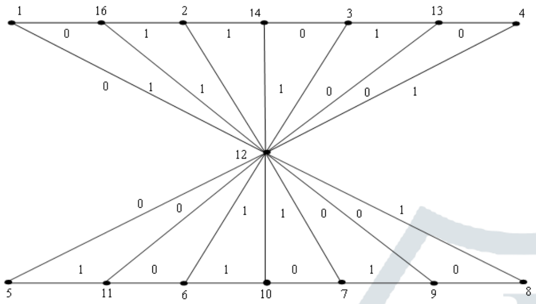

Figure 3.1.4

Theorem 3.2: Triangular Snake (TS_n) is a Near MeanCordial graph.(When n ≡ 0,1,3 (mod 4))

Proof: Let V(G) = {v_i :1≤ i≤ n+1, w_i : 1≤ i≤ n}

Let E(G) = {(v_iv_{i+1}) :1≤ i≤ n } U {(v_iw_i) : 1≤i ≤ n} U {($v_{i+1}w_i$) : 1≤ i ≤ n}

Define f : V(G) → {1,2,3,….., 2n, 2n+2} by

<u>Case(i)</u>:When <u>n ≡ 0 (mod 4)</u> :-

$f(v_{2i-1}) = i$, $1 \le i \le \frac{n+2}{2}$

$f(v_2) = 2n+2$

$f(v_{2i+2}) = 2n - (i-1)$, $1 \le i \le \frac{n-2}{2}$

$f(w_i) = \frac{n}{2}+i+1$, $1 \le i \le n$

<u>Case (ii)</u>:When <u>n ≡ 1 (mod 4)</u> :

$f(v_{2i-1}) = i$, $1 \le i \le \frac{n+1}{2}$

$f(v_2) = 2n+2$

$f(v_{2i+2}) = 2n - (i-1)$, $1 \le i \le \frac{n-1}{2}$

$f(w_i) = \frac{n+3}{2}+(i-1)$, $1 \le i \le n$

<u>Case (iii)</u>:When <u>n ≡ 3 (mod 4) :</u>

$f(v_{2i-1}) = i$, $1 \le i \le \frac{n+1}{2}$

$f(v_2) = 2n+2$

$f(v_{2i+2}) = 2n - (i-1)$, $1 \le i \le \frac{n-1}{2}$

$f(w_i) = \frac{n+3}{2}+(i-1)$, $1 \le i \le n$

The induced edge labelings are,

$$f^*(v_iw_i)=\begin{cases}1 & \text{if } f(v_i)+f(w_i)\equiv 0 \pmod 2\\ 0 & \text{else}\end{cases}, 1 \le i \le n$$

$$f^*(v_iv_{i+1})=\begin{cases}1 & \text{if } f(v_i)+f(v_{i+1})\equiv 0 \pmod 2\\ 0 & \text{else}\end{cases}, 1 \le i \le n$$

$$f^*(w_iv_{i+1})=\begin{cases}1 & \text{if } f(w_i)+f(v_{i+1})\equiv 0 \pmod 2\\ 0 & \text{else}\end{cases}, 1 \le i \le n$$

Edge condition:-

1. Here $e_f(0) = e_f(1) = \frac{3n}{2}$ (when n ≡ 0 (mod 4))

2. Here $e_f(0) = \frac{3n+1}{2}$, $e_f(1) = \frac{3n-1}{2}$ (when n ≡ 1 (mod 4))

3. Here $e_f(0) = \frac{3n-1}{2}$, $e_f(1) = \frac{3n+1}{2}$ (when n ≡ 3 (mod 4))

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, TS_n is a Near Mean Cordial Graph(when n ≡ 0,1,3 (mod 4)).

For example, the Near Mean Cordial labeling of TS_8, TS_9 & $TS_7$are shown in Figures 3.2.1`-3.2.3.

When $n \equiv 0 \pmod 4$:

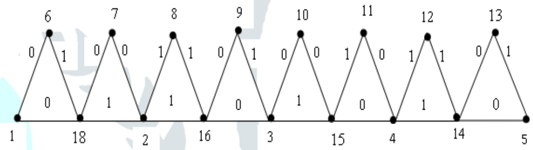

Figure 3.2.1

When $n \equiv 1 \pmod 4$:

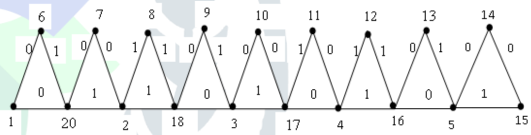

Figure 3.2.2

When $n \equiv 3 \pmod 4$:

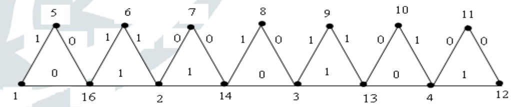

Figure 3.2.3

Theorem 3.3: Triangular snake (TS_n) is not a Near Mean Cordial graph [when n ≡ 2 (mod 4)].

Proof: Let V(G) = {v_i :1≤ i≤ n+1, w_i : 1≤ i≤ n}

Let E(G) = {(v_iv_{i+1}) :1≤ i≤ n } U {(v_iw_i) : 1≤i ≤ n} U {($v_{i+1}w_i$) : 1≤ i ≤ n}

Define f : V(G) → {1,2,3,….., 2n, 2n+2}

Consider TS_6 ,

Now the vertex labels are

1,2,3,4,5,6,7,8,9,10,11,12,14.

Out of which 7 are even numbers and 6 are odd numbers.

If a pair consisting of same parity it gives edge labeling 1. Otherwise the edge labling is 0.

In the example of TS_6,

The path P_7 have 3 ones and 3 zeros

The curved path have 7 ones and 5 zeros.

On the whole ,we get 10 ones and 5 zeros .Clearly in this case$|e_f(0) - e_f(1)| > 1$.If we give any type of labeling ,they do not satisfy the conditions of Near Mean Cordial labeling.

Clearly we have, $|e_f(0) - e_f(1)| > 1$.

Hence TS_n is not a Near Mean Cordial Graph. [when n ≡ 2 (mod 4)].

For example , the Near Mean Cordial Labeling of TS_6 is shown in Figure 3.3.1

When n ≡ 2 (mod 4) :

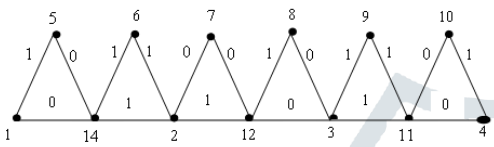

Figure 3.3.1

Theorem 3.4: Jelly Fish J(m,n) is a Near Mean Cordial graph.

Proof: Let V(G) = {u,v,x,y, x_i : 1≤ i ≤ m , y_i : 1≤ i ≤ n}

Let E(G) = {ux}U {uy} U{xy}U {xv} U {yv}U{(ux_i): 1≤ i ≤ m} U {(vy_i): 1≤ i ≤ n}

Define f : V(G) → {1,2,3,……..,m+n+3, m+n+5} by

Case (i) :When m is even, n is odd & When m is odd, n is odd

Fix f(u) = 1, f(v) = 2, f(x) = 3, f(y) = 4

$f(x_i) = 4 + i$, 1≤ i ≤ m

$f(y_i) = m+4+ i$, 1≤ i ≤ n-1

$f(y_n) = 5 + m+n$

Case (ii): When m is even, n is even & When m is odd, n is even

Fix f(u) = m+n+1,f(v) = m+n+2,

f(x) = m+n+3, f(y) = m+n+5

$f(x_i) = i$, 1≤ i ≤ m

$f(y_i) = m + i$, 1≤ i ≤ n

The induced edge labelingsare,

$$f^*(ux_i)=\begin{cases} 1 & \text{if } f(u)+f(x_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le m$$

$$f^*(vy_i)=\begin{cases} 1 & \text{if } f(v)+f(y_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(ux)=\begin{cases} 1 & \text{if } f(u)+f(x) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

$$f^*(uy)=\begin{cases} 1 & \text{if } f(u)+f(y) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

$$f^*(xy)=\begin{cases} 1 & \text{if } f(x)+f(y) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

$$f^*(xv)=\begin{cases} 1 & \text{if } f(x)+f(v) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

$$f^*(yv)=\begin{cases} 1 & \text{if } f(y)+f(v) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

Edge condition:-

When m and n is even:

$e_f(0) = \frac{m+n+4}{2}$, $e_f(1) = \frac{m+n+6}{2}$

When m is odd, n is even:

$e_f(0) = e_f(1) = \frac{m+n+5}{2}$

When m and n is odd:

$e_f(0) = \frac{m+n+6}{2}$, $e_f(1) = \frac{m+n+4}{2}$

When m is even, n is odd:

$e_f(0) = e_f(1) = \frac{m+n+5}{2}$

Hence, it satisfies the condition $|e_f 0) - e_f(1)| \le 1$

Hence, J(m,n) is a Near Mean Cordial Graph. For example, the Near Mean Cordial labeling of J(8,12), J(11,8), J(10,7) & J(9,11) are shown in Figures3.4.1-3.4.4.

When m and n is even:

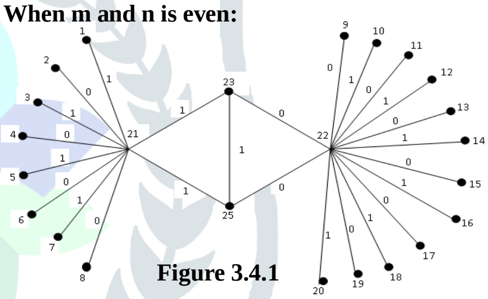

Figure 3.4.1

When m is odd and n is even:

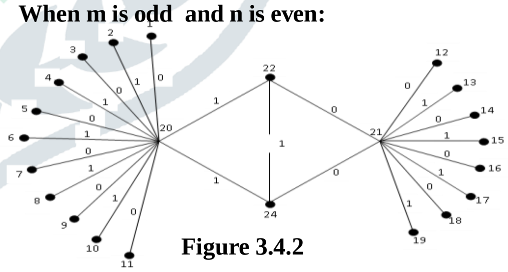

Figure 3.4.2

When m is even and n is odd :

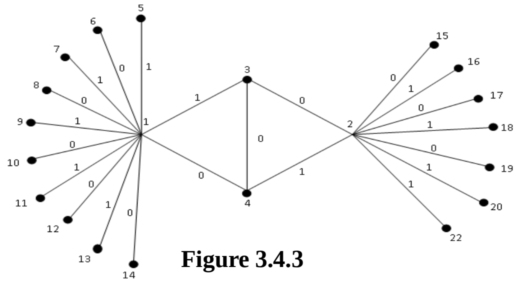

Figure 3.4.3

When m and n is odd:

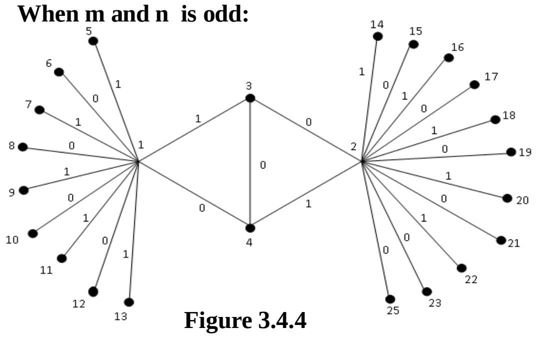

Figure 3.4.4

Theorem 3.5: $K_{1,n}@P_n@K_{1,m}$ is a Near Mean Cordial Graph.

Proof:

Let V(G) = {u_i:1≤ i ≤ n, v_i: 1≤ i ≤ n, w_i : 1≤ i ≤ m}

Let E(G) = {(u_iv_1):1≤ i ≤ n}U{(v_iv_{i+1}):1≤ i≤n-1}U{(v_nw_i) :1≤ i ≤ m}

Define f : V(G) → {1,2,3…..m+2n-1, m+2n+1} by

Case (i):When m ≡ 0 (mod 2) , n϶N

Let f(u_i) = 2i, 1≤ i≤ n

f(v_i) = 2i-1, 1≤ i≤ n

f(w_i) = 2n+i, 1≤ i≤ m-1

f(w_m) = m+2n+1

Case (ii):When m ≡ 1 (mod 2) , n϶N

Let f(u_i) = 2i-1, 1≤ i≤ n

f(v_i) = 2i, 1≤ i≤ n

f(w_i) = 2n+i, 1≤ i≤ m-1

f(w_m) = m+2n+1

The induced edge labelings are

$$f^*(u_iv_1)=\begin{cases} 1 & \text{if } f(u_i)+f(v_1)\equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1\le i\le n$$

$$f^*(v_iv_{i+1})=\begin{cases} 1 & \text{if } f(v_i)+f(v_{i+1})\equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1\le i\le n-1$$

$$f^*(v_nw_i)=\begin{cases} 1 & \text{if } f(v_n)+f(w_i)\equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1\le i\le m$$

Edge condition:-

Let m = 2k+1, (k϶N)

Here $e_f(0) = e_f(1)$ = m-k+n-1

Let m=2k, (k϶N)

Here $e_f(0)$ = m-k+n-1

$e_f(1)$ = m-k+n

So it satisfies the condition $|e_f(0) - e_f(1)| \le 1$. Hence $K_{1,n}@P_n@K_{1,m}$ is a Near Mean Cordial Graph.. For example, the Near Mean Cordial Labeling of $K_{1,8}@P_8@K_{1,9}$,$K_{1,8}@P_8@K_{1,10}$, $K_{1,9}@P_9@K_{1,10}$, $K_{1,9}@P_9@K_{1,11}$ are shown in Figures 3.5.1-3.5.4.

**When n is even : ** m ≡ 1 (mod 2)

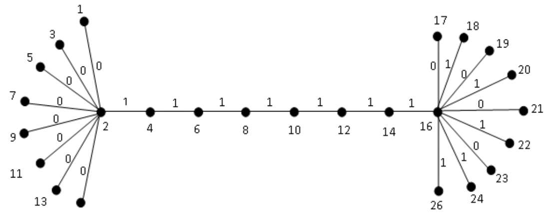

Figure 3.5.1

**When n is even : ** m ≡ 0 (mod 2)

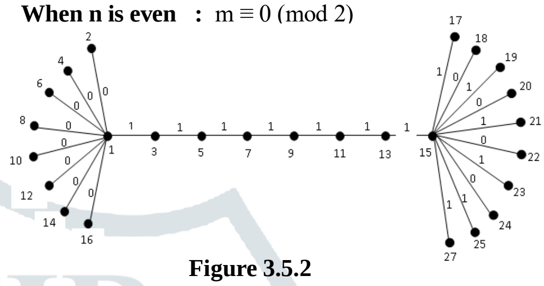

Figure 3.5.2

When n is odd : m ≡ 0 (mod 2)

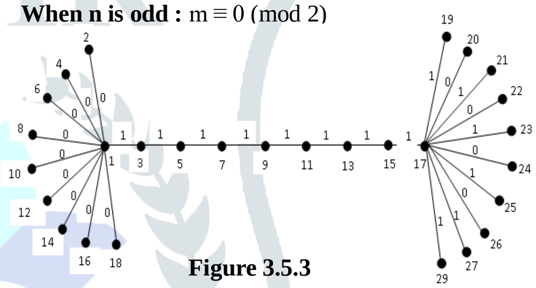

Figure 3.5.3

When n is odd: m ≡ 1 (mod 2)

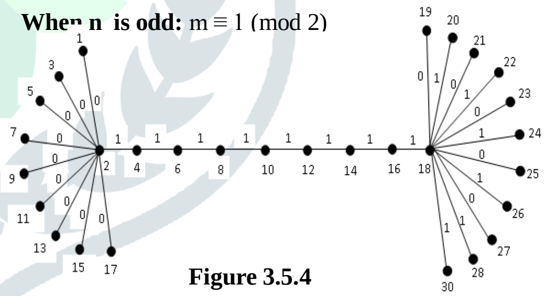

Figure 3.5.4

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