SOME TRANSLATIONS IN (T, S)-INTUITIONISTIC FUZZY SUBFIELD OF A FIELD

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ABSTRACT: In this paper, some translations of (T, S)-intuitionistic fuzzy subfield of a field are given. These translations are giving a new algebraic structure and this type of translations is very useful for convert to the one intuitionistic fuzzy algebraic structure to another intuitionistic fuzzy algebraic structure.

KEY WORDS: (T, S)- norm, fuzzy subset, intuitionistic fuzzy subset, (T, S)-intuitionistic fuzzy subfield, some translations.

INTRODUCTION: After the introduction of fuzzy sets by L.A.Zadeh[7], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[1], as a generalization of the notion of fuzzy set. Hur.K, et.al.[3] have given the idea about the (T, S)-intuitionistic fuzzy ideals of a ring. Jianming Zhan[4] introduced the properties of fuzzy left h-ideals in hemiring with t-norms. Jun.Y.B, et.al.[5] gave the idea in intuitionistic nil radicals of (T, S)-intuitionistic fuzzy ideals and euclidean (T, S)-intuitionistic fuzzy ideals in rings. Vasu.M, et.al[6] have introduced the intuitionistic L-fuzzy subfields of a field. The above papers was useful for developing this paper. In this paper, some translations theorem of (T, S)-intuitionistic fuzzy subfield of a field are given.

1.PRELIMINARIES:

Definition 1.1[3]. A (T, S)-norm is a binary operations $T: [0, 1]\times[0, 1] \rightarrow [0, 1]$ and $S: [0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) $T(0, x) = 0$, $T(1, x) = x$ (boundary condition)
(ii) $T(x, y) = T(y, x)$ (commutativity)
(iii) $T(x, T(y, z)) = T(T(x,y), z)$(associativity)
(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$(monotonicity).
(v) $S(0, x) = x$, $S(1, x) = 1$ (boundary condition)
(vi) $S(x, y) = S(y, x)$(commutativity)
(vii) $S(x, S(y, z)) = S(S(x, y), z)$ (associativity)
(viii) if $x \leq y$ and $w \leq z$, then $S(x, w) \leq S(y, z)$(monotonicity).

Definition 1.2[7]. Let X be a non-empty set. A **fuzzy subset A** of X is a function $A: X\rightarrow[0, 1]$.

Definition 1.3[1]. An **intuitionistic fuzzy subset**(**IFS**) A of a set X is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x\in X \}$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 1.4. Let (F, +, ·) be a field. An intuitionistic fuzzy subset A of F is said to be a **(T, S)-intuitionistic fuzzy subfield** of F if the following conditions are satisfied:

(i) $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F,
(ii) $\mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y))$, for all x and $y\neq e$ in F,
(iii) $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in F,
(iv) $\nu_A(xy^{-1}) \leq S(\nu_A(x), \nu_A(y))$, for all x and $y\neq e$ in F.

Example 1.5. Consider the field Z_5 = { 0, 1, 2, 3, 4 } with addition modulo 5 and multiplication modulo 5 operations. Then A ={ ⟨ 0, 0.7, 0.1 ⟩, ⟨1, 0.5, 0.4⟩, ⟨2, 0.5, 0.4⟩, ⟨3, 0.5, 0.4⟩, ⟨4, 0.5, 0.4⟩ } is a (T, S)-intuitionistic fuzzy subfield of Z_5.

Definition 1.6[2]. Let A and B be intuitionistic fuzzy subsets of X. Then the following translations and operations are defined as

(i) $\Lambda(A) = \{ \langle x, \min\{ ½, \mu_A(x) \}, \max\{ ½, \nu_A(x) \} \rangle$ / for all $x \in X\}$.

(ii) $\Theta(A) = \{ \langle x, \max\{ ½, \mu_A(x) \}, \min\{ ½, \nu_A(x) \} \rangle$ / for all $x \in X\}$.

(iii) $Q_{\alpha,\beta}(A) = \{\langle x, \min\{\alpha, \mu_A(x)\}, \max\{\beta, \nu_A(x)\}\rangle$/for all $x \in X$, $\alpha, \beta \in [0, 1]$ and $\alpha+\beta \leq 1\}$.

(iv) $P_{\alpha,\beta}(A) = \{\langle x, \max\{\alpha, \mu_A(x)\}, \min\{\beta, \nu_A(x)\}\rangle$/for all $x \in X$, $\alpha, \beta \in [0, 1]$ and $\alpha+\beta \leq 1\}$.

(v) $G_{\alpha,\beta}(A) = \{ \langle x, \alpha\mu_A(x), \beta\, \nu_A(x) \rangle$ / for all $x \in X$ and $\alpha, \beta \in [0, 1]$ }.

(vi) $A \cap B = \{ \langle x, \min\{\mu_A(x), \mu_B(x) \}, \max\{\nu_A(x), \nu_B(x) \} \rangle$ / for all $x \in X\}$

2- PROPERTIES:

Theorem 2.1 If A is a (T, S)-intuitionistic fuzzy subfield of a field (F, +, ·), then ΛA is a (T, S)-intuitionistic fuzzy subfield of F.

Proof. For every x, y∈F, then $\mu_{\Lambda A}(x-y) = \min\{ ½, \mu_A(x-y) \} \geq \min\{ ½, T(\mu_A(x), \mu_A(y)) \} \geq T(\min\{½, \mu_A(x)\}, \min\{½, \mu_A(y)\}) = T(\mu_{\Lambda A}(x), \mu_{\Lambda A}(y))$ for all x, y∈F. For every x, y∈F, then $\nu_{\Lambda A}(x-y) = \max\{ ½, \nu_A(x-y) \} \leq \max\{ ½, S(\nu_A(x), \nu_A(y)) \} \leq S(\max\{½, \nu_A(x)\}, \max\{½, \nu_A(y)\}) = S(\nu_{\Lambda A}(x), \nu_{\Lambda A}(y))$ for all x, y∈F. For every x and y≠ e in F, then $\mu_{\Lambda A}(xy^{-1}) = \min\{ ½, \mu_A(xy^{-1}) \} \geq \min\{ ½, T(\mu_A(x), \mu_A(y))\} \geq T(\min\{½, \mu_A(x)\}, \min\{½, \mu_A(y)\}\} = T(\mu_{\Lambda A}(x), \mu_{\Lambda A}(y))$ for all x, y≠ e ∈F. For every x and y≠ e in F, then $\nu_{\Lambda A}(xy^{-1}) = \max\{½, \nu_A(xy^{-1})\} \leq \max\{½, S(\nu_A(x), \nu_A(y))\} \leq S(\max\{½, \nu_A(x)\}, \max\{½, \nu_A(y)\}) = S(\nu_{\Lambda A}(x), \nu_{\Lambda A}(y))$ for all x, y ≠ e∈F. Hence ΛA is a (T, S)-intuitionistic fuzzy subfield of F.

Theorem 2.2 If A is a (T, S)-intuitionistic fuzzy subfield of a field (F, +, ·), then ΘA is a (T, S)-intuitionistic fuzzy subfield of F.

Proof. For every x, y∈F, then $\mu_{\Theta A}(x-y) = \max\{½, \mu_A(x-y)\} \geq \max\{½, T(\mu_A(x), \mu_A(y))\} \geq T(\max\{½, \mu_A(x)\}, \max\{½, \mu_A(y)\}) = T(\mu_{\Theta A}(x), \mu_{\Theta A}(y))$ for all x, y∈F. For every x, y∈F, then $\nu_{\Theta A}(x-y) = \min\{½, \nu_A(x-y)\} \leq \min\{½, S(\nu_A(x), \nu_A(y))\} \leq S(\min\{½, \nu_A(x)\}, \min\{½, \nu_A(y, q)\}) = S(\nu_{\Theta A}(x), \nu_{\Theta A}(y)\}$ for all x, y∈F. For every x, y≠ e∈F, then $\mu_{\Theta A}(xy^{-1}) = \max\{½, \mu_A(xy^{-1})\} \geq \max\{½, T(\mu_A(x), \mu_A(y))\} \geq T(\max\{½, \mu_A(x)\}, \max\{½, \mu_A(y) \}) = T(\mu_{\Theta A}(x), \mu_{\Theta A}(y))$ for all x, y ≠ e∈F. For every x, y≠ e∈F, then $\nu_{\Theta A}(xy^{-1}) = \min\{ ½, \nu_A(xy^{-1}) \} \leq \min\{ ½, S(\nu_A(x), \nu_A(y)) \} \leq S(\min\{½, \nu_A(x)\}, \min\{½, \nu_A(y) \}) = S(\nu_{\Theta A}(x), \nu_{\Theta A}(y))$ for all x, y ≠ e∈F. Hence ΘA is a (T, S)-intuitionistic fuzzy subfield of F.

Theorem 2.3. If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then $Q_{\alpha,\beta}(A)$ is a (T, S)-intuitionistic fuzzy subfield of F.

Proof. For every x, y∈F and α, β ∈[0, 1] and α+β ≤ 1, then $\mu_{Q_{(\alpha,\beta)}(A)}(x-y) = \min\{\alpha, \mu_A(x-y) \} \geq \min\{\alpha, T(\mu_A(x), \mu_A(y)) \} \geq T(\min\{\alpha, \mu_A(x)\}, \min\{\alpha, \mu_A(y)\}) = T(\mu_{Q_{(\alpha,\beta)}(A)}(x), \mu_{Q_{(\alpha,\beta)}(A)}(y))$ for all x, y∈F. And $\nu_{Q_{(\alpha,\beta)}(A)}(x-y) = \max\{\beta, \nu_A(x-y)\} \leq \max\{\beta, S(\nu_A(x), \nu_A(y))\} \leq S(\max\{\beta, \nu_A(x)\}, \max\{\beta, \nu_A(y)\}) = S(\nu_{Q_{(\alpha,\beta)}(A)}(x), \nu_{Q_{(\alpha,\beta)}(A)}(y))$ for all x, y∈E. For every x, y≠ e∈F and α, β ∈[0, 1] and α+β ≤ 1, then $\mu_{Q_{(\alpha,\beta)}(A)}(xy^{-1}) = \min\{\alpha, \mu_A(xy^{-1}) \} \geq \min\{\alpha, T(\mu_A(x), \mu_A(y)) \} \geq T(\min\{\alpha, \mu_A(x)\}, \min\{\alpha, \mu_A(y)\}) = T(\mu_{Q_{(\alpha,\beta)}(A)}(x), \mu_{Q_{(\alpha,\beta)}(A)}(y))$ for all x, y≠ e∈F. And $\nu_{Q_{(\alpha,\beta)}(A)}(xy^{-1}) = \max\{\beta, \nu_A(xy^{-1})\} \leq \max\{\beta, S(\nu_A(x), \nu_A(y))\} \leq S(\max\{\beta, \nu_A(x)\}, \max\{\beta, \nu_A(y)\}) = S(\nu_{Q_{(\alpha,\beta)}(A)}(x), \nu_{Q_{(\alpha,\beta)}(A)}(y))$ for all x, y ≠ e∈F. Hence $Q_{\alpha,\beta}(A)$ is a (T, S)-intuitionistic fuzzy subfield of F.

Theorem 2.4 If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then $P_{\alpha,\beta}(A)$ is a (T, S)-intuitionistic fuzzy subfield of F.

Proof. For every x, y∈F and α, β ∈[0, 1] and α+β ≤ 1, then $\mu_{P_{(\alpha,\beta)}(A)}$(x−y) = max{α, μ_A(x−y)}≥ max{α, T(μ_A(x), μ_A(y))} ≥ T(max{α, μ_A(x)}, max{α, μ_A(y)}) = T($\mu_{P_{(\alpha,\beta)}(A)}$(x), $\mu_{P_{(\alpha,\beta)}(A)}$(y)) for all x, y∈F. And $\nu_{P_{(\alpha,\beta)}(A)}$(x−y) = min{β, ν_A(x−y)} ≤ min{β, S(ν_A(x), ν_A(y))}≤ S(min{β, ν_A(x)}, min{β, ν_A(y)}) = S($\nu_{P_{(\alpha,\beta)}(A)}$(x), $\nu_{P_{(\alpha,\beta)}(A)}$(y)} for all x, y∈F. For every x, y≠ e∈F, then $\mu_{P_{(\alpha,\beta)}(A)}$($xy^{-1}$) = max{α, μ_A(xy^{-1})} ≥ max { α, T(μ_A(x), μ_A(y))} ≥ T(max{α, μ_A(x)}, max{α, μ_A(y)}) = T($\mu_{P_{(\alpha,\beta)}(A)}$(x), $\mu_{P_{(\alpha,\beta)}(A)}$(y)) for all x, y≠ e∈F. For every x, y≠ e∈F, then $\nu_{P_{(\alpha,\beta)}(A)}$($xy^{-1}$) = min{β, ν_A(xy^{-1})} ≤ min{β, S(ν_A(x), ν_A(y))}≤ S(min{β, ν_A(x)}, min{β, ν_A(y)}) = S($\nu_{P_{(\alpha,\beta)}(A)}$(x), $\nu_{P_{(\alpha,\beta)}(A)}$(y)) for all x, y≠ e∈F. Hence $P_{\alpha,\beta}(A)$ is a (T, S)-intuitionistic fuzzy subfield of F.

Theorem 2.5. If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then $G_{\alpha,\beta}$(A) is a (T, S)-intuitionistic fuzzy subfield of F.

Proof. For every x, y∈F and α, β ∈[0, 1] and α+β ≤ 1, then $\mu_{G_{(\alpha,\beta)}(A)}$(x−y) = α μ_A(x−y) ≥ α (T(μ_A(x), μ_A(y))) = T(αμ_A(x), αμ_A(y)) = T($\mu_{G_{(\alpha,\beta)}(A)}$(x), $\mu_{G_{(\alpha,\beta)}(A)}$(y)) for all x, y∈F. And $\nu_{G_{(\alpha,\beta)}(A)}$(x−y) = β ν_A(x−y) ≤ β(S(ν_A(x), ν_A(y))) = S(βν_A(x), βν_A(y)) = S($\nu_{G_{(\alpha,\beta)}(A)}$(x), $\nu_{G_{(\alpha,\beta)}(A)}$(y)) for all x, y∈F. For every x, y≠ e∈F, then $\mu_{G_{(\alpha,\beta)}(A)}$($xy^{-1}$) = α μ_A(xy^{-1}) ≥ α (T(μ_A(x), μ_A(y)) = T(αμ_A(x), α μ_A(y)) = T($\mu_{G_{(\alpha,\beta)}(A)}$(x), $\mu_{G_{(\alpha,\beta)}(A)}$(y)) for all x, y≠ e∈F. And $\nu_{G_{(\alpha,\beta)}(A)}$($xy^{-1}$) = β ν_A(xy^{-1}) ≤ β(S(ν_A(x), ν_A(y))) = S(βν_A(x), βν_A(y)) = S($\nu_{G_{(\alpha,\beta)}(A)}$(x), $\nu_{G_{(\alpha,\beta)}(A)}$(y)) for all x, y≠ e∈F. Hence $G_{\alpha,\beta}$(A) is a (T, S)-intuitionistic fuzzy subfield of F.

Theorem 2.6. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field F, then *A*∩*B* is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.7. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field F, then Θ(*A*∩*B*) = Θ(*A*) ∩Θ(*B*) is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.8. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field F, then Λ(*A*∩*B*) = Λ(*A*) ∩ Λ(*B*) is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.9. If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then Θ(Λ(*A*)) = Λ(Θ(*A*)) is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.10. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field G, then $P_{\alpha,\beta}(A\cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$ is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.11. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field F, then $Q_{\alpha,\beta}(A\cap B) = Q_{\alpha,\beta}(A) \cap Q_{\alpha,\beta}(B)$ is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.12. If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then $P_{\alpha,\beta}(Q_{\alpha,\beta}(A)) = Q_{\alpha,\beta}(P_{\alpha,\beta}(A))$ is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.13. If *A* and *B* are (T, S)-intuitionistic fuzzy subfields of a field F, then $G_{\alpha,\beta}(A\cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$ is also a (T, S)-intuitionistic fuzzy subfield of F.

Corollary 2.14. If A is a (T, S)-intuitionistic fuzzy subfield of a field F, then $Q_{\alpha,\beta}$(◊(*A*)) = ◊($Q_{\alpha,\beta}$(*A*)) is also a (T, S)-intuitionistic fuzzy subfield of F.

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