EOQ model for deteriorating items with time-varying demand and partial backlogging

S. Kanji[1, *] **& S. K. Manna**[2, *]

[1] *M. Sc. Student, Department of Mathematics, Narasinha Dutt College, Howrah, W.B., India.*

[2] *Supervisor & Faculty, Department of Mathematics, Narasinha Dutt College, Howrah, W.B., India.*

Abstract

In this project work, we develop an EOQ model for deteriorating items with time-varying demand. In the model, shortages are allowed and partially backlogged. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Analytical results show that the optimal re-order time of the proposed model is unique and is independent of the form of the demand rate. Results are illustrated with the help of numerical examples. Computational results show that a decrease in the backlogging parameter causes the lower average total cost per unit time. Sensitivity of the solution to changes in the value of input parameters of the base example is also carried out.

Keywords: Deterioration, Time-varying demand, Partial backlogging.

1. Introduction

Most of the physical goods undergo decay or deterioration overtime. Commodities such as fruits, vegetables, foodstuffs, etc., suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as gasoline, alcohol, turpentine, etc. undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through

[*] Authors e-mail addresses: *skanji553@gmail.com* (S. Kanji), *skmanna_5@hotmail.com* (S. K. Manna)

a gradual loss of potential or utility with the passage of time. Thus decay or deterioration of physical goods in stock is a very realistic feature and inventory modellers felt the need to take this factor into consideration.

In recent years, inventory problems for deteriorating items have been widely studied after Ghare and Schrader (1963). They presented an EOQ model for an exponentially decaying items. Later, Covert and Philip (1973) formulated the model with variable deterioration rate with two-parameter Weibull distribution. Philip (1974) then developed the inventory model with a three-parameter Weibull distribution rate and no shortages. Shah and Jaiswal (1977) extended Philip's (1974) model and considered that shortages are allowed. In different times, inventory researchers developed various features of inventory models with a time-dependent deterioration rate. Interested reader may consult the researchers by Mishra (1975), Fujiwara (1993), Hariga and Benkherouf (1994), Wee (1995), Su et al. (1996), Lin et al. (2000), Wu and Ouyang (2000), Manna and Chaudhuri (2001, 2006), Mukhopadhyay et al. (2004) and Goyal and Giri (2001).

In the above literatures, almost all the inventory models for deteriorating items assume that the deterioration occurs as soon as the retailer receives the commodities. However, in real life, most of the physical goods would have a span of maintaining quality or the original condition(e.g. vegetables, fruit, fish, meat and so on) namely, during that period, there was no deterioration occuring. We term the phenomenon as "non-instantaneous deterioration". In this regard, Wu et al. (2006) developed an optimal replenishment policy for non-instantaneous constant deteriorating items.

The assumption of constant demand is not always applicable to real situations. For instance, it is usually observed in the super market that display of the consumer goods in large quantities attracts more customers and generates higher demand. This observation has influenced researchers to introduce a time-varying demand pattern in inventory modelling. Donaldson (1977) was the first to solve analytically the EOQ model, where demand was assumed to be a linearly increasing function of time. Resh et al. (1976) derived an algorithm to determine the optimal number of replenishments and timing for a linearly increasing demand pattern. Barbosa and Friedman (1978) then generalised the solutions for power form demand functions.

Furthermore, Henery (1979) extended the demand pattern to be of any log concave form. Dave and Patel (1981) considered an inventory model for deteriorating items with time-proportional demand when shortages are prohibited. Silver (1979) formulated a very simple inventory replenishment decision rule for the special case of positive trended demand. Wu (2001, 2002) further investigated the inventory model with ramp type demand rate. However, he did not guarantee the existence and uniqueness of his solution. Recently, Giri et al. (2003) extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution.

Furthermore, when the shortages occur, some customers are willing to wait for backorder and others would turn to buy from other sellers. Many researchers such as Park (1982), Hollier and Mak (1983) and Wee (1995) consider the constant partial backlogging rates during the shortage period in their inventory models. For fashionable commodities and high-tech products with short product life cycle, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period is declined with the length of the waiting time. To reflect this phenomenon, Chang and Dye (1999) developed an inventory model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time. Recently, many researchers have modified inventory policies by considering the "time-proportional partial backlogging rate" such as Abad (2000), Papachristos and Skouri (2000), Chang and Dye (2001), Wang (2002), Dye and Ouyang (2005), etc.

In the present paper, the EOQ model is developed for time-dependent deteriorating items. In addition, we also assumed that demand rate is time-varying and backlogging rate is variable and dependent on the waiting time for the next replenishment. Results are illustrated with the help of numerical examples. Finally, sensitivity of the solution to changes in the value of input parameters associated with the model is discussed.

2. Assumptions

The mathematical model with a infinite rate of replenishment is developed with the following assumptions.

(i) Lead time is zero.

(ii) Replenishment size is constant.

(iii) Shortages are allowed and only a fraction of demand is backlogged.

(iv) During the shortage period, the backlogging rate is variable.

(iv) Backlogging rate is dependent on the length of the wait time for the next replenishment. The longer the wait time is, the production of customers who would like to accept backlogging at time t is decreases with the wait time waiting for the next replenishment.

3. Notations

C_1: Inventory holding cost per unit per unit of time.

C_2: Shortage cost per unit per unit of time.

C_3: Opportunity cost due to lose sales per unit time.

C_4: Cost of each deteriorated units.

T: Fixed length of each ordering cycle.

$D(t)$: Demand rate at any instant t.

$\theta(t)$: Inventory deterioration rate.

In addition, we make the following assumptions and notations:

$\theta(t) = \alpha e^{\beta t}$ is the deterioration rate, where $\alpha(>0)$ and $\beta(\geq 0)$ are respectively scale and shape parameters. (For $\beta = 0$, deterioration rate is constant.)

$B(t) = \frac{1}{1+\delta t}$, where backlogging parameter δ is a positive constant. The longer the waiting time is the proportion of customers who would like to accept backlogging at time t is decreases with the wait time $(T-t)$ waiting for the next replenishment. Thus the demand rate at time t is partially backlogged at fraction $B(T-t)$.

4. Model Formulation

In this paper, the replenishment problem of non-instantaneous deteriorating item with partial backlogging is considered. Replenishment is made at time $t = 0$ when inventory level is its maximum, S. The inventory level decreasing until at time t_1 when it reaches the zero level. The decrease in inventory during the time interval $[0, t_1]$, occurs mainly to meet demand and partly for deterioration. Shortages are allowed to occur during the time interval $[t_1, T]$ and some part of shortage is backlogged and other part of it is the lost sales. Only the backlogging items are replaced by the next replenishment. Behaviour of the inventory system is depicted in Figure-1.

Let $I(t)$ be the inventory level at any time t $(0 \leq t \leq T)$ the differential equations governing the instantaneous states of $I(t)$ in the interval $[0, T]$ are given by,

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t), \qquad 0 \leq t \leq t_1 \tag{1}$$

with the condition $I(0) = S$

$$\frac{dI(t)}{dt} = -D(t)B(T-t), \qquad t_1 \leq t \leq T \tag{2}$$

with the condition $I(t_1) = 0$

putting $\theta(t) = \alpha e^{\beta t}$ in (1) we get,

$$\frac{dI(t)}{dt} + \alpha e^{\beta t} I(t) = -D(t), \qquad 0 \leq t \leq t_1$$

This is 1st order linear differential equation. It's solution is,

$$I(t) = e^{-\frac{\alpha}{\beta}e^{\beta t}}[Se^{\frac{\alpha}{\beta}} - \int_0^t D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx] \tag{3}$$

Again from (2) we get,

$$\begin{aligned}\frac{dI(t)}{dt} &= -D(t)B(T-t), \qquad t_1 \leq t \leq T \\ &= -\frac{D(t)}{1+\delta(T-t)} \qquad [\text{Since } B(t) = \tfrac{1}{1+\delta t}]\end{aligned}$$

Therefore,
$$I(t) = -\int_{t_1}^{t} \frac{D(x)}{1+\delta(T-x)}dx \tag{4}$$

The solutions of (1) and (2) are as follows:-

$$I(t) = \begin{cases} e^{-\frac{\alpha}{\beta}e^{\beta t}}[Se^{\frac{\alpha}{\beta}} - \int_0^t D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx], & 0 \leq t \leq t_1 \\ -\int_{t_1}^t \frac{D(x)}{1+\delta(T-x)}dx, & t_1 \leq t \leq T \end{cases} \quad (5)$$

where,

$$S = e^{-\frac{\alpha}{\beta}}\int_0^{t_1} D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx \quad (6)$$

Accumulated inventory over the period $[0, t_1]$ is expressed by,

$$\begin{aligned} H_T &= \int_0^{t_1} I(t)dt & (7) \\ &= \int_0^{t_1}\{e^{-\frac{\alpha}{\beta}e^{\beta t}}[Se^{\frac{\alpha}{\beta}} - \int_0^t D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx]\}dt \\ &= \int_0^{t_1}\{e^{-\frac{\alpha}{\beta}e^{\beta t}}[\int_0^{t_1} D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx - \int_0^t D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx]\}dt \quad \text{[using (6)]} & (8) \\ &= \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}}[\int_t^{t_1} D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx]dt & (9) \end{aligned}$$

Amount of shortage during the period $[t_1, T]$ is given by,

$$\begin{aligned} B_T &= -\int_{t_1}^T I(t)dt \\ &= \int_{t_1}^T[\int_{t_1}^t \frac{D(x)}{1+\delta(T-x)}dx]dt \\ &= \int_{t_1}^T \frac{(T-t)D(t)}{1+\delta(T-t)}dt \quad \text{[See } \textbf{appendix I}\text{]} \end{aligned}$$

The amount of lost sales during the time interval$[t_1, T]$ is given by,

$$\begin{aligned} L_T &= \text{Demand in } [t_1, T] \text{ - partial backlog amount in } [t_1, T] \\ &= \int_{t_1}^T D(t)dt - \int_{t_1}^T D(t)B(T-t)dt \\ &= \int_{t_1}^T D(t)dt - \int_{t_1}^T \frac{D(t)}{1+\delta(T-t)}dt \\ &= \delta\int_{t_1}^T \frac{(T-t)D(t)}{1+\delta(T-t)}dt & (10) \end{aligned}$$

Total number of deteriorated items during $[0, t_1]$ is written by,

$$\begin{aligned} D_T &= S \text{ - total demand in } [0, t_1] \\ &= e^{-\frac{\alpha}{\beta}}\int_0^{t_1} D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx - \int_0^{t_1} D(t)dt \\ &= e^{-\frac{\alpha}{\beta}}\int_0^{t_1} D(t)e^{\frac{\alpha}{\beta}e^{\beta t}}dt - \int_0^{t_1} D(t)dt & (11) \end{aligned}$$

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Average total cost AC during the time interval $[0, T]$ is expressed by,

$$\begin{aligned} AC(t_1) &= \frac{C_1 H_T + C_2 B_T + C_3 L_T + C_4 D_T}{T} \\ &= \frac{1}{T}\{C_1 \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}}[\int_t^{t_1} D(x) e^{\frac{\alpha}{\beta}e^{\beta x}} dx]dt \\ &\quad +(C_2 + C_3\delta) \int_{t_1}^{T} \frac{(T-t)D(t)}{1+\delta(T-t)}dt \\ &\quad +C_4[e^{-\frac{\alpha}{\beta}} \int_0^{t_1} D(t) e^{\frac{\alpha}{\beta}e^{\beta t}} dt - \int_0^{t_1} D(t)dt]\} \end{aligned} \tag{12}$$

The first and second order derivative of $AC(t_1)$ with respect to t_1 are given by,

$$\begin{aligned} \frac{dAC(t_1)}{dt_1} &= \frac{D(t_1)}{T}[C_1 e^{\frac{\alpha}{\beta}e^{\beta t_1}} \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}} dt - (C_2 + C_3\delta)\frac{(T-t_1)}{1+\delta(T-t_1)} \\ &\quad +C_4(e^{-\frac{\alpha}{\beta}} e^{\frac{\alpha}{\beta}e^{\beta t_1}} - 1)] \qquad \text{[See } \textbf{appendix II}\text{]} \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{d^2 AC(t_1)}{dt_1^2} &= \frac{D^{/}(t_1)}{T}[C_1 e^{\frac{\alpha}{\beta}e^{\beta t_1}} \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}} dt - (C_2 + C_3\delta)\frac{(T-t_1)}{1+\delta(T-t_1)} \\ &\quad +C_4(e^{-\frac{\alpha}{\beta}} e^{\frac{\alpha}{\beta}e^{\beta t_1}} - 1)] \\ &\quad +\frac{D(t_1)}{T}[C_1 + \frac{C_2 + C_3\delta}{\{1+\delta(T-t_1)\}^2} + C_4 \alpha e^{-\frac{\alpha}{\beta}} e^{\frac{\alpha}{\beta}e^{\beta t_1}} \\ &\quad +C_1 \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}} \alpha e^{\beta t_1} e^{\frac{\alpha}{\beta}e^{\beta t_1}} dt] \end{aligned} \tag{14}$$

Now $\dfrac{dAC(t_1)}{dt_1} = 0$ gives,

$$C_1 e^{\frac{\alpha}{\beta}e^{\beta t_1}} \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}} dt - (C_2 + C_3\delta)\frac{(T-t_1)}{1+\delta(T-t_1)} + C_4(e^{-\frac{\alpha}{\beta}} e^{\frac{\alpha}{\beta}e^{\beta t_1}} - 1) = 0 \tag{15}$$

Here $\dfrac{dAC(t_1)}{dt_1} = 0$, gives the necessary condition for $AC(t_1)$ to be minimum. Therefore, the sufficient condition for minimum average total cost is satisfied. Optimal S is given by,

$$S^* = e^{-\frac{\alpha}{\beta}} \int_0^{t_1^*} D(t) e^{\frac{\alpha}{\beta}e^{\beta t}} dt \tag{16}$$

and optimal ordering units Q is expressed as,

$$Q^* = S^* + \int_{t_1^*}^{T} \frac{D(t)}{1+\delta(T-t)}dt \tag{17}$$

Moreover, from equation (12), the minimum average total cost per unit time is $AC(t_1^*)$.

5. Computational Results

The total average cost is the function of single variable t_1. Our objective is to determine t_1 which minimise the cost function $AC(t_1)$. Using subroutine Find root in **Mathematica 4.1**, we solve equation (13) to find t_1 satisfying the proposition for given input parameters. Minimum average total cost AC, optimal S and optimal order quantity Q are calculate from (12), (16) and (17).

To illustrate, consider the base example $C_1 = 3$, $C_2 = 15$, $C_3 = 20$, $C_4 = 5$, $\delta = 0.5$, $\alpha = 0.2$, $\beta = 0.9$, $T = 1$ and $D(t) = 20 + 2t$ in appropriate units. The optimal solution is $t_1^* = 0.817492$ and the corresponding optimal S, Q and AC are $S^* = 18.9988$, $Q^* = 22.8098$ and $AC^* = 40.7805$. For $D(t) = 60e^{-0.98t}$ in the base example, the optimal values of S, Q and AC are given by $S^* = 37.0242$, $Q^* = 41.3253$ and $AC^* = 77.3779$ respectively. For $\delta = 0$ in the above two base examples, the optimal solution for t_1^* are given by (0.750754, 0.750754) and optimal values of S, Q and AC are (17.1909, 34.6918), (22.6122, 41.0494) and (36.781, 69.8209) respectively.

It is noted that the average total cost per unit is an increasing function of the parameter δ. This implies that the model with this type of partial backlogging always has smaller average total cost per unit time than that with complete backlogging.

To study the effect of change in the input parameters C_1, C_2, C_3, C_4, δ, α, β, T on the optimal value of $t_1(t_1^*)$, optimal on hand inventory (S^*), optimal order quantity (Q^*), optimal average system cost (AC^*) derived from the proposed model, a sensitivity analysis is performed by considering two numerical examples for the case of partial backlogging given above. Sensitivity analysis is done by changing (increasing or decreasing) the parameters by 25% and 50% and taking one parameter at a time. Keeping the remaining parameters at their original values. From Tables 1-2, it is seen that the percentage change in the cost is almost equal for both positive and negative changes of all the parameters. The average optimal cost is highly sensitive to T.

Table 1: Sensitivity analysis for $D(t) = 20 + 2t$ in the base example

changing parameter	(%) change	t_1^*	S^*	Q^*	AC^*	(%) change in AC^*
C_1	+50	0.769424	17.6907	22.4422	51.3831	+25.999
	+25	0.792842	18.3241	22.6205	46.2484	+13.4079
	-25	0.843452	19.7187	23.0113	34.9503	-14.2966
	-50	0.870808	20.4879	23.2261	28.7252	-29.5616
C_2	+50	0.85364	20.0039	23.091	42.7589	+4.85122
	+25	0.837553	19.5543	22.9654	41.8761	+2.68649
	-25	0.791777	18.2951	22.6123	39.3847	-3.42272
	-50	0.757628	17.3744	22.353	37.5458	-7.93215
C_3	+50	0.843295	19.7143	23.6101	42.1907	+3.45802
	+25	0.831375	19.3826	22.9173	41.5381	+1.85759
	-25	0.801117	18.5497	22.6839	39.8907	-2.18215
	-50	0.781515	18.0168	22.534	38.8304	-4.78201
C_4	+50	0.793802	18.3502	22.6278	45.6129	+11.8496
	+25	0.805437	18.6678	22.717	43.2404	+6.03182
	-25	0.829995	19.3444	22.9066	38.2279	-6.25941
	-50	0.842976	19.7055	23.0076	35.5764	-12.7613
δ	+50	0.838353	19.5766	22.909	42.0132	+3.02273
	+25	0.828599	19.3057	22.8604	41.4383	+1.61281
	-25	0.804704	18.6477	22.7578	40.0199	-1.86512
	-50	0.789791	18.2411	22.7056	39.1289	-4.05021
α	+50	0.783219	19.0585	23.5426	47.0017	+15.2552
	+25	0.800231	19.0415	23.1931	43.9246	+7.70975
	-25	0.834933	18.9285	22.392	37.5749	-7.86085
	-50	0.852478	18.8289	21.9396	34.3142	-15.8564

Table 1: continued...

changing parameter	(%) change	t_1^*	S^*	Q^*	AC^*	(%) change in AC^*
β	+50	0.800791	18.857	22.9976	42.8642	+5.10932
	+25	0.809558	18.9359	22.9039	41.7813	+2.45401
	-25	0.824625	19.0476	22.7169	39.859	-2.25968
	-50	0.831002	19.0841	22.6262	39.013	-4.33429
T	+50	1.17017	29.7349	36.6586	67.7193	+66.0579
	+25	1.00037	24.2905	29.5257	53.5725	+31.3677
	-25	0.623934	13.9108	16.5239	29.188	-28.4267
	-50	0.421951	9.04868	10.6506	18.6293	-54.3183

Table 2: Sensitivity analysis for $D(t) = 60e^{-0.98t}$ in the base example

changing parameter	(%) change	t_1^*	S^*	Q^*	AC^*	(%) change in AC^*
C_1	+50	0.769424	35.3534	40.8557	99.0847	+28.0529
	+25	0.792842	36.1731	41.0864	88.6335	+14.5463
	-25	0.843452	37.9079	41.5726	65.2222	-15.7096
	-50	0.870808	38.8254	41.8288	52.056	-32.725
C_2	+50	0.85364	38.2512	41.6686	81.3464	+5.12872
	+25	0.837553	37.7082	41.5168	79.5688	+2.83142
	-25	0.791777	36.1361	41.076	74.6102	-3.57693
	-50	0.757628	34.9362	40.7381	71.0021	-8.23983
C_3	+50	0.843295	37.9026	41.5711	80.2011	+3.64856
	+25	0.831375	37.4984	41.4581	78.8911	+1.9551
	-25	0.801117	36.4602	41.1671	75.6104	-2.28433
	-50	0.781515	35.778	40.9753	73.5181	-4.98831

Table 2: continued...

changing parameter	(%) change	t_1^*	S^*	Q^*	AC^*	(%) change in AC^*
C_4	+50	0.793802	36.2065	41.6958	84.3805	+9.04977
	+25	0.805437	36.6095	41.209	80.918	+4.57505
	-25	0.829995	37.4514	41.4449	73.7577	-4.67863
	-50	0.842976	37.8918	41.5681	70.0547	-9.46419
δ	+50	0.838353	37.7353	41.4526	79.7608	+3.07951
	+25	0.828599	37.4039	41.3908	78.6456	+1.63828
	-25	0.804704	36.5842	41.2565	75.9218	-1.8818
	-50	0.789791	36.0669	41.1855	74.2283	-4.07048
α	+50	0.783219	37.5139	42.6684	86.796	+12.1716
	+25	0.800231	37.2885	42.0174	82.1211	+6.12987
	-25	0.834933	36.7201	40.5927	72.5705	-6.21239
	-50	0.852478	36.3758	39.8213	67.7054	-12.5004
β	+50	0.800791	36.937	41.652	80.1153	+3.53771
	+25	0.809558	36.99	41.4873	78.6962	+1.70371
	-25	0.824625	37.0425	41.1679	76.1562	-1.57898
	-50	0.831002	37.0475	41.0164	75.026	-3.03955
T	+50	1.17017	48.081	53.0328	102.369	+32.2979
	+25	1.00037	42.9956	47.6816	90.7592	+17.2933
	-25	0.623934	29.9967	33.7381	62.0202	-19.8477
	-50	0.421951	21.6898	24.6139	44.3509	-42.6828

6. Managerial Implications

The assumptions of constant demand is not always applicable to real situations. For instance, it is usually observed in the super market that display of the customer goods in large quantities attracts more customers and generates higher demand. This observation has influenced researchers to introduce a time-varying demand pattern in inventory modelling. When the shortages occur, some customers are willing to wait for back-order and others would turn to buy from others sellers. For fashionable commodities and high-tech products with short product life cycle, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period is declined with the length of the waiting time.

7. Concluding Remarks

In this paper, a deterministic inventory model has been developed for deteriorating items and time varying demand. Shortages are allowed. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Analytical results indicate that the optimal re-order time of the proposed model is unique and independent of the form of demand rate. Computational results show that a decrease in backlogging parameter causes the lower average total cost per unit. The effect of the scale(α) and shape(β) parameter are also discussed. Average total cost per unit time is an increasing function of the parameter δ which implies that the model for such kind of partial backlogging always has smaller average total cost per unit time than that of complete backlogging. The proposed model can be used in inventory control of certain non-instantaneous deteriorating items such as electronic components, food items, fashionable commodities and others.

Acknowledgment

I am indebted to my M. Sc. project supervisor for giving helpful suggestions and comments for developing this project work. The authors express their thanks to Department of Mathematics, Narasinha Dutt College, Howrah, for providing infrastructural support to carry out this work.

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Appendix I.

From Fundamental theorem of integral calculus, we have following result

$$\frac{d}{d\alpha}\int_{a(\alpha)}^{b(\alpha)} F(x,\alpha)dx = \int_a^b \frac{\partial F}{\partial \alpha}dx + F(b,\alpha)\frac{db}{d\alpha} - F(a,\alpha)\frac{da}{d\alpha} \tag{18}$$

$$\begin{aligned}\text{Now,}\quad B_T &= \int_{t_1}^{T}[\int_{t_1}^{t}\frac{D(x)}{1+\delta(T-x)}dx]dt \\ &= \int_{t_1}^{T} u(t)dt\end{aligned}$$

where,

$$u(t) = \int_{t_1}^{t}\frac{D(x)}{1+\delta(T-x)}dx$$

Therefore,

$$\begin{aligned}\frac{du(t)}{dt} &= \frac{D(t)}{1+\delta(T-t)} && \text{[Using (18).]}\\ \text{or,}\quad \frac{(T-t)D(t)}{1+\delta(T-t)} &= (T-t)\frac{du(t)}{dt}\\ \text{or,}\quad \int_{t_1}^{T}\frac{(T-t)D(t)}{1+\delta(T-t)}dt &= \int_{t_1}^{T}[(T-t)\frac{du(t)}{dt}]dt\\ \text{or,}\quad \int_{t_1}^{T}\frac{(T-t)D(t)}{1+\delta(T-t)}dt &= [(T-t)u(t)]_{t_1}^{T} + \int_{t_1}^{T}u(t)dt\\ &= \int_{t_1}^{T}[\int_{t_1}^{t}\frac{D(x)}{1+\delta(T-x)}dx]dt\end{aligned}$$

Therefore,

$$B_T = \int_{t_1}^{T}\frac{(T-t)D(t)}{1+\delta(T-t)}dt$$

Appendix II.

Let, $F_1(t,t_1) = e^{-\frac{\alpha}{\beta}e^{\beta t}}[\int_t^{t_1} D(x)e^{\frac{\alpha}{\beta}e^{\beta x}}dx]$

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Here, $F_1(t,t_1)=0$

$$\begin{aligned}\frac{\partial F_1}{\partial t_1} &= e^{-\frac{\alpha}{\beta}e^{\beta t}}D(t_1)e^{\frac{\alpha}{\beta}e^{\beta t_1}} && \text{[Using (18).]}\\ \text{or,} \quad \int_0^{t_1}\frac{\partial F_1}{\partial t_1} &= \int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}}D(t_1)e^{\frac{\alpha}{\beta}e^{\beta t_1}}dt\\ &= D(t_1)e^{\frac{\alpha}{\beta}e^{\beta t_1}}\int_0^{t_1} e^{-\frac{\alpha}{\beta}e^{\beta t}}dt\end{aligned}$$

Let,

$$\begin{aligned}F_2(T,t_1) &= \int_{t_1}^{T}\frac{(T-t)D(t)}{1+\delta(T-t)}dt\\ &= -\int_{T}^{t_1}\frac{(T-t)D(t)}{1+\delta(T-t)}dt\\ \frac{\partial F_2}{\partial t_1} &= -\frac{(T-t_1)D(t_1)}{1+\delta(T-t_1)} && \text{[Using (18).]}\end{aligned}$$

Here, $F_2(t,t_1)=0$

Let,

$$\begin{aligned}F_3(t_1) &= e^{-\frac{\alpha}{\beta}}\int_0^{t_1}D(t)e^{\frac{\alpha}{\beta}e^{\beta t}}dt-\int_0^{t_1}D(t)dt\\ \frac{\partial F_3}{\partial t_1} &= D(t_1)e^{-\frac{\alpha}{\beta}e^{\beta t_1}}dt-D(t_1)dt && \text{[Using (18).]}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dAC(t_1)}{dt_1} &= \frac{D(t_1)}{T}[C_1e^{\frac{\alpha}{\beta}e^{\beta t_1}}\int_0^{t_1}e^{-\frac{\alpha}{\beta}e^{\beta t}}dt-(C_2+C_3\delta)\frac{(T-t_1)}{1+\delta(T-t_1)}\\ &\quad +C_4(e^{-\frac{\alpha}{\beta}}e^{\frac{\alpha}{\beta}e^{\beta t_1}}-1)]\\ \frac{d^2AC(t_1)}{dt_1^2} &= \frac{D^/(t_1)}{T}[C_1e^{\frac{\alpha}{\beta}e^{\beta t_1}}\int_0^{t_1}e^{-\frac{\alpha}{\beta}e^{\beta t}}dt-(C_2+C_3\delta)\frac{(T-t_1)}{1+\delta(T-t_1)}\\ &\quad +C_4(e^{-\frac{\alpha}{\beta}}e^{\frac{\alpha}{\beta}e^{\beta t_1}}-1)]\\ &\quad +\frac{D(t_1)}{T}[C_1+\frac{C_2+C_3\delta}{\{1+\delta(T-t_1)\}^2}+C_4\alpha e^{-\frac{\alpha}{\beta}}e^{\frac{\alpha}{\beta}e^{\beta t_1}}\\ &\quad +C_1\int_0^{t_1}e^{-\frac{\alpha}{\beta}e^{\beta t}}\alpha e^{\beta t_1}e^{\frac{\alpha}{\beta}e^{\beta t_1}}dt]\end{aligned}$$

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