Anti Fuzzy Bi-Ideals in Boolean Like Semi-Rings

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Abstract

Boolean like semi-rings were introduced by K.Venkatesawarlu, B.V.N.Murthy and N.Amarnath. A Boolean like semi-ring is a commutative ring with unity and is of characteristic 2. The concept of a fuzzy subset of a non-empty set was introduced by zadeh. In this paper, we introduce the notion of anti-fuzzy bi-ideals in Boolean like semi-ring R. Let R be a Boolean like semi-ring and μ be the fuzzy set of R. Then μ is said to be a anti fuzzy bi-ideal of R if

1) $\mu(x-y) \leq \max\{ \mu(x),\mu(y) \}$, $x,y \in R$
2) $\mu(xyz) \leq \max\{ \mu(x),\mu(z) \}$, $x,y,z \in R$ and also obtain some of their properties.

Keywords

Boolean like semi-ring, fuzzy set, fuzzy bi-ideal, Anti-fuzzy bi-ideal.

1.Introduction

Boolean like semi-rings were introduced in role by K.Venkatesawarlu, B.V.N.Murthy and N.Amarnath[7] during 2011. Boolean like rings of A.L.Foster arise naturally from general ring dulity considerations and preserve many of the formal properties of Boolean ring. A Boolean like ring is a commutative ring with unity & is of characteristic 2. The concept of a fuzzy subset of a non-empty set was introduced by zadeh[8]. Fuzzy ideals of rings were introduced by Ziu, and it has been studied by several authors.Fuzzy bi-ideals in Boolean like semi-rings was introduced by N.Meenakumari and R.Rajeswari[6]. In this paper, we introduce the concept of anti fuzzy bi-ideals in Boolean like semi-rings and study the some properties of anti fuzzy bi-ideals.

2.Preliminaries:

Definition 2.1

A non-empty set R with two binary operations '+' and '·' is called a **near-ring** if

i) (R,+) is a group (not necessarily abelian)
ii) (R, ·) is a semigroup
iii) $x.(y+z) = x.y+x.z$ for all $x,y,z \in R$

Definition 2.2

A subgroup B of (N,+) is said to be a bi-ideal of N if $BNB \cap (BN)* B \subseteq B$.

Definition 2.3

A system (R,+,·) a **Boolean semi-ring** iff the following properties hold

i) (R,+) is a additive (abelian) group(whose 'zero' will be denoted by '0')
ii) (R, ·) is a semigroup of idempotent in the sense aa=a for all $a \in R$.
iii) a(b+c)=ab+ac and

iv) abc=bac, for all a,b,c∈R.

Example 2.4

Let (G,+) be any abelian group define ab=b for all a,b∈G. Then (G,+,·) is a Boolean semi-ring.

Definition 2.5

A nonempty set R together with two binary operations + and · satisfying the following conditions is called a **Boolean like semi- ring.**

i) (R,+) is an ableian group.

ii) (R, ·) is a semi group.

iii) a.(b+c) = a.b+a.c for all a,b,c∈R

iv) a+a = 0 for all a in R.

v) ab(a+b+ab) = ab for all a,b∈R

Definition 2.6

A nonempty I of R is said to be an **ideal** if

i) (I,+) is a subgroup of (R,+), (ie)., for a,b∈R ⇒a+b∈R.

ii) ra∈R for all a∈I, r∈R (ie)., RI⊆I.

iii) (r+a)s+rs∈I for all r,s∈R, a∈I.

Definition 2.7

Let μ be a fuzzy set defined on R. Then μ is said to be **a fuzzy ideal** of R if

i) $\mu(x-y) \geq \min\{ \mu(x), \mu(y)\}$, x,y∈R.

ii) $\mu(ra) \geq \mu(a)$ for all r,a∈R.

iii) $\mu((r+a)s+rs) \geq \mu(a)$ for all r,a,s∈R.

Definition2.8

A fuzzy set μ in a Boolean like semi-ring R is called an anti fuzzy left ideal of M, if

i) $\mu(x-y) \leq \max\{ \mu(x), \mu(y)\}$, x,y∈R.

ii) $\mu(ra) \leq \mu(a)$ for all r,a∈R.

iii) $\mu((r+a)s+rs) \leq \mu(a)$, for all r,a,s∈R.

Definition2.9

If ϑ is a fuzzy set in f(M), then the fuzzy set μ = ϑ ∘ f in M (ie)., the fuzzy set defined by μ(x) = ϑ (f(x)) for all x in M is called the pre-image of ϑ under f.

Definition 2.10

Let μ be a fuzzy set defined on R. Then μ is said to be a fuzzy bi-ideal of R if

1) $\mu(x-y) \geq \min\{ \mu(x),\mu(y) \}$, x,y ϵ R

2) $\mu(xyz) \geq \min\{ \mu(x),\mu(z) \}$, x,y,z ϵ R

3. Anti Fuzzy Bi-ideals in Boolean like semi-rings:

In this section we define an anti fuzzy bi-ideal in Boolean like semi-rings and some theorems are proved.

Definition 3.1

Let μ be a fuzzy set defined on R**.** Then μ said to be **anti fuzzy bi-idea**l of R, if

1) $\mu(x-y) \leq \max\{ \mu(x),\mu(y) \}$, x,y ϵ R

2) $\mu(xyz) \leq \max\{ \mu(x),\mu(z) \}$, x,y,z ϵ R

Example :3.2

Consider a Boolean like semi – ring R, Let μ be an Anti-fuzzy bi-ideal defined on R by μ(0) = 0.4 μ(a) = 0.5 μ(b) = 0.6 μ(c) = 0.7 for every x ϵ M.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	a	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	b	c

Then μ is an anti-fuzzy bi-ideal of M.

Theorem 3.3

Let R be a Boolean like semi-ring and μ be an anti-fuzzy bi-ideal of R. Then the set R_μ= { xϵR / μ(x) = μ(0) } is a bi-ideal of R.

Proof :

Let μ be an anti-fuzzy bi-ideal.

i)Let x,y ϵ R_μ implies μ(x) = μ(0) and μ(y) = μ(0).

Then μ(x-y) ≤ max { μ(x),μ(y) }

μ(x-y) ≤ max { μ(0),μ(0) }

= max { 0,0 }=0

μ(x-y) = μ(0).

Hence, x-y ϵ R_μ.

ii)Now,Let x,y,z ϵ R_μ implies μ(x) = μ(y) = μ(z) = μ(0).

Then,μ(xyz) ≤ max { μ(x),μ(z) }

μ(xyz) ≤ max { μ(0),μ(0) }

μ(xyz) = μ(0).

Hence, xyzϵ R_μ.

Therefore, R_μ is a bi-ideal of R.

Theorem 3.4

If { μ_i / i ϵ **Λ** } is a family of anti fuzzy bi-ideals of Boolean like semi-ring R then so is $\cup_{i\epsilon\Lambda} \mu_i$

Proof

Let { μ_i / i ϵ **Λ** } be a family of anti fuzzy bi-ideals of R and let x,y ϵ R.

Then,

($\cup_{i\epsilon\Lambda} \mu_i$)(x – y)

= sup{μ_i(x – y) / iϵ **Λ**}

≤ sup { max { μ_i(x), μ_i(y) / i ϵ **Λ** }}

= max{sup{ μ_i(x) / i ϵ **Λ** }, sup{ μ_i(y) / i ϵ **Λ** }}

= max { ($\cup_{i\epsilon\Lambda} \mu_i$) (x) , ($\cup_{i\epsilon\Lambda} \mu_i$) (y) }

And let x,y,zϵ R. Then,

($\cup_{i\epsilon\Lambda} \mu_i$) (xyz) = sup { μ_i (xyz) / i ϵ **Λ** }

≤ sup { max { μ_i(x), μ_i(z) / i ϵ **Λ** }

= max{sup{ μ_i(x) / i ϵ **Λ** }, sup{ μ_i(z) / i ϵ Λ }}

= max { ($\cup_{i\epsilon\Lambda} \mu_i$) (x) , ($\cup_{i\epsilon\Lambda} \mu_i$) (z) }

Theorem 3.5

Intersection of a non-empty collection of anti fuzzy bi-ideals of a Boolean like semi-ring R is an anti fuzzy bi-ideal of R.

Proof

Let R be a Boolean like semi-ring. Let{ μ_i / i ϵ I } be the family of anti fuzzy bi-ideal of R and let x,y ϵ R.

Then, we have

i)($\cap_{i\in I} \mu_i$) (x- y) = $\inf_{i\,\epsilon\, I}$ { μ_i(x- y)}

≤ $\inf_{i\,\epsilon\, I}$ { max { μ_i(x) , μ_i(y) } }

$= \max\{ \inf_{i \in I} \mu_i(x) , \inf_{i \in I} \mu_i(y) \}$

$= \max [(\cap_{i \in I} \mu_i)(x) ,(\cap_{i \in I} \mu_i)(y)]$

Now let x,y,z ϵ R.

Then we have,

ii) $(\cap_{i \in I} \mu_i)(xyz) = \inf_{i \in I}\{ \mu_i(xyz) \}$

$\leq \inf_{i \in I}\{ \max\{ \mu_i(x),\mu_i(z) \} \}$

$= \max \{ \inf_{i \in I}\mu_i(x) , \inf_{i \in I}\mu_i(z) \}$

$= \max [(\cap_{i \in I} \mu_i)(x),(\cap_{i \in I} \mu_i) (z)]$

Theorem 3.6

Let R be a Boolean like semi-ring. Then a fuzzy set μ is an anti fuzzy bi-ideal of R iff μ^c is a fuzzy bi-ideal of R.

Proof

Let x,y∈ R and μ be an anti fuzzy bi-ideal of R then we have,

i) $\mu^c (x – y) = 1- \mu(x – y)$

$\geq 1- \max \{ \mu(x) , \mu(y) \}$

$= \max \{1- \mu(x) , 1- \mu(y)\}$

$= \max\{ \mu^c (x) , \mu^c (y) \}$

Now let x,y,zϵ R. Then,

ii) $\mu^c (xyz) = 1- \mu(xyz)$

$\geq 1- \max \{ \mu(x) , \mu(z) \}$

$= \max \{1- \mu(x) , 1- \mu(z)\}$

$= \max\{ \mu^c (x) , \mu^c (z) \}$

Hence μ^c is a fuzzy bi-ideal of R. similarly the converse follows.

Theorem 3.7 :

A Boolean like semi-ring homomorphic pre-image of an anti fuzzy bi-ideal is an anti fuzzy bi-ideal.

Proof

Let R & S be Boolean like semi-rings. Let f : R → S be a Boolean like semi-ring homomorphism ϑ be an anti fuzzy bi-ideal of S and μ be the pre image of ϑ under f. Let x,y,z ϵ R. Then,

i) $\mu(x – y) = \vartheta(f(x-y))$

$= \vartheta (f(x) – f(y))$

$\leq \max \{\vartheta(f(x)), \vartheta(f(y))\}$

$= \max \{ \mu(x) , \mu(y) \}$

ii) $\mu(xyz) = \vartheta(f(xzy))$

$= \vartheta (f(x) ,f(y),f(z))$

$\leq \max \{\vartheta(f(x)) , \vartheta(f(z)\}$

$= \max \{ \mu(x) , \mu(z) \}$

Hence μ is an anti fuzzy bi-ideal of R.

Theorem 3.8

Let μ be an anti fuzzy bi-ideal of a Boolean like semi-ring R and μ^+ be a fuzzy set in R given by $\mu^+(x) = \mu(x) + 1 - \mu(1)$ for all x ϵ R. Then μ^+ is an anti fuzzy bi-ideal of R.

Proof

Let μ be an anti fuzzy bi-ideal of a Boolean like semi-ring R for all x,y,zϵ R. Then,

i) $\mu^+(x – y)= \mu(x – y) + 1 - \mu(1)$

$\leq \max \{\mu(x) , \mu(y) \} + 1 - \mu(1)\}$

$= \max\{\mu(x)+1-\mu(1),\mu(y)+1-\mu(1)\}$

$= \max \{ \mu^+(x) , \mu^+(y) \}$

ii) $\mu^+(xyz) = \mu(xyz) + 1 - \mu(1)$

$\leq \max \{ \mu(x) , \mu(z) \} + 1 - \mu(1)$

$= \max\{\mu(x)+1-\mu(1),\mu(z)+1-\mu(1)\}$

$= \max \{ \mu^+(x) , \mu^+(z) \}$

Hence μ^+ is an anti fuzzy bi-ideal of a Boolean like semi-ring R.

Theorem 3.9

Let μ be an anti fuzzy bi-ideal of a Boolean like semi-ring R then $(\mu^+)^+ = \mu^+$

Proof

For any xϵ R ,

we have $(\mu^+)^+(x) = \mu^+(x) + 1 - \mu(1)$

(by Theorem 3.8)

$= \mu(x) + 1 - \mu(1)$

$= \mu^{+}(x)$

Hence $(\mu^{+})^{+} = \mu^{+}$

Theorem 3.10

Let μ be an anti fuzzy bi-ideal of a Boolean like semi-ring R and $\phi:[0,\mu(0)]\rightarrow[0,1]$ be an increasing function. Let μ_ϕ be a fuzzy set in R defined by $\mu_\phi(x) = \phi(\mu(x))$ for all $x \in R$. Then μ_ϕ is an anti fuzzy bi-ideal of R.

Proof

Let $x,y,z \in R$. Then

i) $\mu_\phi(x - y) = \phi(\mu(x - y))$

$\leq \phi(\max\{\mu(x), \mu(y)\})$

$= \max\{\phi(\mu(x)), \phi(\mu(y))\}$

$= \max\{\mu_\phi(x), \mu_\phi(y)\}$

ii) $\mu_\phi(xyz) = \phi(\mu(xyz))$

$\leq \phi(\max\{\mu(x), \mu(z)\})$

$= \max\{\phi(\mu(x)), \phi(\mu(z))\}$

$= \max\{\mu_\phi(x), \mu_\phi(z)\}$

Hence μ_ϕ is an anti fuzzy bi-ideal of R.

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