On Generalized Semi $b^{\#}$-Closed sets in Topological spaces

[1]K. Bala Deepa Arasi, [2]G. Gomathi, [3]S.Jackson

[1]Assistant Professor of Mathematics,
[2]M.Phil Scholar,
[1,2]PG & Research Department of Mathematics,
A.P.C.Mahalaxmi College for Women, Thoothukudi- 628002, Tamil Nadu.
baladeepa85@gmail.com, gomathiganesan96@gmail.com
[3]Assistant Professor of Mathematics,
PG & Research Department of Mathematics
V.O.Chidambaram College, Thoothukudi, Tamil Nadu.
jackvocmaths@gmail.com

Abstract: A subset A of a topological space X is said to $b^{\#}$-closed if A=cl(int(A)) ∩ int(cl(A)). The $b^{\#}$-closure of A is the intersection of all $b^{\#}$-closed sets containing A and is denoted by $b^{\#}$-cl(A). The main objective of this paper, is to be introduced a new class of closed sets namely generalized semi $b^{\#}$-closed set (briefly gsb$^{\#}$-closed) in topological spaces by using $b^{\#}$-closure and semi open. Also, we characterize their properties and we study the relationship between the gsb$^{\#}$-closed set with the existing class of stronger and weaker form of closed sets.

Keywords: $b^{\#}$-closed set, $b^{\#}$-closure, gb$^{\#}$-closed set, gsb$^{\#}$-closed set, gsb$^{\#}$-open set, Semi open.

1.INTRODUCTION:

Topology is the Mathematical study of shapes and spaces. A major area of Mathematics concerns with the most basic properties of space such as connectedness, continuity and boundary. In topology properties that are preserved under continuous deformations including stretching and bending, but not tearing or gluing are studied. Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension, and transformation. By the middle of the 20th century, topology had become an important area of study within Mathematics. Topology has many subfields like point-set topology, algebraic topology and geometric topology. The sets in the topology τ for a set X are defined as open. A set is defined as closed if its complement with respect to X are defined as open. There are other equivalent ways to define a topology for a set besides open and closed sets.

The generalized closed sets, semi closed, semi-generalized closed sets respectively introduced and studied by Levine (1970), Biswas (1970), Bhattacharyya & Lahiri (1987). The following sets called semi-open, α-open and pre-open are introduced by to Levine (1963), Njastad (1965), Mashhour et al (1982), respectively. On the other, N.Levine[7] generalized the concepts of closed sets to generalized closed sets. After the works of N.Levine[8], P.Bhattacharya and P.K.Lahiri[6] introduced the concept of semi-generalized closed set. S.P.Arya and T.Nours[3] defined the generalized semi-open sets and studied some of their properties. Recently, Parameswari et al.[10] introduced notions of $b^{\#}$-open sets and $b^{\#}$-closed sets by taking equality in the definitions of b-open sets and b-closed sets.

Vithya et al.[11] introduced the notion of generalized $b^{\#}$-closed sets. K.Bala Deepa Arasi et al.[4] defined the $sb\hat{g}$-closed sets and $b^*\hat{g}$ –closed sets.

In this paper, is to be introduced a new class of closed sets namely generalized semi $b^{\#}$-closed set (briefly $gsb^{\#}$-closed) in topological spaces by using $b^{\#}$-closure and semi open. Also, we characterize their properties and we study the relationship between the $gsb^{\#}$-closed set with the existing class of stronger and weaker form of closed sets.

2.PRELIMINARIES:

Definition 2.1: A subset A of a topological space (X,τ) is called

i) semi open[8] if A⊆ cl(int(A)).
ii) pre-semi open (α open set)[9] if A ⊆ int(cl(int(A))).
iii) $b^{\#}$-closed[10] if A= cl(int(A)) ∩ int(cl(A)).
iv) b-open[1] if A⊆ cl(int(A)) ∪int(cl(A)) and b-closed[1] if cl(int(A))∩ int(cl(A))⊆ A.

Definition 2.2: A subset A of a topological space (X,τ) is called

i) generalized closed (briefly g-closed)[7] if cl(A)⊆U whenever A⊆ U and U is open.
ii) g*-closed[13] if cl(A)⊆U whenever A⊆ U and U is g-open.
iii) generalized $b^{\#}$-closed (briefly $gb^{\#}$-closed)[12] if $b^{\#}cl(A)$⊆ U whenever A⊆ U and U is open.
iv) generalized semi closed (briefly gs-closed) [3] if scl(A)⊆U whenever A⊆ U and U is open.
v) semi generalized closed (briefly sg-closed)[6] if scl(A)⊆U whenever A⊆ U and U is semi open.
vi) generalized b-closed(briefly gb-closed)[14] if bcl(A)⊆ U whenever A⊆U and U is open.

The collection of all semi-closed, α-closed, $b^{\#}$-closed, g-closed,g*-closed, $gb^{\#}$-closed, gs-closed, sg-closed, gb-closed sets in topological space (X,τ) is denoted by SC(X), αC(X), $b^{\#}$-C(X), g-C(X), g*-C(X), $gb^{\#}$-C(X), gs-C(X), sg-C(X), gb- C(X) respectively.

Theorem 2.3: [10] Let A be a subset of a topological space X. Then cl(int(A))∩ int(cl(A)) ⊆ bcl(A)⊆ $b^{\#}cl(A)$

Lemma 2.4: [2] Let A be a subset of a space X. Then bcl(A) = scl(A) ∪ pcl(A)

3.MAIN RESULTS

Definition 3.1: Let X be a topological space. A subset A of a topological space X is called **generalized semi $b^{\#}$-closed (briefly $gsb^{\#}$-closed)** if $b^{\#}cl(A)$ ⊆ U whenever A⊆ U and U is semi open. The collection of all $gsb^{\#}$-closed sets is denoted by $gsb^{\#}$-C(X).

Example 3.2:

i) Let X={a, b, c} with τ = {X, ∅, {a}}. Then $gsb^{\#}$-C(X)= {X, ∅, {b, c}}
ii) Let X={a, b, c, d} with τ = {X, ∅, {a}, {d}, {a,d}}. Then $gsb^{\#}$-C(X) = {X, ∅, {a},{d}}

Definition 3.3: The complement of the $gsb^{\#}$-closed set is called $gsb^{\#}$-open set. The collection of all $gsb^{\#}$-open sets in topological space (X,τ) is denoted by $gsb^{\#}$-O(X).

Example 3.4: Let X={a, b, c} with τ = {X, ∅, {a}}. Then $gsb^{\#}$-O(X) = {X, ∅, {a}}

Theorem 3.5: Every $gsb^{\#}$-closed set is $gb^{\#}$-closed.

Proof: Let A be any $gsb^{\#}$-closed set in X and U be any open set in X such that A⊆U.

Since every open set is semi-open and A is $gsb^{\#}$-closed, $b^{\#}cl(A) \subseteq U$. Therefore, A is $gb^{\#}$-closed.

Remark 3.6: The converse of the above theorem is not true as shown in the following example.

Example: Let X={a, b, c}, τ ={X, ∅, {a}, {b}, {a,b}}, $gb^{\#}C(X)$ ={X,∅, {a}, {b}, {c},{b,c},{a,c}}, $gsb^{\#}$-C(X)= {X, ∅, {a},{b}}. Here {c},{b,c},{a,c} are $gb^{\#}$-closed sets but not $gsb^{\#}$-closed.

Theorem 3.7: Every $gb^{\#}$- closed set is $gsb^{\#}$-closed if open and semi-open sets are equal.

Proof: Let A be any $gb^{\#}$-closed set in X and U be any semi open set in X such that $A \subseteq U$. Since A is $gb^{\#}$-closed and by hypothesis, $b^{\#}cl(A) \subseteq U$. Therefore, A is a $gsb^{\#}$-closed.

Theorem 3.8: Every $b^{\#}$-closed set is $gsb^{\#}$-closed.

Proof: Let A be any $b^{\#}$-closed set in X and U be any semi open set in X such that $A \subseteq U$. Since A is $b^{\#}$-closed and by theorem 2.3, $A = cl(int(A)) \cap int(cl(A)) \subseteq bcl(A) \subseteq b^{\#}cl(A) \subseteq U$. Therefore, A is $gsb^{\#}$-closed.

Remark 3.9: The converse of the above theorem is not true as shown in the following example.

Example: Let X={a, b, c}, τ ={X, ∅, {b}, {a,c}}. $b^{\#}$-C(X)={X,∅,{b},{a,c}},$gsb^{\#}$-C(X) = {X, ∅, {a},{b},{c}, {a,b},{a,c},{b,c}}. Here {a},{c},{a,b},{b,c} are $gsb^{\#}$-closed sets but not $b^{\#}$-closed set.

Remark 3.10: $gsb^{\#}$-closedness is independent from α-closedness.

Example: Let X={a, b, c}, τ ={X, ∅,{a}, {b}, {a,b}},$gsb^{\#}$-C(X)= {X, ∅, {a},{b}}, α-C(X)= {X, ∅, {c},{b,c}, {a,c}}. Here {a},{b} are $gsb^{\#}$-closed sets but not α-closed sets. Also {c},{b,c},{a,c} are α-closed sets but not $gsb^{\#}$-closed sets. So they are independent.

Theorem 3.11: Every $gsb^{\#}$-closed set is gs-closed.

Proof: Let A be any $gsb^{\#}$-closed set in X and U be any open set in X such that $A \subseteq U$. Since every open set is semi open and A is $gsb^{\#}$-closed and by 2.3&2.4, $scl(A) \subseteq b^{\#}cl(A) \subseteq U$. Therefore, A is gs-closed.

Remark 3.12: The converse of the above theorem is not true as shown in the following example.

Example: Let X={a, b, c}, τ ={X, ∅, {a},{b},{a,b}},gs-C(X)={X, ∅,{a},{b}, {c}, {a,c},{b,c}}, $gsb^{\#}$-C(X)= {X, ∅, {a},{b}}. Here {c},{a,c},{b,c} are gs-closed sets but not $gsb^{\#}$-closed set.

Theorem 3.13: Every $gsb^{\#}$-closed set is sg-closed.

Proof: Let A be any $gsb^{\#}$-closed set in X and U be any semi open set in X such that $A \subseteq U$. Since A is $gsb^{\#}$-closed and by 2.3&2.4, $scl(A) \subseteq b^{\#}cl(A) \subseteq U$ where U is semi open. Therefore, A is sg-closed.

Remark 3.14: The converse of the above theorem is not true as shown in the following example.

Example: Let X={a, b, c}, τ ={X, ∅, {a},{b}, {a,b}} sg-C(X)={X, ∅,{a},{b}, {c},{a,c},{b,c}}, $gsb^{\#}$-C(X)= {X, ∅, {a},{b}}. Here {c},{a,c},{b,c} are sg-closed sets but not $gsb^{\#}$-closed sets.

Remark 3.15: $gsb^{\#}$-closedness is independent from g-closedness.

Example: Let X={a, b, c}, τ ={X, ∅,{a}, {b}, {a,b}}. $gsb^{\#}$-C(X)= {X, ∅, {a},{b}}, g-C(X)= {X, ∅, {c},{b,c}, {a,c}}. Here {a},{b} are $gsb^{\#}$-closed sets but not g-closed sets and also {c},{b,c},{a,c} are g-closed sets but not $gsb^{\#}$-closed sets. So they are independent.

Remark 3.16: $gsb^{\#}$-closedness is independent from g*-closedness.

Example: Let X={a, b, c}, τ ={X, ∅,{a},{b},{a,b}},gsb$^{\#}$-C(X)={X,∅,{a},{b}}, g*-C(X)={X,∅,{c},{b,c},{a,c}}. Here {a},{b} are gsb$^{\#}$-closed sets but not g*closed sets and also {c},{b,c},{a,c} are g*-closed sets but not gsb$^{\#}$-closed sets. So they are independent.

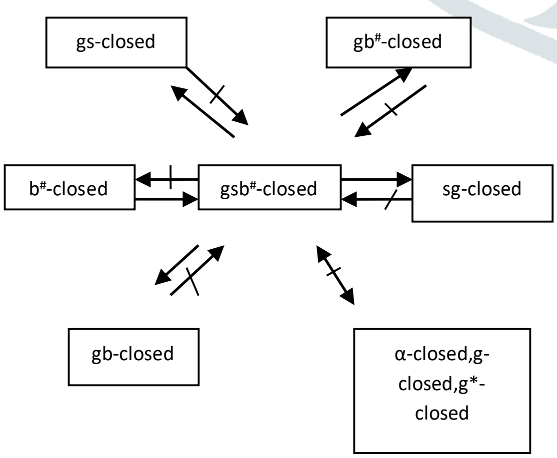
Theorem 3.17: Every gsb$^{\#}$-closed set is gb-closed.

Proof: Let A b any gsb$^{\#}$-closed set in X and U be any open set in X such that A⊆U. Since A is gsb$^{\#}$-closed and every open is semi-open and by theorem 2.3, bcl(A)⊆b$^{\#}$cl(A)⊆U. Therefore, A is gb-closed.

Remark 3.18: The converse of the above theorem is not true as shown in the following example.

Example: Let X={a,b,c}, τ={X,∅,{a}}, gbC(X)={X,∅,{b},{c},{a,b},{a,c},{b,c}}, gsb$^{\#}$-C(X)={X,∅,{b,c}}. Here {b},{c}, {a,b}, {a,c} are gb-closed sets but not gsb$^{\#}$-closed sets.

Remark 3.19: The following diagram demonstrate the relationship of gsb$^{\#}$-closed set with other existing closed sets.



4. CHARACTERIZATION OF gsb$^{\#}$-CLOSED SETS:

Result 4.1: Union of any two gsb-closed sets need not be gsb-closed set as shown as in the following example.

Example: Let X={a,b,c} with τ ={X,∅,{a},{b},{a,b}} and gsb$^{\#}$-C(X)={X,∅, {a},{b}}Then the set A={a}, B={b}, A∪B={a,b} is not gsb$^{\#}$-closed set in X.

Result 4.2: Intersection of any two gsb$^{\#}$-closed sets need not be an gsb$^{\#}$-closed set as shown in the following example.

Example: Let X={a,b,c,d} with τ ={X,∅,{b},{c,d},{b,c,d}} and gsb$^{\#}$-C(X) ={X,∅,{b},{c},{d},{c,d}, {a,b,c}, {a,b,d}}. Then the set A={a,b,c}, B={a,b,d}, A∩B={a,b} is not gsb$^{\#}$-closed set in X.

Theorem 4.3: If A is gsb$^{\#}$-closed set of X, then b$^{\#}$cl(A)\ A does not contain a non-empty semi-closed set.

Proof: Suppose A is an gsb$^{\#}$-closed set. Let F be a semi-closed set contained in b$^{\#}$cl(A)\ A. Then F^c is a semi-open set of X and A⊆F^c. Since A is gsb$^{\#}$-closed, b$^{\#}$cl(A)⊆ F^c. Thus,F⊆[b$^{\#}$cl(A)]^c. Also, F⊆ b$^{\#}$cl(A)\ A. So, F⊆ b$^{\#}$cl(A)∩[b$^{\#}$cl(A)]^c=∅. Therefore, F must be ∅. Hence b$^{\#}$cl(A)\A does not contain a non-empty semi-closed set.

Theorem 4.4: If A is semi-open and gsb$^{\#}$-closed set of X, then A is b$^{\#}$-closed.

Proof: Since A is semi-open and gsb$^{\#}$-closed, b$^{\#}$cl(A)⊆A. Hence A is b$^{\#}$-closed.

Theorem 4.5: The intersection of a gsb$^{\#}$-closed set and a b$^{\#}$-closed set of X is always gsb$^{\#}$-closed set.

Proof: Let A be an gsb$^{\#}$-closed set and B be a b$^{\#}$-closed set. Let A⊆U and U be semi-open. Since A is gsb$^{\#}$-closed, b$^{\#}$cl(A)⊆U whenever U is semi-open. Let B be such that A∩B ⊆ U where U is semi-open. Now, b$^{\#}$cl(A∩B) ⊆ b$^{\#}$cl(A)∩ b$^{\#}$cl(B) ⊆ U∩B ⊆ U. Therefore, A∩B is an gsb$^{\#}$-clsoed set. Hence, intersection of any gsb$^{\#}$-closed set and a b$^{\#}$-closed set of X is always gsb$^{\#}$-closed set.

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