APPLICATION ON b$\hat{g}$-CLOSEDSETS IN TOPOLOGICAL SPACES

[1] K.Bala Deepa Arasi, [2] A.Balavickhneswari, [3] V.Maheswari
[1,3] Assistant Professor of Mathematics, PG & Research Department of Mathematics,
A.P.C.Mahalaxmi College For Women, Thoothukudi, TN, India.
baladeepa85@gmail.com
[2] M.Phil Scholar, PG & Research Department of Mathematics,
A.P.C.Mahalaxmi College For Women, Thoothukudi, TN, India.
balavig1091996@gmail.com

Abstract:

In 1970, Levine defined generalized closed sets in Topological Spaces. In 2013, R.Subasree and M.Maria Singam introduced a new set namely b$\hat{g}$-closed set which is defined as bcl(A)⊆U whenever A⊆U and U is $\hat{g}$-open in X. Later, they have established function such as b$\hat{g}$-continous function that is the inverse image of every closed set is b$\hat{g}$-closed in .b$\hat{g}$-irrresolute function if the inverse image every b$\hat{g}$-closed set is b$\hat{g}$-closed; b$\hat{g}$-open map means the image of every open set is b$\hat{g}$-open and b$\hat{g}$-closed map means the image of every closed set is b$\hat{g}$-closed. In 1973, Das defined semi-interior point and semi-limit point of a subset. Further, the semi-derived set of a topological space was defined and studied by him. Following this concept, we define b$\hat{g}$-limit point,b$\hat{g}$-derived set, b$\hat{g}$-border, b$\hat{g}$-frontier and b$\hat{g}$-exterior of a subset of topological spaces using the concept of b$\hat{g}$-closed sets which is defined as and we studied some of its properties.

Keywords: b$\hat{g}$-limit point, b$\hat{g}$-derived set, b$\hat{g}$-border, b$\hat{g}$-frontier, b$\hat{g}$-exterior, b$\hat{g}$-closed set.

INTRODUCTION:

In 2013, R. Subasree and M. Maria Singam introduced[3] a new set namely b$\hat{g}$-closed set. In 1973, Das[2] defined semi-interior point and semi-limit point of a subset. Further, the semi-derived set of a topological space was defined and studied by him. Following this concept, we define b$\hat{g}$-limit point, b$\hat{g}$-derived set, b$\hat{g}$-border, b$\hat{g}$-frontier and b$\hat{g}$-exterior of a subset of topological spaces using the concept of b$\hat{g}$-closed sets which is defined as and we studied some of its properties.

PRELIMINARIES:

Throughout this paper (X,τ) (or simply X) represents topological space on which no separation axioms are assumed unless otherwise mentioned . For a subset A of (X,τ), Cl(A), Int(A), D(A), b(A) and Ext(A) denote the closure, interior, derived, border and exterior of A respectively. We are giving some basic definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1) *b$\hat{g}$-closed set* if bcl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open in X. The collection of all b$\hat{g}$-closed sets in (X,τ) is denoted by b$\hat{g}$-C(X,τ).
2) *b$\hat{g}$-open set* if X \A is b$\hat{g}$-closed in A . The collection of all b$\hat{g}$-open sets in (X,τ) is denoted by b$\hat{g}$-O(X,τ).

Definition 2.2: Let A be the subset of a space (X,τ). Then

1) The *border of A* is defined as b(A) = A\Int(A).
2) The *frontier of A* is defined as Fr(A) = Cl (A)\Int (A).
3) The *exterior of A* is defined as Ext (A) = Int (X\A).

Theorem 2.3: [2]

1) Every closed set is b$\hat{g}$-closed.
2) Every open set is b$\hat{g}$-open.

3. PROPERTIES OF $b\hat{g}$-INTERIOR AND $b\hat{g}$-CLOSURE:

Definition 3.1: The *$b\hat{g}$-interior of A* is defined as the union of all $b\hat{g}$-open sets of X contained in A. It is denoted by $b\hat{g}$ Int(A).

Definition 3.2: A point $x \in X$ is called *$b\hat{g}$-interior point* of A if A contains a $b\hat{g}$-open sets containing x.

Definition 3.3: The *$b\hat{g}$-closure of A* is defined as the intersection of all $b\hat{g}$-closed sets of X containing A.It is denoted by $b\hat{g}$Cl(A).

Theorem 3.4: *If A is a subset of X, then $b\hat{g}$Int(A) is the set of all $b\hat{g}$-interior points of A.*

Proof: If $x \in b\hat{g}$ Int(A), then x belongs to some $b\hat{g}$-open subset U of A. That is, x is a $b\hat{g}$-interior point of A.

Remark 3.5: If A is any subset of X, $b\hat{g}$ Int(A) is $b\hat{g}$-open. In fact $b\hat{g}$ Int(A) is the largest $b\hat{g}$-open set contained in A.

Remark 3.6: A subset A of X is $b\hat{g}$-open$\Leftrightarrow b\hat{g}$ Int(A)=(A).

Result 3.7: *For the subset A of a topological space (X,τ), Int(A)$\subseteq b\hat{g}$ Int(A).*

Proof: Since Int(A) is the union of open sets and theorem 2.3(2), Int(A) is $b\hat{g}$-open. It is clear from the definition 3.1 that Int(A)$\subseteq b\hat{g}$Int(A).

Theorem 3.8: *Let A and B be the subsets of a topological space (X,τ), then the following result holds:*

1) *$b\hat{g}$ Int($\emptyset$)=$\emptyset$;*
2) *$b\hat{g}$ Int(X)=X;*
3) *$b\hat{g}$ Int(A)$\subseteq$ A;*
4) *A$\subseteq$ B$\Longrightarrow b\hat{g}$ Int(A)$\subseteq b\hat{g}$ Int(B);*
5) *$b\hat{g}$Int(A$\cup$ B)$\supseteq b\hat{g}$ Int(A)$\cup$ $b\hat{g}$Int(B);*
6) *$b\hat{g}$Int(A$\cap$ B)$\subseteq b\hat{g}$Int(A)$\cap$ $b\hat{g}$Int(B);*
7) *$b\hat{g}$ Int(Int(A)) = Int(A);*
8) *Int($b\hat{g}$ Int(A))$\subseteq$ Int(A);*
9) *$b\hat{g}$ Int ($b\hat{g}$ Int(A)) = $b\hat{g}$ Int(A)*

Proof: (1), (2) and (3) follows from definition 3.1.

(4) From definition 3.1 we have, $b\hat{g}$ Int(A)$\subseteq$ A. Since A$\subseteq$ B, $b\hat{g}$ Int(A)$\subseteq$ B. But $b\hat{g}$ Int(B)$\subseteq$ B. By remark 3.5 $b\hat{g}$ Int(A) $\subseteq b\hat{g}$ Int(B).

(5) Since A$\subseteq$A$\cup$ B;B$\subseteq$ A$\cup$ B and by (4) we have, $b\hat{g}$ Int(A)$\subseteq b\hat{g}$ Int(A$\cup$B) and $b\hat{g}$ Int(B)$\subseteq b\hat{g}$ Int(A$\cup$ B). Therefore $b\hat{g}$ Int(A)$\cup$ $b\hat{g}$ Int(B)$\subseteq$ $b\hat{g}$ Int(A$\cup$ B).

(6) Since A$\cap$ B$\subseteq$ A;A$\cap$ B$\subseteq$ B and by (4) we have,$b\hat{g}$ Int(A$\cap$ B)$\subseteq b\hat{g}$ Int(A) and $b\hat{g}$ Int(A$\cap$ B) $\subseteq b\hat{g}$ Int(B). Therefore $b\hat{g}$Int(A$\cap$ B)$\subseteq b\hat{g}$ Int(A)$\cap$ $b\hat{g}$ Int(B).

(7) Since Int(A) is an open set and by theorem 2.3 (2), Int(A) is $b\hat{g}$-open. By remark 3.6, $b\hat{g}$Int(Int(A)) = Int (A).

(8) From definition 3.1 we have, $b\hat{g}$ Int(A)$\subseteq$ A. Clearly, it follows that Int($b\hat{g}$ Int(A))$\subseteq$ Int(A);

(9) Follows from remark 3.6 and 3.5.

Remark 3.9: If A is any subset of X, $b\hat{g}$Cl(A) is closed. In fact $b\hat{g}$Cl(A) is smallest $b\hat{g}$-closed set containing A.

Remark 3.10: A subset A of X is $b\hat{g}$-closed $\Leftrightarrow$ $b\hat{g}$Cl(A) = A.

Theorem 3.11: *Let A and B be the subsets of a topological space (X,τ), then the following result holds:*

1) *$b\hat{g}$ Cl($\emptyset$)=$\emptyset$;*
2) *$b\hat{g}$ Cl(X) = X;*
3) *A$\subseteq b\hat{g}$ Cl(A);*
4) *A$\subseteq$ B $b\hat{g}$ Cl(A)$\subseteq b\hat{g}$ Cl(B);*
5) *$b\hat{g}$ Cl($b\hat{g}$ Cl(A)) = $b\hat{g}$ Cl(A);*
6) *$b\hat{g}$Cl(A$\cup$ B)$\supseteq b\hat{g}$ Cl(A)$\cup$ $b\hat{g}$ Cl(B);*
7) *$b\hat{g}$Cl(A$\cap$ B)$\subseteq b\hat{g}$ Cl(A)$\cap$ $b\hat{g}$ Cl(B);*
8) *$b\hat{g}$ Cl(Cl(A))= Cl(A);*
9) *Cl($b\hat{g}$ Cl(A))= Cl (A);*

Proof: (1), (2) and (3) follows from definition:3.3.

(4) From definition 3.3 we have, A$\subseteq$ $b\hat{g}$ Cl(A).Since A$\subseteq$ B, $b\hat{g}$Cl(A)$\subseteq$ B. Therefore, $b\hat{g}$Cl(A) $\subseteq b\hat{g}$ Cl(B).

(5) Follows from remark 3.9 and 3.10.

(6) Since A$\subseteq$ A$\cup$ B; B $\subseteq$ A$\cup$ B and by (4) we have, $b\hat{g}$ Cl(A) $\subseteq b\hat{g}$ Cl(A$\cup$ B) and $b\hat{g}$ Cl(B)$\subseteq b\hat{g}$ Cl(A$\cup$ B). Therefore, $b\hat{g}$ Cl(A) $\cup$ $b\hat{g}$ Cl(B)$\subseteq b\hat{g}$ Cl(A$\cup$ B).

(7) Since A$\cap$ B$\subseteq$ A; A$\cap$ B$\subseteq$ B and by (4) we have, $b\hat{g}$Cl(A$\cap$ B)$\subseteq b\hat{g}$ Cl(A) and $b\hat{g}$ Cl(A$\cap$ B) $\subseteq b\hat{g}$ Cl(B). Therefore, $b\hat{g}$Cl(A$\cap$ B) $b\hat{g}$ Cl(A)$\cap$ $b\hat{g}$ Cl(B).

(8) Since Cl(A) is a closed set and by theorem 2.3 (1), Cl(A) is b$\hat{g}$-closed. Therefore by remark 3.10, b$\hat{g}$Cl(Cl(A)) = Cl(A).

(9) Follows from remark 3.9 and 3.10.

4. APPLICATION OF b$\hat{g}$-OPEN SETS

Definition 4.1: Let A be a subset of a topological space X. A point $x \in X$ is said be*b$\hat{g}$-limit point of A* if for every b$\hat{g}$-open set U containing , U∩ (A\ $\{x\}$) ≠ ∅ . The set of all b$\hat{g}$-limit points of A is called anb$\hat{g}$-derived set of A and is denoted by b$\hat{g}$ (A).

Example 4.2: Let X ={a,b,c} with property τ ={ X, ∅ ,{a},{a,b}} and b$\hat{g}$O(X) ={ X, ∅, {a},{b},{a,c},{a,b}}. If A={b}, then b$\hat{g}$D(A) = {b}.

Result 4.3: *Let A be a subset of a topological space X. Then ,*

1) *b$\hat{g}$ Cl(X \A) = X \b$\hat{g}$ Int(A)*
2) *b$\hat{g}$ Int (X \A) = X \b$\hat{g}$ Cl(A)*

Proof:

1) Let $x \in X$ \ b$\hat{g}$ Int(A). Then, $x \notin$ b$\hat{g}$ Int(A). This implies that x does not belong to any b$\hat{g}$-open subset of A. Let F be a b$\hat{g}$-closed set containing X \A. Then X \F is b$\hat{g}$-open set contained in A. Therefore, $x \notin X$ \F and so x ∈ F. Hence, x ∈ b$\hat{g}$Cl(X \A). This implies X \ b$\hat{g}$ Int(A) ⊆ b$\hat{g}$Cl(X \A). On the other hand, let x ∈b$\hat{g}$ Cl(X \A). Then x belongs to every b$\hat{g}$-closed set containing X \ A. Hence, x does not belongs to any b$\hat{g}$-open subset of A. That is $x \notin$ b$\hat{g}$ Int(A). This implies $x \in X$ \ b$\hat{g}$ Int(A). Therefore, b$\hat{g}$ Cl(X \ A)⊆ X \b$\hat{g}$ Int(A). Thus, b$\hat{g}$ Cl(X \A) = X \b$\hat{g}$ Int(A).

2) can be proved by replacing A by X \A in 1) and using set theoretic properties.

Theorem 4.4: *For subsets A,B of a space X,the following statement holds:*

1) *D(A) ⊆ b$\hat{g}$ D(A), where D(A) is the derived set of A;*
2) *b$\hat{g}$ D(∅) = ∅;*
3) *If A⊂ B,then b$\hat{g}$ D(A)⊆b$\hat{g}$ D(B);*
4) *b$\hat{g}$ D(A∪ B)⊇ b$\hat{g}$ D(A) ∪ b$\hat{g}$ D(B);*
5) *b$\hat{g}$ D(A∩ B) ⊆ b$\hat{g}$ D(A) ∩ b$\hat{g}$ D(B);*
6) *b$\hat{g}$ D(A)⊆ b$\hat{g}$ D(A\ $\{x\}$)*

Proof: 1) Let x ∈ D(A). By the definition of D(A), there exist an open set U containing x such that U∩ (A\ $\{x\}$) ≠ ∅. By theorem 2.3 (2), U is an b$\hat{g}$-open set containing x such that U ∩ (A\ $\{x\}$) ≠ ∅. Therefore, x ∈b$\hat{g}$(A). Hence, D(A)⊆ b$\hat{g}(A)$.

2) For all b$\hat{g}$-open set U and for all x ∈ X,U ∩ (∅ \ $\{x\}$) = ∅. Hence, b$\hat{g}$ D(∅) = ∅.
3) Let x ∈b$\hat{g}$D(A). Then for each b$\hat{g}$-open set U containing x, U ∩ (A \ $\{x\}$) ≠ ∅. Since A⊆ B, U∩ (B\ $\{x\}$)≠ ∅. This implies that x ∈ b$\hat{g}$(B). Hence b$\hat{g}$D(A) ⊆ b$\hat{g}$D(B).
4) Let x ∈b$\hat{g}$ D(A) ∪ b$\hat{g}$ D(B). Then x ∈ b$\hat{g}$ D(A) or x ∈b$\hat{g}$D(B). If x ∈ b$\hat{g}$D(A), then for each b$\hat{g}$-open set U containing x, U ∩ (A \ $\{x\}$) ≠ ∅ . Since A⊆ A∪ B, U∩ (A ∪ B\ $\{x\}$)≠ ∅. This implies that x ∈ b$\hat{g}$ D(A ∪ B). Hence b$\hat{g}$D(A)⊆ b$\hat{g}$ D(A∪ B).⟶ (1)

   Otherwise, if x ∈b$\hat{g}$ D(B), then for each b$\hat{g}$-open set U containing x,U∩ (B \ $\{x\}$) ≠ ∅. Since B ⊆ A ∪ B, U∩ (A ∪ B \ $\{x\}$) ≠ ∅ . This implies that x ∈b$\hat{g}$D(A∪ B). Hence, b$\hat{g}$D(B)⊆ b$\hat{g}$ D(A∪ B).⟶ (2)

   From (1) and (2), b$\hat{g}$ D(A)∪ b$\hat{g}$ D(B) ⊆ b$\hat{g}$ D(A∪ B).
5) Let x ∈b$\hat{g}$ D(A∩ B). Then for each b$\hat{g}$-open set U containing x, U∩ (A∩ B\ $\{x\}$)≠ ∅. Since A∩ B ⊆ A, U∩ (A\ $\{x\}$)≠ ∅. This implies that x ∈b$\hat{g}$ D(A). Also, since A∩ B⊆ B,U∩ (B \ $\{x\}$) ≠ ∅ . This implies that x ∈ b$\hat{g}$D(B). Therefore, x ∈b$\hat{g}$ D(A)∩ b$\hat{g}$ D(B). Thus b$\hat{g}$ D(A∩ B) ⊆ b$\hat{g}$ D(A) ∩ b$\hat{g}$ D(B).
6) Let x ∈b$\hat{g}$D(A).Then for each b$\hat{g}$-open set U containing x, U ∩ (A \ $\{x\}$) ≠ ∅. This implies that U ∩ ((A \ $\{x\}$) \ $\{x\}$)≠ ∅. This implies that x ∈ b$\hat{g}$ D(A\ $\{x\}$). Hence, b$\hat{g}$ D(A) ⊆ b$\hat{g}$ D(A\ $\{x\}$).

Definition 4.5: If A is subset of X, then the *b$\hat{g}$-border* of A is defined by b$\hat{g}$ b(A) =A\ b$\hat{g}$ Int(A).

Theorem 4.6: *For a subset A of a space X, the following statement holds:*

1) *b$\hat{g}$ b(∅) = ∅;*
2) *b$\hat{g}$ b(X) = X;*

3) *$b\hat{g}\, b(A) \subseteq (A)$;*
4) *$b\hat{g}\, b(A) \subseteq b(A)$; where $b(A)$ denotes the border of A ;*
5) *$b\hat{g}\, Int(A) \cup b\hat{g}\, b(A) = A$;*
6) *$b\hat{g}\, Int(A) \cap b\hat{g}\, b(A) = \emptyset$;*
7) *$b\hat{g}\, b(b\hat{g}\, Int(A)) = \emptyset$;*
8) *$b\hat{g}\, Int(b\hat{g}\, b(A)) = \emptyset$;*
9) *$b\hat{g}\, b(b\hat{g}\, b(A)) = b\hat{g}\, b(A)$.*

Proof: 1),2) and 3) follows from definition 4.5.

4) Let $x \in b\hat{g}\, b(A)$.Then by definition 4.5, $x \in A \setminus b\hat{g}\, Int(A)$. This implies that $x \in A$ and $x \notin b\hat{g}\, Int(A)$. By result 3.7, $x \in A$ and $x \notin Int(A)$.This implies that $x \in A \setminus Int(A)$. This implies that $x \in b(A)$. Hence, $b\hat{g}\, b(A) \subseteq (A)$.

5) and 6) follows from definition 4.5

7) $b\hat{g}\, b(b\hat{g}\, Int(A)) = b\hat{g}\, Int(A) \setminus b\hat{g}\, Int(b\hat{g}\, Int(A)) = b\hat{g}\, Int(A) \setminus b\hat{g}\, Int(A)$ (by theorem 3.8(9)) which is $\emptyset$. Hence, $b\hat{g} b(b\hat{g}\, Int(A)) = .\emptyset$.

8) Let $x \in b\hat{g} Int(b\hat{g}\, b(A))$. By theorem 3.8 (3),$x \in b\hat{g} b(A)$. On the other hand, since $b\hat{g}\, b(A) \subseteq A$, $x \in b\hat{g} Int(A)$. Therefore, $x \in b\hat{g}\, b(A) \cap b\hat{g}\, Int(A)$ which is a contradiction to (6) . Hence, $b\hat{g}\, Int(b\hat{g}\, b(A)) = \emptyset$.

9) $b\hat{g}\, b(b\hat{g}\, b(A)) = b\hat{g}\, b(A) \setminus b\hat{g}\, Int(b\hat{g}\, b(A)) = b\hat{g}\, b(A) \setminus \emptyset = b\hat{g}\, b(A)$ (from(8)). Hence, $b\hat{g}\, (b\hat{g}\, b(A)) = b\hat{g}\, b(A)$.

Definition 4.7: If A is a subset of X, then the *$b\hat{g}$-frontier* of A is defined by $b\hat{g}\, Fr(A) = b\hat{g}\, Cl(A) \setminus b\hat{g}\, Int(A)$.

Theorem 4.8: *Let A be a subset of a space X.Then the following statement holds:*

1) *$b\hat{g}\, Fr(\emptyset) = \emptyset$;*
2) *$b\hat{g}\, Fr(X) = \emptyset$;*
3) *$b\hat{g}\, Fr(A) \subseteq b\hat{g}\, Cl(A)$;*
4) *$b\hat{g}\, FrCl(A) = b\hat{g}\, Int(A) \cup b\hat{g}\, Fr(A)$;*
5) *$b\hat{g}\, Int(A) \cap b\hat{g}\, Fr(A) = \emptyset$;*
6) *$b\hat{g}\, b(A) \subseteq b\hat{g}\, Fr(A)$;*
7) *$b\hat{g}\, Fr(b\hat{g}\, Int(A)) \subseteq b\hat{g}\, Fr(A)$;*
8) *$b\hat{g}\, Cl(b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(A)$;*
9) *$b\hat{g}\, Int(A) \subseteq b\hat{g}\, Cl(A)$*
10) *$b\hat{g}\, Int(b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(A)$;*
11) *$X = b\hat{g}\, Int(A) \cup b\hat{g}\, Int(X \setminus A) \cup b\hat{g}\, Fr(A)$;*
12) *$b\hat{g} Fr(A) = b\hat{g}\, Cl(A) \cap b\hat{g}\, Cl(X \setminus A)$;*
13) *$b\hat{g}\, Fr(A) = b\hat{g}\, Fr(X \setminus A)$.*

Proof: (1),(2),(3) and (4) follows from definition:4.7.

5) $b\hat{g}\, Int(A) \cap b\hat{g}\, Fr(A) = b\hat{g}\, Int(A) \cap (b\hat{g}\, Cl(A) \setminus b\hat{g}\, Int(A) \subseteq A \cap (b\hat{g}\, Cl(A) \setminus A)$ (by theorem 3.8(3)). $b\hat{g}\, Int(A) \cap b\hat{g}\, Fr(A) \subseteq b\hat{g}\, Cl(A) \cap (b\hat{g}\, Cl(A) \setminus b\hat{g}\, Cl(A))$ (by theorem 3.11(3)). $b\hat{g}\, Int(A) \cap b\hat{g}\, Fr(A) = b\hat{g}\, Cl(A) \cap \emptyset = \emptyset$. Hence, $b\hat{g} Int(A) \cap b\hat{g}\, Fr(A) = \emptyset$.
6) Let $x \in b\hat{g} b(A)$. Then $x \in A \setminus b\hat{g}\, Int(A)$. By theorem 3.11 (3), $x \in b\hat{g} Cl(A) \setminus b\hat{g}\, Int(A) = b\hat{g}\, Fr(A)$. Hence, $b\hat{g}\, b(A) \subseteq b\hat{g}\, Fr(A)$.
7) $b\hat{g}\, Fr(b\hat{g}\, Int(A)) = b\hat{g}\, Cl(b\hat{g}\, Int(A)) \setminus b\hat{g}\, Int(b\hat{g}\, Int(A)) \subseteq b\hat{g}\, Cl(A)\ b\hat{g}\, Int(A)$ (by theorem 3.8(3),(9)) which is $b\hat{g}\, Fr(A)$. Hence, $b\hat{g}\, (b\hat{g}\, Int(A)) \subseteq b\hat{g}\, Fr(A)$.
8) From(3) we have, $b\hat{g}\, Cl(b\hat{g} Fr(A)) \subseteq b\hat{g}\, Cl(b\hat{g}\, Cl(A)) = b\hat{g}\, Cl(A)$ (by theorem 3.11 (5)). Hence, $b\hat{g}\, (b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(A)$.
9) follows from (4).
10) From (9), $b\hat{g} Int(b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(A)$ (from (8)). Hence, $b\hat{g}\, Int(b\hat{g}\, Fr(A)) \subseteq b\hat{g}\, Cl(A)$.
11) $b\hat{g} Int(A) \cup b\hat{g}\, Int(X \setminus A) \cup b\hat{g} Fr(A) = b\hat{g}\, Cl(A) \cup b\hat{g}\, Int(X \setminus A)$ (from (4)) $= b\hat{g}\, Cl(A) \cup \{X \setminus b\hat{g} Cl(A)\}$ (by result 4.3(2)) which is X. Hence, $X = b\hat{g}\, Int(A) \cup b\hat{g}\, Int(X \setminus A) \cup b\hat{g}\, Fr(A)$.
12) $b\hat{g} Cl(A) \cap b\hat{g} Cl(X \setminus A) = b\hat{g}\, Cl(A) \cap (X \setminus b\hat{g}\, Int(A))$ (by result 4.3 (1)) $= b\hat{g}\, Cl(A) \setminus b\hat{g}\, Int(A)$ (from (9)) $= b\hat{g}\, Fr(A)$.
13) $b\hat{g}\, Fr(X \setminus A) = b\hat{g}\, Cl(X \setminus A) \setminus b\hat{g}\, Int(X \setminus A)$ (by result 4.3). $b\hat{g}\, Fr(X \setminus A) = (b\hat{g}\, Cl(A) \setminus b\hat{g}\, Int(A) = b\hat{g}\, Fr(A)$.

Definition 4.9: If A be a subset of a space X. Then the *$b\hat{g}$-Exterior of A* is defined by $b\hat{g}\, Ext(A) = b\hat{g}\, Int(X \setminus A)$.

Theorem 4.10: *Let A be a subset of a space X. Then the following statement holds:*

1) *$b\hat{g}\, Ext(\emptyset) = X$;*
2) *$b\hat{g}\, Ext(X) = \emptyset$;*
3) *$Ext(A) \subseteq b\hat{g}\, Ext(A)$;*
4) *$b\hat{g}\, Ext(A) = X \setminus b\hat{g}\, Cl(A)$;*
5) *A is closed iff $b\hat{g}\, Ext(A) = X \setminus A$;*
6) *If $A \subseteq B$, then $b\hat{g}\, Ext(A) \supseteq b\hat{g}\, Ext(B)$;*

7) *$b\hat{g}$ Ext (A∪ B) ⊆ $b\hat{g}$ Ext(A)∩ $b\hat{g}$ Ext(B);*
8) *$b\hat{g}$ Ext (A∩B) ⊇ $b\hat{g}$ Ext (A)∪ $b\hat{g}$ Ext (B)*
9) *$b\hat{g}$ Ext(A) is $b\hat{g}$-open;*
10) *$b\hat{g}$ Ext(X \ $b\hat{g}$ Ext(A)) = $b\hat{g}$ Ext(A);*
11) *$b\hat{g}$ Ext($b\hat{g}$ Ext(A)) = $b\hat{g}$ Int($b\hat{g}$ Cl(A));*
12) *$b\hat{g}$ Int(A)⊆ $b\hat{g}$ Ext($b\hat{g}$ Ext(A));*
13) *X= $b\hat{g}$Int(A)∪$b\hat{g}$Ext(A)∪$b\hat{g}$ Fr(A).*

Proof :

1) $b\hat{g}$Ext(∅) = $b\hat{g}$ Int(X \A) = $b\hat{g}$ Int(X) = X(by theorem 3.8 (2)).
2) $b\hat{g}$ Ext(X) = $b\hat{g}$ Int(X \ X) = $b\hat{g}$ Int(∅) =∅ (by theorem 3.8(1))
3) Let x ∈ Ext(A). Then by definition 2.2 (3), x ∈ Int(X\ A). By theorem 3.7, x ∈$b\hat{g}$ Int(\A) = $b\hat{g}$ Ext(A). Hence, Ext(A) ⊆ $b\hat{g}$ Ext(A).
4) Let x ∈ $b\hat{g}$ Ext(A) ⟺ x ∈ $b\hat{g}$ Int(X \A) ⟺ x ∈ X \ $b\hat{g}$ Cl(A) (by result4.3 (2)). Hence , $b\hat{g}$ Ext(A) = X \ $b\hat{g}$ Cl(A).
5) Let A be $b\hat{g}$-closed. Then X \A is $b\hat{g}$-open. By remark 3.6, $b\hat{g}$ Int(X \ A) = X \A. This implies that $b\hat{g}$ Ext(A) = X \A. Conversely, let $b\hat{g}$ Ext(A) = X \A. Then $b\hat{g}$ Int(X \A) = X \A. Again by remark 3.6, X \A is $b\hat{g}$-open. Hence, A is $b\hat{g}$-closed.
6) $b\hat{g}$ Ext(A) = $b\hat{g}$ Int(X \A) =X \$b\hat{g}$ Cl(A) (by result 4.3). X \$b\hat{g}$ Cl(B) (since A⊆B and by theorem 3.11(4)). $b\hat{g}$ Int(X \B) = $b\hat{g}$ Ext(B) (by definition 4.9). Hence, $b\hat{g}$ Ext(A) ⊇ $b\hat{g}$ Ext(B).
7) Since A ⊆ A ∪ B and by (6), $b\hat{g}$ Ext(A∪B)⊆$b\hat{g}$ (A). Similarly since B ⊆ A∪B and by (6), $b\hat{g}$ Ext(A∪B)⊆$b\hat{g}$(B). Hence, $b\hat{g}$ Ext(A ∪ B) ⊆ $b\hat{g}$ Ext (A) ∩ $b\hat{g}$Ext(B).
8) Since A ∩ B ⊆ A and by (6), $b\hat{g}$ Ext(A) ⊆ $b\hat{g}$ (A ∩ B). Similarly since A ∩ B ⊆ B and by (6), $b\hat{g}$ Ext(B) ⊆ $b\hat{g}$ (A ∩ B). Hence, $b\hat{g}$ Ext(A)∪$b\hat{g}$Ext(B)⊆$b\hat{g}$ Ext(A∩B).
9) Follows from definition 4.9 and theorem 3.8(2).
10) $b\hat{g}$ Ext(X \$b\hat{g}$ Ext(A)) = $b\hat{g}$ Ext(X \$b\hat{g}$ Int(X \ A)) = $b\hat{g}$ Int(X \ { X \ $b\hat{g}$ Int(X \A)}) = $b\hat{g}$ Int($b\hat{g}$ Int(X \A)) = $b\hat{g}$ Int(X \A) (by theorem 3.8 (9)) which is $b\hat{g}$ Ext(A). Hence, $b\hat{g}$ (X \ $b\hat{g}$ Ext(A)) = $b\hat{g}$ Ext(A).
11) $b\hat{g}$ Ext($b\hat{g}$ Ext(A)) = $b\hat{g}$ Int(X \ $b\hat{g}$ Ext(A)) = $b\hat{g}$ Int(X \$b\hat{g}$ Int X \A)) = $b\hat{g}$ Int ($b\hat{g}$ Cl(X \(X \A)) = $b\hat{g}$ Int($b\hat{g}$ Cl(A)) (by result 4.3 (1)). Hence, $b\hat{g}$ Ext($b\hat{g}$ Ext(A)) = $b\hat{g}$ Int($b\hat{g}$ Cl(A)).
12) Since A ⊆ $b\hat{g}$ Cl(A), $b\hat{g}$ Int(A) ⊆ $b\hat{g}$ Int($b\hat{g}$ Cl(A)) = $b\hat{g}$ Ext($b\hat{g}$ Ext(A)) (from(11)).
13) $b\hat{g}$ Int(A)∪ $b\hat{g}$ Ext(A)∪ $b\hat{g}$ Fr(A) = $b\hat{g}$ Int(A) ∪ $b\hat{g}$ Int(X / A) ∪ $b\hat{g}$ Fr(A) = X (from theorem 4.8 (12)).

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