

# 2<sup>nd</sup> Neighbourly Edge Irregular Fuzzy Graphs

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## Abstract

In this paper, 2<sup>nd</sup>-neighbourly edge irregular fuzzy graphs, 2<sup>nd</sup>-neighbourly edge totally irregular fuzzy graphs are introduced. Comparative study between 2<sup>nd</sup>-neighbourly edge irregular fuzzy graph and 2<sup>nd</sup>-neighbourly edge totally irregular fuzzy graph is done. Some properties of 2<sup>nd</sup>-neighbourly edge irregular fuzzy graphs are studied and they are examined for 2<sup>nd</sup>-neighbourly edge totally Irregular fuzzy graphs.

Key words: degree and total degree of a vertex in fuzzy graph, 2<sup>nd</sup> neighbourly irregular fuzzy graph, 2<sup>nd</sup> neighbourly totally irregular fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph, edge regular fuzzy graphs, strongly edge irregular fuzzy graphs.

## 1 Introduction

Euler first introduced the concept of graph theory in 1736. In 1965, Lofti A.Zadeh [10] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1957[7]. It has been growing fast and has numerous applications in various fields. The relation between fuzzy sets were also considered by Rosenfeld, and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts [2]. M.Akram and A.Dudek introduced the concept of regular bipolar fuzzy graphs. A.Nagoorani and K.Radha [4] introduced the concept of regular fuzzy graphs and defined degree of a vertex in fuzzy graphs. A. Nagoorgani and S.R.Latha [3] introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular

fuzzy graph in 2008. S.P Nandhini and E.Nandhini introduced the concept of strongly irregular fuzzy graphs and discussed about its properties [5]. K.Radha and N. Kumaravel [6] introduced the concept of edge degree, total edge degree in fuzzy graph and edge irregular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs. N.R.Santhi Maheswari and C.Sekar introduced the concept of strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed about its properties [8]. Also, N.R.Santhi Maheswari and C.Sekar introduced the concept of edge irregular fuzzy graphs and edge totally irregular fuzzy graphs and discussed about its properties [9]. This is the background to introduce 2<sup>nd</sup> - neighbourly edge irregular fuzzy graphs, 2<sup>nd</sup> - neighbourly edge totally irregular fuzzy graphs. comparative study between 2<sup>nd</sup> -neighbourly edge irregular fuzzy graphs and 2<sup>nd</sup> -neighbourly edge totally irregular fuzzy graphs is done. 2<sup>nd</sup> -neighbourly edge irregularity on some fuzzy graphs whose underlying crisp graphs are a path and a cycle is also studied.

## 2 Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper. By graph, we mean a pair  $G = (V, E)$ , where  $V$  is the set and  $E$  is a relation on  $V$ . The elements of  $V$  are vertices of  $G$  and the elements of  $E$  are edges of  $G$ .

Definition 2.1. A fuzzy graph denoted by  $G : (\sigma, \mu)$  on the graph  $G^* : (V, E)$  is a pair of functions  $(\sigma, \mu)$  where  $\sigma : V \rightarrow [0, 1]$  is a fuzzy subset of a set  $V$  and  $\mu : V \times V \rightarrow [0, 1]$  is a symmetric fuzzy relation on  $\sigma$  such that for all  $u, v$  in  $V$  the relation  $\mu(u, v) = \mu(v, u)$  and  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$  is satisfied. A fuzzy graph  $G$  is complete if  $\mu(u, v) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$  where  $uv$  denotes the line joining  $u$  and  $v$ .  $G^* : (V, E)$  is called the underlying crisp graph of the fuzzy graph  $G : (\sigma, \mu)$ , where  $\sigma$  and  $\mu$  are called membership function.

Definition 2.2. Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The degree of a vertex  $u$  is  $d_G(u) = \sum_{uv \in E} \mu(uv)$ , for  $uv \in E$  and  $\mu(uv) = 0$ , for  $uv$  not in  $E$ , this is equivalent to  $d_G(u) = \sum_{uv \in E} \mu(uv)$ ,  $u \neq v$ .

this is equivalent to  $d_G(u) = \sum_{uv \in E} \mu(uv)$ ,  $u \neq v$ .

Definition 2.3. Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . If  $d(v) = k$  for all  $v$ , then  $G$  is said to be a regular fuzzy graph of degree  $k$ .

Definition 2.4. Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The total degree of vertex  $u$  is defined as  $td(u) = \sum_{uv \in E} \mu(uv) + \sigma(u) = d(u) + \sigma(u)$ ,  $uv \in E$ . If each vertex of  $G$  has the same total degree  $k$ , then  $G$  is said to be a totally regular fuzzy graph of degree  $k$  or  $k$ -totally regular fuzzy graph.

Definition 2.5. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be an irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees.

Definition 2.6. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a totally irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total degree.

Definition 2.7. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees.

Definition 2.8. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a neighbourly irregular fuzzy graph if every pair of adjacent vertices have distinct degrees.

Definition 2.9. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a neighbourly totally irregular fuzzy graph if every pair of adjacent vertices have distinct total degrees.

Definition 2.10. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . The degree of an edge  $uv$  is defined as  $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$ . The minimum degree of an edge is  $\delta_E(G) = \wedge \{d_G(uv) : uv \in E\}$ . The maximum degree of an edge is  $\Delta_E(G) = \vee \{d_G(uv) : uv \in E\}$ .

Definition 2.11. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . The total degree of an edge  $uv$  is defined as  $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$ . The minimum degree of an edge is  $\delta_{tE}(G) = \wedge \{td_G(uv) : uv \in E\}$ . The maximum degree of an edge is  $\Delta_{tE}(G) = \vee \{td_G(uv) : uv \in E\}$ .

Definition 2.12. The degree of an edge  $uv$  in the underlying graph is defined as  $d_G(uv) = d_G(u) + d_G(v) + 2$

### 3 $2^{nd}$ -Neighbourly Edge Irregular Fuzzy Graphs And $2^{nd}$ -Neighbourly Edge Totally Irregular Fuzzy Graphs

In this section,  $2^{nd}$  neighbourly edge irregular graphs and  $2^{nd}$  neighbourly edge totally irregular graphs are introduced.

Definition 3.1. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a  $2^{nd}$  neighbourly edge irregular fuzzy graph if every pair of edges which are at a distance two have distinct degrees.

Definition 3.2. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is said to be a  $2^{nd}$  neighbourly edge totally irregular fuzzy graph if every pair of edges which are at a distance two have distinct total degrees.

Remark 3.3. Graph which is both  $2^{nd}$  neighbourly edge irregular fuzzy graph and  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.

Example 3.4. Consider a fuzzy graph on  $G^* (V, E)$ .

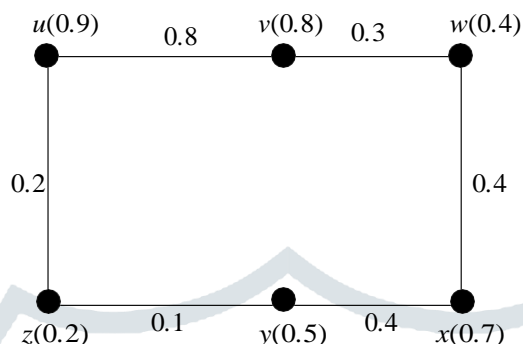


Figure.1

From Figure 1,  $d_G(u) = 1$  ;  $d_G(v) = 1.1$  ;  $d_G(w) = 0.7$  ;  $d_G(x) = 0.8$  ;  $d_G(y) = 0.5$  ;  $d_G(z) = 0.3$  ,Degrees of the edges are calculated below.

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = 1 + 1.1 - 2(0.8) = 0.5 .$$

$$d_G(vw) = d_G(v) + d_G(w) - 2\mu(vw) = 1.1 + 0.7 - 2(0.3) = 2.2 .$$

$$d_G(wx) = d_G(w) + d_G(x) - 2\mu(wx) = 0.7 + 0.8 - 2(0.4) = 0.7 .$$

$$d_G(xy) = d_G(x) + d_G(y) - 2\mu(xy) = 0.8 + 0.5 - 2(0.4) = 0.5 .$$

$$d_G(yz) = d_G(y) + d_G(z) - 2\mu(yz) = 0.5 + 0.3 - 2(0.1) = 0.6 .$$

$$d_G(zx) = d_G(z) + d_G(x) - 2\mu(zx) = 0.3 + 1 - 2(0.2) = 0.9 .$$

It is noted every pair of edges which are at a distance 2 have distinct degrees. Hence G is 2<sup>nd</sup> neighbourly edge irregular fuzzy graph. Total degrees of the edges are calculated below.

$$td_G(uv) = d_G(uv) + \mu(uv) = 0.5 + 0.8 = 1.3 .$$

$$td_G(vw) = d_G(vw) + \mu(vw) = 2.2 + 0.3 = 2.5 .$$

$$td_G(wx) = d_G(wx) + \mu(wx) = 0.7 + 0.4 = 1.1 .$$

$$td_G(xy) = d_G(xy) + \mu(xy) = 0.5 + 0.4 = 0.9 .$$

$$td_G(yz) = d_G(yz) + \mu(yz) = 0.6 + 0.1 = 0.7 .$$

$$td_G(zu) = d_G(zu) + \mu(zu) = 0.9 + 0.2 = 1.1 .$$

It is observed that every pair of edges which are at a distance 2 having distinct total degree. So, G is a 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph. Hence G is both 2<sup>nd</sup> neighbourly edge irregular fuzzy graph and 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph.

Remark 3.5. A 2<sup>nd</sup> neighbourly edge irregular fuzzy graph need not be 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph.

Example 3.6. Consider  $G : (\sigma, \mu)$  be a fuzzy graph such that  $G^* : (V, E)$  where  $V = \{u, v, w, x, y, z\}$  and  $E = \{uv, vw, wx, xy, yz\}$ .

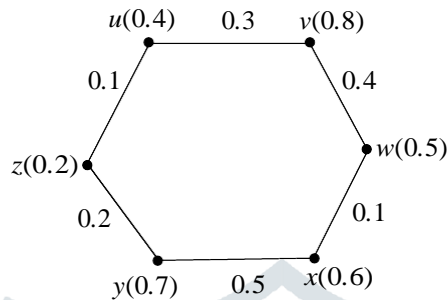


Figure.2

From Figure 2,

$$d_G(u) = 0.4; d_G(v) = 0.7; d_G(w) = 0.5; d_G(x) = 0.6;$$

$$d_G(y) = 0.7; d_G(z) = 0.3$$

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = 0.4 + 0.7 - 2(0.3) = 0.5.$$

$$d_G(vw) = d_G(v) + d_G(w) - 2\mu(vw) = 0.7 + 0.5 - 2(0.4) = 0.4.$$

$$d_G(wx) = d_G(w) + d_G(x) - 2\mu(wx) = 0.5 + 0.6 - 2(0.1) = 0.9.$$

$$d_G(xy) = d_G(x) + d_G(y) - 2\mu(xy) = 0.6 + 0.7 - 2(0.5) = 0.3.$$

$$d_G(yz) = d_G(y) + d_G(z) - 2\mu(yz) = 0.7 + 0.3 - 2(0.2) = 0.6.$$

$$d_G(zu) = d_G(z) + d_G(u) - 2\mu(zu) = 0.3 + 0.4 - 2(0.1) = 0.5.$$

It is noted every pair of edges which are at a distance 2 have distinct degrees.

Hence G is a 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.

Total degrees of the edges are calculated below.

$$td_G(uv) = d_G(uv) + \mu(uv) = 0.5 + 0.3 = 0.8.$$

$$td_G(vw) = d_G(vw) + \mu(vw) = 0.4 + 0.4 = 0.8.$$

$$td_G(wx) = d_G(wx) + \mu(wx) = 0.9 + 0.1 = 1.0.$$

$$td_G(xy) = d_G(xy) + \mu(xy) = 0.3 + 0.5 = 0.8.$$

$$td_G(yz) = d_G(yz) + \mu(yz) = 0.6 + 0.2 = 0.8.$$

$$td_G(zu) = d_G(zu) + \mu(zu) = 0.5 + 0.1 = 0.6.$$

It is observed that pair of edges yz, uv are at a distance 2 away from wx which have same total degree. So, G is not 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph. Hence G is 2<sup>nd</sup> neighbourly edge irregular fuzzy graph. But G is not 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph.

Remark 3.7. A 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph need not be 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.

Consider  $G^* : (V, E)$  where  $V = \{p, q, r, s, t\}$  and  $E = \{pq, qr, rs, st, ps, qt\}$

From Figure 3,

$$d_G(p) = 0.9; d_G(q) = 1; d_G(r) = 0.5; d_G(s) = 1.2; d_G(t) = 1.5$$

$$d_G(pq) = d_G(p) + d_G(q) - 2\mu(pq) = 0.9 + 1 - 2(0.4) = 1.1.$$

$$d_G(qr) = d_G(q) + d_G(r) - 2\mu(qr) = 1 + 0.5 - 2(0.2) = 1.1.$$

$$d_G(rs) = d_G(r) + d_G(s) - 2\mu(rs) = 0.5 + 1.2 - 2(0.3) = 1.1.$$

$$d_G(st) = d_G(s) + d_G(t) - 2\mu(st) = 1.2 + 1.3 - 2(0.7) = 1.1.$$

$$d_G(ps) = d_G(p) + d_G(s) - 2\mu(ps) = 0.9 + 1.2 - 2(0.5) = 1.1.$$

$$d_G(qt) = d_G(q) + d_G(t) - 2\mu(qt) = 1 + 1.3 - 2(0.6) = 1.1.$$

$$\text{Here, } d_G(pq) = d_G(qr) = d_G(rs) = d_G(st) = d_G(ps) = d_G(qt).$$

Hence  $G$  is not  $2^{nd}$  neighbourly edge irregular fuzzy graph. Total degrees of the edges are calculated below.

$$td_G(pq) = d_G(pq) + \mu(pq) = 1.1 + 0.4 = 1.5.$$

$$td_G(qr) = d_G(qr) + \mu(qr) = 1.1 + 0.2 = 1.3.$$

$$td_G(rs) = d_G(rs) + \mu(rs) = 1.1 + 0.3 = 1.4.$$

$$td_G(st) = d_G(st) + \mu(st) = 1.1 + 0.7 = 1.8. \quad td_G(ps) =$$

$$d_G(ps) + \mu(ps) = 1.1 + 0.5 = 1.6. \quad td_G(qt) = d_G(qt) +$$

$$\mu(qt) = 1.1 + 0.6 = 1.7.$$

$$d_G(pq) \quad d_G(qr) \quad f = d_G(rs) \quad f = d_G(st) \quad f = d_G(ps) \quad d_G(qt) \quad d_G(qt).$$

Hence  $G$  is  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.

Hence  $G$  is not  $2^{nd}$  neighbourly edge irregular fuzzy graph. But  $G$  is  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.

**Theorem 3.8.** : Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  and  $\mu$  is a constant function. If  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph. Then  $G$  is  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.

*Proof.* Assume that  $\mu$  is a constant function. Let  $\mu(uv) = c$ , for all  $uv$  in  $E$ , where  $c$  is a constant. Suppose that  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph = every pair of edges which are at a distance 2 have distinct degrees  $d_G(uv) = d_G(wx)$ , where  $uv$  and  $wx$  are pair of edge which are at a distance 2 in  $E$ .

$$\text{Now, } d_G(uv) \neq d_G(wx) \Rightarrow d_G(uv) + c \neq d_G(wx) + c$$

$$\Rightarrow d_G(uv) + \mu(uv) \neq d_G(wx) + \mu(wx)$$

$$\Rightarrow td_G(uv) \neq td_G(wx), \text{ where } uv \text{ and } wx \text{ are pair of edges which are at a distance two in } E. \text{ This is true for every vertex } v \text{ in } G. \text{ Hence } G \text{ is } 2^{nd} \text{ neighbourly edge totally irregular fuzzy graph. } \square$$

**Theorem 3.9.** Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  and  $\mu$  is a constant function. If  $G$  is  $2^{nd}$  neighbourly edge totally irregular fuzzy graph, then  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph.

*Proof.* Assume that  $\mu$  is a constant function, let  $\mu(uv) = c$  for all  $uv$  in  $E$ , where  $c$  is a constant. Suppose that  $G$  is  $2^{nd}$  neighbourly edge totally irregular fuzzy graph = every pair of edges  $s$  which are at a distance 2 have distinct total degree  $td_G(uv) \neq td_G(wx)$ , where  $uv$  and  $wx$  are pair of edges which are at a distance 2 in  $E$ . Now,  $td_G(uv) \neq td_G(wx)$

$$\Rightarrow d_G(uv) + 4\mu(uv) \neq d_G(wx) + \mu(wx)$$

$$\Rightarrow d_G(uv) + c \neq d_G(wx) + c$$

$\Rightarrow d_G(uv) \neq d_G(wx)$ , where  $uv$  and  $wx$  are pair of edges which are at a distance 2 in  $E$ . This is true for every vertex  $v$  in  $G$ . Hence  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph.  $\square$

**Remark 3.10.** : Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$ . If  $G$  is both  $2^{nd}$  neighbourly edge irregular fuzzy graph and  $2^{nd}$  neighbourly edge totally irregular fuzzy graph. Then  $\sigma$  need not be constant function.



Example 3.11. Consider  $G : (\sigma, \mu)$  be a fuzzy graph such that  $G^* : (V, E)$  where  $V = \{u, v, w, x, y, z\}$  and  $E = \{uv, vw, wx, xy, yz\}$ .

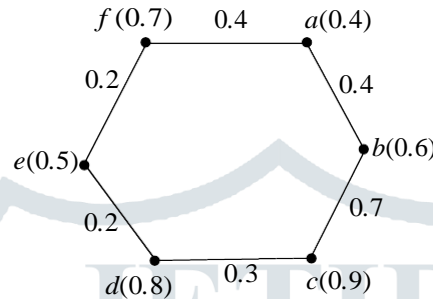


Figure.4

$$d_G(a) = 0.8, d_G(b) = 1.1, d_G(c) = 1.0, d_G(d) = 0.5, d_G(e) = 0.4, \\ d_G(f) = 0.6. d_G(ab) = 1.1, d_G(bc) = 0.7, d_G(cd) = 0.9, d_G(de) = 0.5, \\ d_G(ef) = 0.6, d_G(fa) = 0.6. td_G(ab) = 1.5, td_G(bc) = 1.4, td_G(cd) = 1.2, \\ td_G(de) = 0.7, td_G(ef) = 0.8, td_G(fa) = 1.0.$$

Theorem 3.12. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  and  $\mu$  is constant function. If  $G$  is strongly irregular fuzzy graph, then  $G$  is 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.

Proof. let  $G : (\sigma, \mu)$  be a connected graph on  $G^* : (V, E)$ . Assume that  $\mu$  is constant function. Let  $\mu(uv) = c$  for all  $uv$  in  $E$ , where  $c$  is constant. Let  $uv$  and  $wx$  are any two adjacent edges which are at a distance 2 in  $G$ . Let us suppose that  $G$  is strongly irregular fuzzy graph  $\Rightarrow$  every pair of vertices  $G$  having distinct degrees

$$\Rightarrow d_G(u) \neq d_G(v) \neq d_G(w) \neq d_G(x) \\ \Rightarrow d_G(u) + d_G(v) - 2c \neq d_G(w) + d_G(x) - 2c \\ \Rightarrow d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(w) + d_G(x) - 2\mu(wx) \\ \Rightarrow d_G(uv) \neq d_G(wx)$$

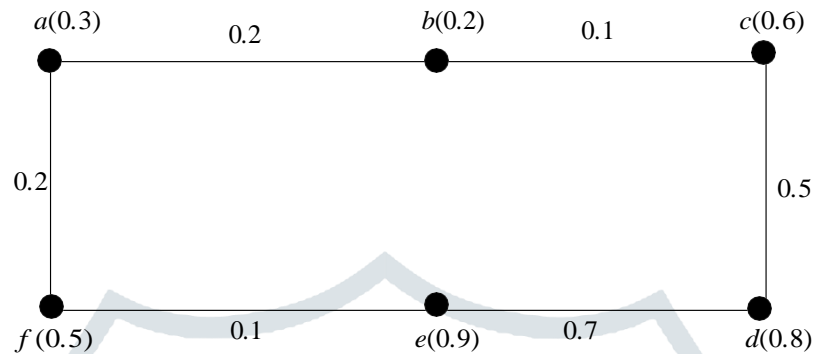
every pair of adjacent edges which are at a distance 2 have distinct degrees. Hence  $G$  is a 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.  $\square$

Theorem 3.13. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  and  $\mu$  is constant function. If  $G$  is strongly irregular fuzzy graph, then  $G$  is 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph.

Proof. proof is similar to the above theorem 3.9.  $\square$

Remark 3.14. Converse of the above theorems need not be true.

Example 3.15. Consider a fuzzy graph on  $G^* (V, E)$ .



From Figure 5, Figure.5  
 $d_G(a) = 0.4; d_G(b) = 0.3; d_G(c) = 0.6; d_G(d) = 1.2; d_G(e) = 0.8;$   
 $d_G(f) = 0.3$ . Here, G is not strongly irregular fuzzy graph.  
 $d_G(ab) = 0.3. td_G(ab) = 0.5. d_G(bc) = 0.7. td_G(bc) = 0.8. d_G(cd) = 0.8.$   
 $td_G(cd) = 1.3. d_G(de) = 0.6. td_G(de) = 1.3. d_G(ef) = 0.9. td_G(ef) = 1.0.$   
 $d_G(fg) = 0.3. td_G(fg) = 0.5.$

It is noted that every pair of edges which are at a distance 2 have distinct degrees. Hence G is both 2<sup>nd</sup> neighbourly edge irregular fuzzy graph and 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph. But G is not strongly irregular fuzzy graph.

Theorem 3.16. Every strongly irregular fuzzy graph is 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.

Theorem 3.17. Every strongly irregular fuzzy graph is 2<sup>nd</sup> neighbourly edge totally irregular fuzzy graph.

### 4 Neighbourly Edge Irregularity on a Path, Cycle With Some Specific Function

Theorem 4.1. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  a path on  $3m+1$  vertices. If consecutive three edges takes same membership values, then G is 2<sup>nd</sup> neighbourly edge irregular fuzzy graph.

Proof. Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  a path on  $3m+1$  vertices. If consecutive three edges takes same membership values,

$$\text{then } \mu(e_i) = \begin{cases} c_1 & \text{if } i = 1, 4, 7, \dots, 3m-2 \\ c_2 & \text{if } i = 2, 5, 8, \dots, 3m-1 \\ c_3 & \text{if } i = 3, 6, 9, \dots, 3m \end{cases}$$

For  $i=1, 3m$   
 $de_i = c_1 + c_3$   
 For  $i = 2, 5, 8, \dots, 3m - 1$   
 $de_i = c_2 + c_1$   
 For  $i = 3, 6, 9, \dots, 3m$



$$de_i = c_3 + c_2$$

For  $i = 4, 7, 10, \dots, 3m + 1$

We observed that every two edges which are at a distance 2 away from each other have distinct degrees. Hence  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph.  $\square$

Remark 4.2. *The above theorem does not hold for  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.*

Theorem 4.3. : *Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  a cycle on  $3m$  vertices. If consecutive three edges takes same membership values, then  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph.*

*Proof.* Let  $G : (\sigma, \mu)$  be a connected fuzzy graph on  $G^* : (V, E)$  a cycle on  $3m$  vertices. If consecutive three edges takes same membership values, then

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i = 1, 4, 7, \dots, 3m - 2 \\ c_2 & \text{if } i = 2, 5, 8, \dots, 3m - 1 \\ c_3 & \text{if } i = 3, 6, 9, \dots, 3m \end{cases}$$

$$de_i = c_3 + c_2$$

For  $i = 1, 4, 7, \dots, 3m - 2$

$$de_i = c_1 + c_3$$

For  $i = 2, 5, 8, \dots, 3m - 1$

$$de_i = c_2 + c_1$$

For  $i = 3, 6, 9, \dots, 3m$

We observed that every two edges which are at a distance 2 away from each other have distinct degrees. Hence  $G$  is  $2^{nd}$  neighbourly edge irregular fuzzy graph.  $\square$

Remark 4.4. *The above theorem does not hold for  $2^{nd}$  neighbourly edge totally irregular fuzzy graph.*

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