BETWEEN STRONGLY g-CLOSED SETS AND STRONGLY g**- CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT: In this dissertation, we study the notions of g-closed sets and g*-closed sets and strongly g*-closed sets. Also , we study strongly g-closed sets and sg**-closed sets in topological spaces and also some examples and Every closed set is strongly g*-closed set. the above theorem is need not be true we explain some examples.

Keywords: g-closed sets, g*-closed sets, strongly g-closed sets, strongly g**- closed sets.

1. INTRODUCTION

The word topology derived from two areakwords,topos meaning discovers or study topology thus literally means the study of surface. modern topology depends strongly on the ideas of theory, developed by Georg Cantor in the later part of the 19th century during the period up to 1960's,researchs in the field of general topology flourished and settled many important. Since the 1960's researches in general topology has moved into several new areas that invoice intricate mathematical tools, including set theoretic methods. In the late 1960's research worked to generalized some of the topological properties of infinite dimensional Hilbert space. Maurice freched(1878-1973) was the first to expand topological consider beyond Euclidean space. He introduced metric space in 1960 in a context the permitted one to consider abstract objects and not just real numbers on n tuples of real numbers topology emerged as a concerned discipline in1914 when felixhausdroff (1868-1942)published his classical treatise grunzuge.Hausdroaff defined topological space in term of neighborhood of members of a set. These concepts where introduce immediately after georg cantor(1845-1918) had developed a general theory of sets in the general theory of sets in the general theory of sets in the 1880's but even before cantor, Bernard Riemann (1826-1866)had fore seen study of abstract space.

2. PRELIMINARIES

2.1:BASIC DEFINITIONS

Definition 2.1.1:A topology on a set X is collection $\tau$of a subsets of X having the following properties

1.$\emptyset$and X are in τ .

2.The union of elements of any collection of τ is in τ .

3.The intersection of the elements of any finite sub collection of τ is in τ

The elements of τ are known as open set and the elements of τ^c are known as closed set.

Definition 2.1.2:A set X to gether with a topology τ defined on it is called a topological space. And it is denoted by (X,τ)

Definition 2.1.3: Let X be any set. The collection of all subsets of X is a topology on X it is called a Discrete Topology.

Definition 2.1.4: The collection consisting of$\emptyset$ and X. only is also a topology on X it is called a Indiscrete Topology.

Definition 2.1.5:A subset of A of a topological space X is said to be open if the set X-A is closed.

Definition 2.1.6: Let X be a topological space and let A$\subset$X.then ,the interior of A [denoted by A^0 (or) int(A)] is defined as the union of all open sets contained in A.

Definition 2.1.7: Let X be a topological space and let A $\subset$X.then ,the closur of A [denoted by A(OR)cl(A)] is defined as the intersection of all closed set containing A.

Definition 2.1.8:A subset A of X is said to be semi open if A $\subseteq$cl(int(A)).

Definition 2.1.9:A subset of A of X is said to be semi closed if int(cl(A)) ⊆ A

3. g-closed and g∗ - closed sets in topological spaces:

In this chapter,we study the concept of g-closed and g* -closed sets in topological spaces.

3.1:GENERALIZED CLOSED SETS

Definition 3.1.1: A subset of a topological space(x,τ) is called generalized closed set(brifly g-closed) if cl(A) ⊆ UWhenever A ⊆ U andU is open in X.

Definition 3.1.2:A subset A of a topological space(X,τ) is called

1. A semi- Generalized closed set (briefly sg-closed) if scl(A)⊆ U, Whenever A ⊆ U and U is semiopen in(X,τ).

2. A Generalized semi-closed set (briefly gs-closed) if scl(A) ⊆ U, Whenever A ⊆ U and U is open in (X,τ).
3. A Generalizedα-closed set (briefly gα-closed) if αcl(A)⊆ U, Whenever A ⊆ U and U is$\alpha -$ open in(X,τ).
4. $\alpha -$ Generalizedclosed set (briefly αg-closed) if αcl(A) ⊆U, Whenever A ⊆ U and U is$\alpha -$ open in(X,τ).

5. A $\alpha ** -$ Generalizedclosed set (briefly $\alpha ** g$-closed) if αcl(A)⊆int(cl(U)), Whenever A ⊆U and U is open in(X,τ).

6. A$g\alpha * -$ closed set if αcl(A)⊆int(U), Whenever A ⊆ U and U is $\alpha -$ open in(X,τ).

7. A Generalized semi-preclosed set (briefly gsp-closed) if sαcl(A)⊆ U, Whenever A ⊆ U and U is open in(X,τ).

8.A regular generalized closed set (briefly r-g-closed)if cl(A)⊆ U, whenever A ⊆U and U is regular open in (X,τ)

9. A Generalized preclosed set (briefly gp-closed) if pcl(A)⊆ U, Whenever A ⊆U and U is open in(X,τ).

10. A Generalized pre$regular$closed set (briefly gpr-closed) if pcl(A)⊆ U, Whenever A ⊆ U and U is Regular- open in(X,τ).

4.BASIC PROPERTIES OF g*-CLOSED SETS:

4.1:Definition

A subset A of(x,τ) is called g*-closed set if cl(A)⊆U, whenever A⊆U and U is g-open in(X,τ)

THEOREM 4.1.1:Every closed set is α g*-closed set.

The following examples supports that α g*-closed set need not be closed in general.

EXAMPLE 4.1.2: Let X={a,b,c} andτ={∅,{a},{a,c},X}. Here let A={a,b} A is α g*-closed set but not a closed set of(X,τ).

THEOREM 4.1.3: If A is α g*-closed set of(X,τ) such that A⊆ B ⊆cl(A),then B is also α g*-closed set in(X,τ).

PROOF: Let U be α g-open set of(X,τ) such that B⊆U.then A⊆ U. since A is g*-closed ,then cl(A)⊆ U. now

cl(B) ⊆ cl(cl(A))=cl(A)⊆U.therefore B is also α g*-closed set of (X,τ)

4.2.STRONGLY g*-CLOSED SETS IN TOPOLOGICAL SPACES

In this chapter, we study the concept of strongly g*-closed sets in topological spaces.

4.2.1:Definition

Let (X,τ) be a topological space and A be its subset, then A is strongly g*-closed set if cl(int(A))⊆ U, when ever A⊆U and U is g-open.

4.2.2: THEOREM Every closed set is strongly g*-closed set.

PROOF: The proof is immediate from the definition of closed set.

4.2.3: REMARK The converse of the above theorem is need not be true.

EXAMPLE 4.2.4: Let X={a,b,c} and τ={∅,{a},{a,c},X}. and A={a,b}.Then clearly A is strongly g*-closed set of (X,τ)

4.3:STRONGLY g-CLOSED SETS AND STRONGLY g-CLOSED SETS**

In this chapter,we study the concept of strongly g-closed and strongly g**-closed sets.

4.3.1:DefinitionA subset A of a space (X,τ) is called a regular generalized closed(brifly rg-closed) set if cl(A)⊆ U whenever A⊆ U U is regular open in (X,τ)

4.3.2:EXAMPLE Let X={a,b,c} andτ={∅,{a},{a,c},X}.then regular generalized closed sets {∅,{a},{a,b},{b,c},{a,c},X}.

4.3.3:Definition A subset A of a space (X,τ) is called a generalized g star closed(brifly g*-closed) set if cl(A)⊆U whenever A⊆U U is g open in (X,τ)

4.3.4:EXAMPLE Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. then generalized g star closed sets $\{\emptyset,\{b\},\{a,b\},\{b,c\},X\}$.

4.3.5:Definition A subset A of a space (X,τ) is called a generalized g star star closed(brifly g**-closed) set if cl(A)⊆ U whenever A⊆ U U is g* open in (X,τ)

4.3.6:EXAMPLE

Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. then generalized g star star closed sets $\{\emptyset,\{b\},\{a,b\},\{b,c\},X\}$.

5.BASIC PROPERTIES OF STRONGLY g-CLOSED SETS AND STRONGLY g**-CLOSED SETS.

5.1:STRONGLY g**-CLOSED SETS

5.1.1:Definition

Let (X,τ) be a topological space and A be its subset .Then A is said to be a strongly g-closed set if cl(int(A))⊆U whenever A⊆ U U is open in X.

5.1.2:EXAMPLE Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$ then strongly g-closed sets is $\{\emptyset,\{a,b\},\{b,c\},\{b\},X\}$

5.1.3:Definition Let (X,τ) be a topological space and A be its subset. Then A is said to be a strongly g**-closed set if Cl(int(A))⊆U whenever A⊆ U and U is g* open in X.

5.1.4: EXAMPLE Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. then strongly g**-closed sets is $\{\emptyset,\{a,b\},\{b,c\},\{b\},X\}$

5.1.5: REMARK Every closed set is strongly g-closed.

5.1.6: EXAMPLE Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. then strongly g-closed sets is $\{\emptyset,\{a,b\},\{b,c\},\{b\},X\}$

5.1.7: REMARK Every closed set is strongly g**-closed.

5.1.8: EXAMPLE Let $X=\{a,b,c\}$ and$\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. then strongly g**-closed sets is $\{\emptyset,\{a,b\},\{b,c\},\{b\},X\}$

5.1.9: REMARK Every strongly g*-closed set is strongly g-closed set.

5.1.10: REMARK Every strongly g**-closed set is strongly g-closed set.

5.1.11: REMARK Every strongly g*-closed set is strongly g**-closed set but not conversely.

5.1.12: EXAMPLE Let $X=\{a,b,c,d\}$ and$\tau=\{\emptyset,\{a\},X\}$. then A={b} strongly g**-closed but not g*-closed in(X,τ).

5.1.13: REMARK Every strongly g-closed set is strongly g-closed set but not conversely.

5.1.14: EXAMPLE5: Let $X=\{a,b,c,\}$ and$\tau=\{\emptyset,\{a\},\{b\},X\}$. then A={b} strongly g-closed but not g-closed in(X,τ).

5.1.15: REMARK Every strongly g**-closed set is strongly g**-closed set but not conversely.

5.1.16:EXAMPLE Let $X=\{a,b,c,\}$ and$\tau=\{\emptyset,\{a\},\{a,b\},X\}$. then A={b} strongly g**-closed but not g-closed in(X,τ).

5.1.17: REMARK5: Every strongly g*-closed set is strongly g-closed set but not conversly.

5.1.18: EXAMPLE Let $X=\{a,b,c,\}$ and$\tau=\{\emptyset,\{a\},\{a,c\},X\}$. then A={b} strongly g-closed but not g*-closed in(X,τ).

5.1.19: REMARK Every strongly g*-closed set is strongly g**-closed set but not conversly.

5.1.20: EXAMPLE Let $X=\{a,b,c,\}$ and$\tau=\{\emptyset,\{a\},,X\}$. then A={b} strongly g**-closed but not g*-closed in(X,τ).

5.1.21: REMARK Every strongly g**-closed set is strongly g-closed set but not conversly.

5.1.22: EXAMPLE: Let $X=\{a,b,c,\}$ and$\tau=\{\emptyset,\{a\},\{a,b\},X\}$. then A={b} strongly g-closed but not g**-closed in(X,τ).

5.1.23 :THEROM: If A subset of a topological space (X,τ) is both open and strongly g-closed then it is closed.

PROOF: Suppose A is both open and strongly g-closed.Since A is strongly g closed cl(int(A))⊆A That is cl(A)=cl(int(A)) ⊆A.A is closed.

5.1.24:THEROM: If A subset of topological space (X,τ) is both strongly g-closed and semi open then it is g-closed.

PROOF: Since A is strongly g-closed cl(int(A)) ⊆ A whenever A ⊆ U and U is open X.cl(int(A)) ⊆ A since A is semi-open.Then cl(A) ⊆ cl(int(A)) ⊆U.Hence A is g-closed.

VIII. CONCLUSION

In this chapter we study g-closed sets and strongly g**- closed sets in topological spaces. Whether you are looking for your first g-closed sets and strongly or returning to the workforce after an extended g**-closed sets in topological spaces will provides best platform for that. It will provides ease of work, multiple clients. G**- closed sets in topological spaces will reduce the cost of hiring process and save the time and eliminates the process.

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