Fuzzy M -continuity Mappings in ˆ Sostak's Fuzzy Topological Spaces

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Abstract

We introduce and investigate some new classes of mappings called fuzzy M -continuous, fuzzy θ-continuous and fuzzy θ-semicontinuous to the fuzzy topological spaces in $\hat{S}$ ostakÿ sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy continuous, fuzzy θ-semicontinuous, fuzzy θ-continuous, fuzzy δ- semicontinuous, fuzzy δ-precontinuous, fuzzy a-continuous, fuzzy M - continuous, fuzzy e-continuous and fuzzy e^*-continuous mappings.

Keywords and phrases: fuzzy continuous, fuzzy θ-semicontinuous, fuzzy θ-continuous, fuzzy δ-semicontinuous, fuzzy δ-precontinuous, fuzzy a-continuous, fuzzy M - continuous, fuzzy e-continuous and fuzzy $e*$-continuous mappings.
AMS (2000) subject classification: 54A40.

1. Introduction

$\hat{S}$ ostak [30] introduced the fuzzy topology as an extension of Chang's fuzzy topology[4]. It has been developed in many directions [12,13,27]. Weaker forms of fuzzy continuity between fuzzy topological spaces have been considered by many authors [2, 3, 5, 9, 11, 20, 23] using the concepts of fuzzy semi-open sets[2] , fuzzy regular open sets[2], fuzzy preopen sets, fuzzy strongly semiopen sets [3] , fuzzy γ-open sets[11], fuzzy δ-semiopen sets[1] , fuzzy δ -preopen sets[1], fuzzy semi δ-preopen sets [34] and fuzzy e-open sets [29] Ganguly and Saha [10] introduced the notions of fuzzy δ-cluster points in Chang's [4] fuzzy topological spaces. Kim and Park [14] introduced r-δ-cluster points and δ-closure operators in $\hat{S}$ ostak's fuzzy topological spaces.

It is a good extension of the notions of Ganguly and Saha[10]. Park et al. [14] introduced the fuzzy semi-preopen. In 2008, the initiations of e-open sets, e^*-open sets and a-open sets in topological spaces are due to Erdal Ekici[[7],[8]]. Sobana et al. [33] defined ι-fuzzy e-open sets in $\hat{S}$ ostak's fuzzy topological space. Vadivel et al. [36] introduced ι-fuzzy e^*-open sets in $\hat{S}$ ostak's fuzzy topological space. In 1968, Velicko studied θ-open sets [35] and δ-open sets for the purpuse of investigating the characherizations of H-closed topological spaces. semi-open set [17] were initiated by Levine in 1963. In 1993, Raychaudhuri and Mukherjee defined δ-preopen sets[26]. In 1997, δ-semiopen sets was obtained by Park [19] and θ -semi-open sets were obtained by Caldas in 2008[6]. Shafei introduced fuzzy θ-closed [31] and fuzzy θ-open sets in 2006. Maghrabi et al.[21] introduced the notion of M -open sets in topological spaces in 2011.

In this paper fuzzy M-continuous, fuzzy θ-continuous and fuzzy θ-semicontinuous to the fuzzy topological spaces in $\hat{S}$ostak's sense are introduced and some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy continuous, fuzzy θ-semicontinuous, fuzzy θ-continuous, fuzzy δ- semicontinuous, fuzzy δ-precontinuous, fuzzy a-continuous, fuzzy M- continuous, fuzzy e-continuous and fuzzy e^*-continuous mappings.

1. Preliminaries

Throughout this article, we denote nonempty sets by X, Y etc., $I = [0,1]$ and $I_0 = (0,1]$. For $\alpha \in I, \overline{\alpha}(x) = \alpha, \quad \forall x \in X$. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that

$$x_t(y) = \begin{cases} t & \text{if y is equal to x} \\ 0 & \text{if y is not equal to x.} \end{cases}$$

Let $Pt(X)$ denote the set of all fuzzy points in X. A fuzzy point $x_t \in \mu$ iff $t < \mu(x)$. $\mu \in I^X$ is quasi-coincident with ν, denoted by $\mu q \nu$, if $\exists \quad x \in X$ such that $\mu(x) + \nu(x) > 1$.

If μ is not quasi-coincident with ν, we denoted $\mu \bar{q} \nu$. If A is a subset of X, we define the characteristic function χ_A on X by $\chi_A(x) = \{1$ x A , 0if x A. All notations and definitions will be standard in the fuzzy set theory.

Lemma 1.1 *[30] Consider X be a nonempty set and $\mu, \nu \in I^X$. Then*

(i) $\mu q \nu$ iff there exists $x_t \in \mu$ such that $x_t q \nu$.

(ii) $\mu q \nu$, then $\mu \wedge \nu \neq \overline{0}$.

(iii) $\mu \bar{q} \nu$ iff $\mu \leq \overline{1} - \nu$.

(iv) $\mu \leq \nu$ iff $x_t \in \mu$ implies $x_t \in \nu$ iff $x_t q \mu$ implies $x_t q \nu$ implies $x_t \bar{q} \mu$.

(v) $x_t \bar{q} \bigvee_{i \in \mu} \nu_i$ iff there exists $i_0 \in \mu$ such that $x_t \bar{q} \nu_{i_0}$.

Definition 1.1 *[30] A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:*

(1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$,

(2) $\tau(\bigvee_{i \in \Gamma} \nu_i) \geq \bigwedge_{i \in \Gamma} \tau(\nu_i)$, for any $\{\nu_i\}_{i \in \Gamma} \subset I^X$,

(3) $\tau(\nu_1 \wedge \nu_2) \geq \tau(\nu_1) \wedge \tau(\nu_2)$, for any $\nu_1, \nu_2 \in I^X$.

The pair (X, τ) is called a fuzzy topological space or $\hat{S}$ostak's fuzzy topological space or smooth topological space (for short, fts, sfts, sts).

Remark 1.1 *[25] Let (X, τ) be a sfts. Then, for every $\iota \in I_0$, $\tau_r = \{\nu \in I^X : \tau(\nu) \geq \iota\}$ is a Change's fuzzy topology on X.*

Theorem 1.1 *[27] Let (X, τ) be a sfts. Then for each $\mu \in I^X, \iota \in I_0$, we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows:*

$C_\tau(\mu, \iota) = \bigwedge \{\nu \in I^X : \mu \leq \nu, \tau(\overline{1} - \nu) \geq \iota\}$.

For $\mu, \nu \in I^X$ and $\iota, s \in I_0$, the operator C_τ satisfies the following conditions:

(1) $C_\tau(\bar{0},\iota)=\bar{0}$,

(2) $\mu\le C_\tau(\mu,\iota)$,

(3) $C_\tau(\mu,\iota)\vee C_\tau(\nu,\iota)=C_\tau(\mu\vee\nu,\iota)$,

(4) $C_\tau(\mu,\iota)\le C_\tau(\mu,s)$ if $\iota\le s$,

(5) $C_\tau(C_\tau(\mu,\iota),\iota)=C_\tau(\mu,\iota)$.

Theorem 1.2 *[27] Let (X,τ) be a sfts. Then for each $\iota\in I_0,\mu\in I^X$ we define an operator $I_\tau: I^X\times I_0\to I^X$ as follows:*

$I_\tau(\mu,\iota)=\bigvee\{\nu\in I^X:\mu\ge\nu,\tau(\nu)\ge\iota\}$.

For $\mu,\nu\in I^X$ and $\iota,s\in I_0$, the operator I_τ satisfies the following conditions:

(i) $I_\tau(\bar{1},\iota)=\bar{1}$,

(ii) $\mu\ge I_\tau(\mu,\iota)$,

(iii) $I_\tau(\mu,\iota)\wedge I_\tau(\nu,\iota)=I_\tau(\mu\wedge\nu,\iota)$,

(iv) $I_\tau(\mu,\iota)\le I_\tau(\mu,s)$ if $s\le\iota$,

(v) $I_\tau(I_\tau(\mu,\iota),\iota)=I_\tau(\mu,\iota)$,

(vi) $I_\tau(\bar{1}-\mu,\iota)=\bar{1}-C_\tau(\mu,\iota)$ and $C_\tau(\bar{1}-\mu,\iota)=\bar{1}-I_\tau(\mu,\iota)$

Definition 1.2 *[15] Let (X,τ) be a sfts. Then for each $\nu\in I^X, x_t\in P_t(X)$ and $\iota\in nI_0$, ν is called*

(i) ι-open Q_τ-neighbourhood of x_t if $x_t q\nu$ with $\tau(\nu)\ge\iota$.

(ii) ι-open R_τ-neighbourhood of x_t if $x_t q\nu$ with $\nu=I_\tau(C_\tau\ (\mu,\iota),\iota)$.

We denote $Q_\tau(x_t,\iota)=\{\nu\in I^X: x_t q\nu,\tau(\nu)\ge\iota\}$, $R_\tau(x_t,\iota)\ =\{\nu\in I^X: x_t q\nu=I_\tau(C_\tau(\mu,\iota),\iota)\}$.

Definition 1.3 *[15] Let (X,τ) be a sfts. Then for each $\mu\in I^X, x_t\in P_t(X)$ and $\iota\in nI_0$, x_t is called*

(i) ι-τ cluster point of μ if for every $\nu\in Q_\tau(x_t,\iota)$, we have $\nu q\mu$.

(ii) ι-δ cluster point of μ if for every $\nu\in R_\tau(x_t,\iota)$, we have $\nu q\mu$.

(iii) An δ-closure operator is a mapping $D_\tau: I^X\times I\to I^X$ defined as follows: $\delta C_\tau(\mu,\iota)$ or $D_\tau(\mu,\iota)=\bigvee\{x_t\in P_t(X): x_t$ is r-δ-cluster point of $\mu\}$

Definition 1.4 *[17] Let (X,τ) be a sfts. For $\mu,\nu\in I^X$ and $\iota\in I_0$, μ is called an*

(i) ι-fuzzy δ-semiopen (resp. ι-fuzzy δ-semiclosed)**Error! Reference source not found.** set if $\mu\le C_\tau(\delta I_\tau(\mu,\iota),\iota)$ (resp. $I_\tau(\delta C_\tau(\mu,\iota),\iota)\le\mu$).

(ii) ι-fuzzy δ-preopen (resp. ι-fuzzy δ-preclosed)**Error! Reference source not found.** set if $\mu\le I_\tau(\delta C_\tau(\mu,\iota),\iota)$ (resp. $C_\tau(\delta I_\tau(\mu,\iota),\iota)\le\mu$).

(iii) ι-fuzzy a-open (resp. ι-fuzzy a-closed)**Error! Reference source not found.** set if $\mu\le I_\tau(C_\tau(\delta I_\tau(\mu,\iota),\iota),\iota)$

(resp. $C_\tau(I_\tau(\delta C_\tau(\mu,\iota),\iota),\iota)\le\mu$).

(iv) ι-fuzzy e-open (resp. ι-fuzzy e-closed)**Error! Reference source not found.** set if $\mu\le C_\tau(\delta I_\tau(\mu,\iota),\iota)\vee I_\tau(\delta C_\tau(\mu,\iota),\iota)$
(resp. $C_\tau(\delta I_\tau(\mu,\iota),\iota)\wedge I_\tau(\delta C_\tau(\mu,\iota),\iota)\le\mu$).

(v) ι-fuzzy e^*-open (resp. ι-fuzzy e^*-closed)**Error! Reference source not found.** set if $\mu\le C_\tau(I_\tau(\delta C_\tau(\mu,\iota),\iota),\iota)$
(resp. $I_\tau(C_\tau(\delta I_\tau(\mu,\iota),\iota),\iota)\le\mu$).

(v) ι-fuzzy semiopen (resp. ι-fuzzy semi-closed) **Error! Reference source not found.** set if $\mu\le C_\tau(I_\tau(\mu,\iota),\iota)$
(resp. $I_\tau(C_\tau(\mu,\iota),\iota)\le\mu$).

Definition 1.5 *[37]*

(i) ι-fuzzy θ-interior (resp. ι-fuzzy θ-semi-interior and ι-fuzzy θ-pre-interior) of a subset μ in a sfts (X,τ), $\forall\,\iota\in I_0$, denoted by $\theta I_\tau(\mu,\iota)$ (resp. $\theta sI_\tau(\mu,\iota)$ and $\theta pI_\tau(\mu,\iota)$) defined as $\theta I_\tau(\mu,\iota)=\bigvee\{I_\tau(\nu):\mu\ge\nu,\tau(\bar{1}-\nu)\ge\iota\}$; $\theta sI_\tau(\mu,\iota)=\bigvee\{sI_\tau(\nu):\mu\ge\nu,\nu$ is r-fsc$\}$; $\theta pI_\tau(\mu,\iota)=\bigvee\{pI_\tau(\nu):\mu\ge\nu,\nu$ is r-fpc $\}$.

(ii) ι-fuzzy θ-closure (resp. ι-fuzzy θ-semi-closure and ι-fuzzy θ-pre-closure) of a subset μ in a sfts (X,τ), $\forall\,\iota\in I_0$, denoted by $\theta C_\tau(\mu,\iota)$ (resp. $\theta sC_\tau(\mu,\iota)$ and $\theta pC_\tau(\mu,\iota)$) defined as $\theta C_\tau(\mu,\iota)=\bigwedge\{C_\tau(\nu):\mu\le\nu,\tau(\nu)\ge\iota\}$; $\theta sC_\tau(\mu,\iota)=\bigwedge\{sC_\tau(\nu):\mu\le\nu,\nu$ is ι-fso $\}$; $\theta pC_\tau(\mu,\iota)=\bigwedge\{pC_\tau(\nu):\mu\le\nu,\nu$ is ι-fpo $\}$.

Definition 1.6 *[37] Let (X,τ) be a sfts. For $\mu,\nu\in I^X$ and $\iota\in I_0$, μ is called an*

(i) ι-fuzzy θ-open (resp. ι-fuzzy θ-closed) set if $\mu=\theta I_\tau(\mu,\iota)$ (resp. $\mu=\theta C_\tau(\mu,\iota)$).

(ii) ι-fuzzy θ-semiopen (resp. ι-fuzzy θ-semiclosed) set if $\mu\le C_\tau(\theta I_\tau(\mu,\iota),\iota)$ (resp. $I_\tau(\theta C_\tau(\mu,\iota),\iota)\le\mu$).

(iii) ι-fuzzy θ-preopen (resp. ι-fuzzy θ-preclosed) set if $\mu\le I_\tau(\theta C_\tau(\mu,\iota),\iota)$ (resp. $C_\tau(\theta I_\tau(\mu,\iota),\iota)\le\mu$).

The family of all ι-fuzzy θ-open (resp. ι-fuzzy θ-closed), ι-fuzzy θ-semiopen, (resp. ι-fuzzy θ-semiclosed), ι-fuzzy θ-preopen (resp. ι-fuzzy θ-preclosed) sets will be denoted by ι-fθo (resp. ι-fθc), ι-fθso (resp. ι-fθsc), ι-fθpo (resp. ι-fθpc) sets.

Lemma 1.2 *[37] Let $\mu,\nu\in I^X$ and $\iota\in I_0$ in a sfts (X,τ), then*

(i) μ is ι-fuzzy θ-open iff $\mu=\theta I_\tau(\mu,\iota)$.

(ii) If $\mu<\nu$, then $\theta I_\tau(\mu,\iota)<\theta I_\tau(\nu,\iota)$.

(iii) $\theta I_\tau(\theta I_\tau(\mu,\iota))<\theta I_\tau(\mu,\iota)$.

(iv) For any subset μ of X, $\mu\le C_\tau(\mu,\iota)\le\delta C_\tau(\mu,\iota)\le\theta C_\tau(\mu,\iota)$ (resp. $\theta I_\tau(\mu,\iota)\le\delta I_\tau(\mu,\iota)\le I_\tau(\mu,\iota)\le\mu$.

(v) $\bar{1}-(\theta I_\tau(\mu,\iota))=\theta C_\tau(\bar{1}-\mu,\iota)$.

(vi) $\bar{1}-(\theta C_\tau(\mu,\iota))=\theta I_\tau(\bar{1}-\mu,\iota)$.

(vii) $\theta I_\tau(\mu\wedge\nu,\iota)=\theta I_\tau(\mu,\iota)\wedge\theta I_\tau(\nu,\iota)$ and $\theta I_\tau(\mu,\iota)\vee\theta I_\tau(\nu,\iota)<\theta I_\tau(\mu\vee\nu,\iota)$.

(viii) $\theta C_\tau(\mu\wedge\nu,\iota)=\theta C_\tau(\mu,\iota)\wedge\theta C_\tau(\nu,\iota)$ and $\theta C_\tau(\mu\vee\nu,\iota)=\theta C_\tau(\mu,\iota)\vee\theta C_\tau(\nu,\iota)$.

Proposition 1.1 *[37] Let $\mu\in I^X$ and $\iota\in I_0$ in a sfts (X,τ), then*

(i) $\theta sC_\tau(\mu,\iota)=\mu\vee I_\tau(\theta C_\tau(\mu,\iota),\iota)$ and $\theta sI_\tau(\mu,\iota)=\mu\wedge C_\tau(\theta I_\tau(\mu,\iota),\iota)$.

(ii) $\delta pC_\tau(\mu,\iota)=\mu\vee C_\tau(\delta I_\tau(\mu,\iota),\iota)$ and $\delta pI_\tau(\mu,\iota)=\mu\wedge I_\tau(\delta C_\tau(\mu,\iota),\iota)$.

(iii) $\bar{1}-(\delta I_\tau(\mu,\iota))=\delta C_\tau(\bar{1}-\mu,\iota)$ and $\delta I_\tau(\bar{1}-\mu,\iota)=\bar{1}-\delta C_\tau(\mu,\iota)$

Lemma 1.3 *[37] Let $\nu\in I^X$ and $\iota\in I_0$ in a sfts (X,τ), then*

(i) $\delta pC_\tau(\nu,\iota)=\nu\vee C_\tau(\delta I_\tau(\nu,\iota),\iota)$ and $\delta pI_\tau(\nu,\iota)=\nu\wedge I_\tau(\delta C_\tau(\nu,\iota),\iota)$.

(ii) $\delta pC_\tau(\delta pI_\tau(\nu,\iota),\iota)=\delta pI_\tau(\nu,\iota)\vee C_\tau(\delta I_\tau(\nu,\iota),\iota)$

(iii) $\delta pI_\tau(\delta pC_\tau(\nu,\iota),\iota)=\delta pC_\tau(\nu,\iota)\wedge I_\tau(\delta C_\tau(\nu,\iota),\iota)$.

(iv) $\delta sI_\tau(\nu,\iota)=\nu\wedge C_\tau(\delta I_\tau(\nu,\iota),\iota)$ and $\delta sC_\tau(\nu,\iota)=\nu\vee I_\tau(\delta C_\tau(\nu,\iota),\iota)$.

Lemma 1.4 *[37] Let $\nu\in I^X$ and $\iota\in I_0$ in a sfts (X,τ), then*

(i) $C_\tau(\delta I_\tau(\nu,\iota),\iota)=\delta C_\tau(\delta I_\tau(\nu,\iota),\iota)$.

(ii) $I_\tau(\delta C_\tau(\nu,\iota),\iota)=\delta I_\tau(\delta C_\tau(\nu,\iota),\iota)$.

Definition 1.7 *[37] A subset μ in a sfts (X,τ) is called a ι-fuzzy locally closed set $\forall\,\iota\in I_0$, if $\mu=\nu\wedge\alpha$, where $\tau(\nu)\geq\iota$, α is ι-fuzzy closed in X.*

Definition 1.8 *[37] Let (X,τ) be a sfts. For $\mu\in I^X$ and $\iota\in I_0$, then A sfts (X,τ) is ι-fuzzy extremely disconnected (briefly, ι-FED) if the ι-fuzzy closure of every ι-fuzzy open set of X is ι-fuzzy open.*

Definition 1.9 *[37] A subset $\mu\in I^X$, & $\forall\,\iota\in I_0$ in a sfts (X,τ) is called an ι-fuzzy*

(i) M-open set if $\mu\leq C_\tau(\theta I_\tau(\mu,\iota),\iota)\vee I_\tau(\delta C_\tau(\mu,\iota),\iota)$.

(ii) M-closed set if $\mu\geq I_\tau(\theta C_\tau(\mu,\iota),\iota)\wedge C_\tau(\delta I_\tau(\mu,\iota),\iota)$.

Definition 1.10 *[37] ι-fuzzy M-interior (resp. ι-fuzzy M-closure) of μ in a sfts (X,τ). $\forall\,\iota\in I_0$, denoted by $MI_\tau(\mu,\iota)$ (resp. $MC_\tau(\mu,\iota)$) defined as $MI_\tau(\mu,\iota)=\bigvee\{\nu\in I^X:\mu\geq\nu,\nu$ is a ι-fM o set$\}$; $MC_\tau(\mu,\iota)=\bigwedge\{\nu\in I^X:\mu\leq\nu,\nu$ is a ι-fM c set$\}$.*

Proposition 1.2 *[37] Let (X,τ) be a fuzzy topological space. $\lambda\in I^X$ and $r\in I_0$, then*

(i) Every ι-fuzzy θ-semiopen (resp. ι-fuzzy δ-preopen) set is ι-fuzzy M-open.

(ii) Every ι-fuzzy M-open set is ι-fuzzy e-open.

Proposition 1.3 *[37] If λ is an ι-fuzzy M-open subset of a sfts (X,τ) and $\theta I_\tau(\lambda,\iota)=\bar{0}$, then λ is ι-fuzzy δ-preopen.*

Theorem 1.3 *[37] Let (X,τ) be a sfts. Let $\lambda \in I^X$ and $\iota \in I_o$.*

(i) λ is ι-fMo iff $\lambda = MI_\tau(\lambda,\iota)$.

(ii) λ is ι-fMc iff $\lambda = MC_\tau(\lambda,\iota)$.

Theorem 1.4 *[37] Let (X,τ) be a sfts. For $\lambda \in I^X$ and $\iota \in I_0$ we have*

(i) $MI_\tau(1-\lambda,\iota) = 1-(MC_\tau(\lambda,\iota))$.

(ii) $MC_\tau(1-\lambda,\iota) = 1-(MI_\tau(\lambda,\iota))$.

Theorem 1.5 *[37] Let (X,τ) be a sfts. Let $\lambda \in I^X$ and $\iota \in I_o$, the following statements hold:*

(i) $MC_\tau(0,\iota) = 0$ and $MI_\tau(1,\iota) = 1$.

(ii) $I_\tau(\lambda,\iota) \le MI_\tau(\lambda,\iota) \le \lambda \le MC_\tau(\lambda,\iota) \le C_\tau(\lambda,\iota)$.

(iii) $\lambda \le \mu \Rightarrow MI_\tau(\lambda,\iota) \le MI_\tau(\mu,\iota)$ and $MC_\tau(\lambda,\iota) \le MC_\tau(\mu,\iota)$.

(iv) $MC_\tau(MC_\tau(\lambda,\iota),\iota) = MC_\tau(\lambda,\iota)$ and $MI_\tau(MI_\tau(\lambda,\iota),\iota) = MI_\tau(\lambda,\iota)$.

(v) $MC_\tau(\lambda,\iota) \vee MC_\tau(\mu,\iota) < MC_\tau(\lambda \vee \mu,\iota)$ and $MI_\tau(\lambda,\iota) \vee MI_\tau(\mu,\iota) < MI_\tau(\lambda \vee \mu,\iota)$.

(vi) $MC_\tau(\lambda \wedge \mu,\iota) < MC_\tau(\lambda,\iota) \wedge MC_\tau(\mu,\iota)$ and $MI_\tau(\lambda \wedge \mu,\iota) < MI_\tau(\lambda,\iota) \wedge MI_\tau(\mu,\iota)$.

Theorem 1.6 *[37] Let (X,τ) be a sfts. For $\lambda,\mu \in I^X$ and $\iota \in I_0$.*

(i) λ is ι-fMo iff $1-\lambda$ is ι-fMc.

(ii) If $\tau(\lambda) \ge \iota$, then λ is ι-fMo set.

(iii) $I_\tau(\lambda,\iota)$ is an ι-fMo set.

(iv) $C_\tau(\lambda,\iota)$ is an ι-fMc set.

Definition 1.11 *Let (X,τ_1) and (X,τ_2) be fts's and $f:(X,\tau_1) \to (Y,\tau_2)$ a mapping.*

(i) f is called fuzzy continuous (briefly, f-cts) [25] if $\tau_2(\mu) \le \tau_1(f^{-1}(\mu))$ for each $\mu \in I^Y$.

(ii) f is called fuzzy semicontinuous (briefly, fs-cts) [25] if $f^{-1}(\mu)$ is r-fso for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$.

(iii) f is called fuzzy precontinuous (briefly, fp-cts) [25] if $f^{-1}(\mu)$ is r-fpo for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$.

Definition 1.12 *Let (X,τ_1) and (X,τ_2) be fts's and $f:(X,\tau_1) \to (Y,\tau_2)$ a mapping.*

(i) f is called fuzzy δ-semicontinuous (briefly, fδs-cts) [33] if $f^{-1}(\mu)$ is r-fδso for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$.

(ii) f is called fuzzy δ-precontinuous (briefly, fδp-cts) [33] if $f^{-1}(\mu)$ is r-fδpo (resp. r-fsδpo) for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$. f is called fuzzy a-continuous (or) fuzzy semi δ-precontinuous **Error! Reference source not found.** if $f^{-1}(\mu)$ is r-fao for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$.

(iii) f is called fuzzy e-continuous (briefly, fe-cts) [33] if $f^{-1}(\mu)$ is r-feo for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$.

(iv) f is called fuzzy e^*-continuous (briefly, fe^*-cts) [33] if $f^{-1}(\mu)$ is r-fe^*o for each

$\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \geq \iota$.

2. Fuzzy M-continuous Mappings

Definition 2.1 *Let (X, τ_1) and (X, τ_2) be sfts's and $f:(X, \tau_1) \to (Y, \tau_2)$ be a mapping. Then f is called*

(i) fuzzy M-continuous (briefly, fM-cts) if $f^{-1}(\mu)$ is ι-fMo for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \geq \iota$.

(ii) fuzzy θ-continuous (briefly, fθ-cts) if $f^{-1}(\mu)$ is ι-fθo for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \geq \iota$.

(iii) fuzzy θ-semicontinuous (briefly, fθs-cts) if $f^{-1}(\mu)$ is ι-fθso for each $\mu \in I^X, \iota \in I_0$ with $\tau_2(\mu) \geq \iota$.

Remark 2.1 *The following implications are true for $\iota \in I_0$.*

From the above definitions, it is clear that every f δp-cts map is fM-cts map and every fuzzy θs-cts map is fM - cts map. Also, it is clear that every fM-cts map is fe-cts map and fe^*-cts map. Also, every fθ-cts map, fδ-cts map, fa-cts map is fM-cts map. The converses need not be true in general, it is shown in the succeeding examples.

Example 2.1 *Consider the identity mapping $f:(X, \tau) \to (Y, \eta)$., where $X = Y = \{x, y, z\}$, λ and μ defined as follows*

$\lambda(x) = 0.4$, $\lambda(y) = 0.5$, $\lambda(z) = 0.2$ $\mu(x) = 0.5$, $\mu(y) = 0.4$, $\mu(z) = 0.7$. Then τ, η : $I^X \to I$ defined as

$$\tau(\lambda) = \{1,$$

$= 0$ *or* $1, \frac{1}{2}$, if $=$, 0, otherwise, $\eta(\mu) = \{1,$ $= 0$ *or* $1, \frac{1}{2}$, if $=$, 0, otherwise, are fuzzy topologies on X and Y. Take $\iota = \frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set μ in (Y, η), $f^{-1}(\mu) = \mu$ is $\frac{1}{2}$-fe*o set in (X, τ). Then f is fe^*-cts, but f is not fM-cts, since $f^{-1}(\mu)$ is not $\frac{1}{2}$-fMo in (X, τ).

Example 2.2 *Let λ and μ be fuzzy subsets of $X = Y = \{x, y, z\}$ defined as follows*

$\lambda(x) = 0.5$, $\lambda(y) = 0.3$, $\lambda(z) = 0.2$;

$\mu(x) = 0.5$, $\mu(y) = 0.4$, $\mu(z) = 0.4$.

Then τ, η : $I^X \to I$ defined as

$$\tau(\lambda) = \{1,$$

$= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, $\eta(\mu) = \{1,$ $= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota = \frac{1}{2}$. For any $\frac{1}{2}$-fo set μ in (Y,η), $f^{-1}(\mu)=\mu$ is $\frac{1}{2}$-feo set in (X,τ). Then f is fe-cts, but f is not fM-cts, since $f^{-1}(\mu)$ is not $\frac{1}{2}$-fMo in (X,τ).

Example 2.3 *Let* λ *and* μ *be fuzzy subsets of* $X = Y = \{x, y, z\}$ *defined as follows*

$\lambda(x) = 0.1, \quad \lambda(y) = 0.1, \quad \lambda(z) = 0.1;$

$\mu(x) = 0.9, \quad \mu(y) = 0.9, \quad \mu(z) = 0.9.$

Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda) = \{1,$$

$= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, $\eta(\mu) = \{1,$ $= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota = \frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set μ in (Y,η), $f^{-1}(\mu)=\mu$ is $\frac{1}{2}$-fMo set in (X,τ). Then f is fM-cts, but f is not fδp-cts, fδ-cts and fa-cts, since $f^{-1}(\mu)$ is not $\frac{1}{2}$-fδpo, $\frac{1}{2}$-fδo and $\frac{1}{2}$-fao sets.

Example 2.4 *Let* λ *and* μ *be fuzzy subsets of* $X = Y = \{x, y, z\}$ *defined as follows*

$\lambda(x) = 0.1, \quad \lambda(y) = 0.1, \quad \lambda(z) = 0.1;$

$\mu(x) = 0.9, \quad \mu(y) = 0.9, \quad \mu(z) = 0.9.$

Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda) = \{1,$$

$= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, $\eta(\mu) = \{1,$ $= 0\, or\, 1, \frac{1}{2}$, if $=$, 0, otherwise, are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota = \frac{1}{2}$. For any $\frac{1}{2}$-fo set μ in (Y,η), $f^{-1}(\mu)=\mu$ is $\frac{1}{2}$-fδso set in (X,τ). Then f f is fuzzy δs-cts, but f is not fδ-cts, since $f^{-1}(\mu)$ is not $\frac{1}{2}$-fδo set.

Example 2.5 *Let* λ, μ *and* ω *be fuzzy subsets of* $X = Y = \{x, y, z\}$ *defined as follows*

$\lambda(x) = 0.3, \quad \lambda(y) = 0.4, \quad \lambda(z) = 0.5;$

$\mu(x) = 0.6, \quad \mu(y) = 0.9, \quad \mu(z) = 0.5;$

$\omega(x) = 0.7, \quad \omega(y) = \bar{1}, \quad \omega(z) = 0.5.$

Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda)=\{1,$$

$= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ , \ ,0,\text{otherwise},\eta(\mu)=\{1,$ $= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ ,0,\text{otherwise},$ are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota=\frac{1}{2}$. For any $\frac{1}{2}$-fo set ω in (Y,η), $f^{-1}(\omega)=\omega$ is $\frac{1}{2}$-fMo set in (X,τ). Then f is fM-cts, but f is neither fθs-cts nor fδs-cts, since $f^{-1}(\omega)$ is neither $\frac{1}{2}$-fθso nor $\frac{1}{2}$-fδso set.

Example 2.6 *Let* λ, μ *and* ω *be fuzzy subsets of* $X=Y=\{x,y,z\}$ *defined as follows*

$\lambda(x)=0.3,\quad \lambda(y)=0.4,\quad \lambda(z)=0.5;$

$\mu(x)=0.6,\quad \mu(y)=0.5,\quad \mu(z)=0.5;$

$\omega(x)=0.7,\quad \omega(y)=0.6,\quad \omega(z)=0.5.$

Then $\tau,\eta \ : \ I^X\to I$ defined as

$$\tau(\lambda)=\{1,$$

$= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ , \ ,0,\text{otherwise},\eta(\mu)=\{1,$ $= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ ,0,\text{otherwise},$ are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota=\frac{1}{2}$. For any $\frac{1}{2}$-fo set ω in (Y,η), $f^{-1}(\omega)=\omega$ is $\frac{1}{2}$-fMo and $\frac{1}{2}$-fθso set in (X,τ). Then f is fM-cts and fθs-cts, but f is not fθ-cts, since $f^{-1}(\omega)$ is not $\frac{1}{2}$-fθo set.

Example 2.7 *Let* λ, μ *and* ω *be fuzzy subsets of* $X=Y=\{x,y,z\}$ *defined as follows*

$\lambda(x)=0.3,\quad \lambda(y)=0.5,\quad \lambda(z)=0.5;$

$\mu(x)=0.5,\quad \mu(y)=0.5,\quad \mu(z)=0.5;$

$\omega(x)=0.7,\quad \omega(y)=0.6,\quad \omega(z)=0.5.$

Then $\tau,\eta \ : \ I^X\to I$ defined as

$$\tau(\lambda)=\{1,$$

$= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ , \ ,0,\text{otherwise},\eta(\mu)=\{1,$ $= 0$ *or* 1 $,\frac{1}{2},\text{if} \ = \ ,0,\text{otherwise},$ are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $\iota=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set λ in (Y,η), $f^{-1}(\lambda)=\lambda$ is $\frac{1}{2}$-fuzzy open in (X,τ). Then f is f-cts, but f is not fθ-cts and fδ-cts, since $f^{-1}(\lambda)$ is neither $\frac{1}{2}$-fθo nor $\frac{1}{2}$-fδo set.

Theorem 2.1 *Let* (X,τ_1) *and* (Y,τ_2) *be fts's and* $f:X\to Y$ *be a mapping. Then the following statements are equivalent:*

(i) f is fM-cts mapping.

(ii) $f^{-1}(\mu)$ is ι-fM c in X for each $\mu\in I^Y,\iota\in I_0$ with $\tau_2(1-\mu)\geq\iota$..

(iii) $f(MC_{\tau_1}(\lambda,\iota)) \le C_{\tau_2}(f(\lambda),\iota), \ \forall \ \lambda \in I^X$ and $r \in I_0$.

(iv) $MC_{\tau_1}(f^{-1}(\mu),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota)), \forall \mu \in I^Y$ and $r \in I_0$.

(v) $I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota)), \ \forall \ \mu \in I^Y$ and $r \in I_0$.

(vi) $f^{-1}(I_{\tau_2}(\mu,\iota)) \le MI_{\tau_1}(f^{-1}(\mu),\iota)$, for each $\mu \in I^Y$ and $r \in I_0$.

Proof. (i) $\Rightarrow$ (ii): Let $\mu \in I^Y, \iota \in I_0$ with $\tau_2(\bar{1}-\mu) \ge \iota$. Since f is fM-cts mapping, $f^{-1}(\bar{1}-\mu)$ is an ι-fM o set of X. But $f^{-1}(\bar{1}-\mu) = \bar{1} - f^{-1}(\mu)$. Therefore $f^{-1}(\mu)$ is an ι-fM c set of X.

(ii)$\Rightarrow$(iii): Let $\lambda \in I^X$, $r \in I_0$, since $\tau_2(\bar{1} - C_{\tau_2}(f(\lambda),\iota)) \ge \iota$. Then by (ii), $f^{-1}(C_{\tau_2}(f(\lambda),\iota))$ is an ι-fM c set of X. Since

$$\lambda \le f^{-1}(f(\lambda)) \le f^{-1}(C_{\tau_2}(f(\lambda),\iota)),$$

we have $MC_{\tau}(\lambda,\iota) \le f^{-1}(C_{\tau_2}(f(\lambda),\iota))$. Hence $f(MC_{\tau}(\lambda,\iota)) \le C_{\tau_2}(f(\lambda),\iota)$.

(iii)$\Rightarrow$(iv): For all $\mu \in I^Y, \iota \in I_0$, let $\lambda = f^{-1}(\mu)$. By (iii), we have

$$f(MC_{\tau_1}(f^{-1}(\mu),\iota)) \le C_{\tau_2}(f(f^{-1}(\mu)),\iota) \le C_{\tau_2}(\mu,\iota).$$

It implies $MC_{\tau_1}(f^{-1}(\mu),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota))$.

(iv)$\Rightarrow$(i): Let $\mu \in I^Y, \iota \in I_0$ with $\tau_2(\mu) \ge \iota$. By (iv),

$MC_{\tau_1}(f^{-1}(\bar{1}-\mu),\iota) \le f^{-1}(C_{\tau_2}(\bar{1}-\mu,\iota)) = f^{-1}(\bar{1}-\mu).$

By Theorem 1.4 , we have $f^{-1}(\bar{1}-\mu) \ge \bar{1} - (MI_{\tau_1}(f^{-1}(\mu),\iota))$. Hence $f^{-1}(\mu)$ is ι-fM o set in X.

(ii) $\Rightarrow$ (v): For all $\mu \in I^Y, \iota \in I_0$, since $\tau_2(\bar{1} - C_{\tau_2}(\mu,\iota)) \ge \iota$. Then by (ii), we see that $f^{-1}(C_{\tau_2}(\mu,\iota))$ is ι-fM c in X. Hence

$$\begin{aligned} f^{-1}(C_{\tau_2}(\mu,\iota)) &\ge I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(C_{\tau_2}(\mu,\iota)),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(C_{\tau_2}(\mu,\iota)),\iota),\iota) \\ &\ge I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota)),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota). \end{aligned}$$

(v)$\Rightarrow$(ii): For all $\mu \in I^Y$, $r \in I_0$, with $\tau_2(\bar{1}-\mu) \ge \iota$. Then by (v),

$I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota)) = f^{-1}(\mu).$

Hence $f^{-1}(\mu)$ is ι-fM c in X.

(iv)$\Rightarrow$(vi): It is easily proved from Theorem (1.4)

(vi)$\Rightarrow$(i): Let μ be ι-fuzzy open set of Y. Then $\mu = I_{\eta}(\mu,\iota)$.

By (vi), $f^{-1}(\mu) \le MI_{\tau_1}(f^{-1}(\mu),\iota)$.

On the other hand, by Theorem(1.5),

$$f^{-1}(\mu) \ge MI_{\tau_1}(f^{-1}(\mu),\iota).$$

Thus, $f^{-1}(\mu) = MI_{\tau}(f^{-1}(\mu),\iota)$, that is, $f^{-1}(\mu)$ is ι-fMo set.

Theorem 2.2 *For a map $f:(X,\tau_1) \to (Y,\tau_2)$ the succeeding statements are equivalent:*

(i) f is fM-cts mapping.

(ii) $f^{-1}(\mu)$ is ι-fM c in X for each $\lambda \in I^Y, \iota \in I_0$ with $\tau_2(\bar{1}-\mu) \ge \iota$. .

(iii) $I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota)), \ \forall \ \mu \in I^Y$ and $r \in I_0$.

(iv) $f^{-1}(I_{\tau_2}(\mu,\iota)) \le C_{\tau_1}(\theta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \vee I_{\tau_1}(\delta C_{\tau_1}(f^{-1}(\mu),\iota),\iota)$ for each $\mu \in I^Y$ and $r \in I_0$.

Proof. (i) $\Rightarrow$ (ii): Let $\mu \in I^Y, \iota \in I_0$ with $\tau_2(\bar{1}-\mu) \ge \iota$. Since f is fuzzy M-continuous mapping, $f^{-1}(\bar{1}-\mu)$ is an ι-fM o set of X. But $f^{-1}(\bar{1}-\mu) = 1 - f^{-1}(\mu)$. Therefore $f^{-1}(\mu)$ is an ι-fM c set of X.

(ii) $\Rightarrow$ (iii): For all $\mu \in I^Y, \iota \in I_0$, since $\tau_2(\bar{1}-C_{\tau_2}(\mu,\iota)) \ge \iota$. Then by (ii), we see that $f^{-1}(C_{\tau_2}(\mu,\iota))$ is ι-fM c in X. Hence

$$f^{-1}(C_{\tau_2}(\mu,\iota)) \ge I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(C_{\tau_2}(\mu,\iota)),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(C_{\tau_2}(\mu,\iota)),\iota),\iota)$$
$$\ge I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota)),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota).$$

(iii)$\Rightarrow$(ii): For all $\mu \in I^Y$, $r \in I_0$, with $\tau_2(\bar{1}-\mu) \ge \iota$. Then by (iii),

$$I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \le f^{-1}(C_{\tau_2}(\mu,\iota)) = f^{-1}(\mu).$$

Hence $f^{-1}(\mu)$ is ι-fM c in X.

(iii)$\Rightarrow$(iv): For all $\mu \in I^Y$, $r \in I_0$, with $\tau_2(\bar{1}-\mu) \ge \iota$. Then by (iii),

$$I_{\tau_1}(\theta C_{\tau_1}(f^{-1}(\bar{1}-\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(f^{-1}(\bar{1}-\mu),\iota),\iota) \le f^{-1}(C_{\tau_2}(\bar{1}-\mu,\iota)).$$
$$I_{\tau_1}(\theta C_{\tau_1}(\bar{1}-f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\delta I_{\tau_1}(\bar{1}-f^{-1}(\mu),\iota),\iota) \le \bar{1}-f^{-1}(I_{\tau_2}(\mu,\iota)).$$
$$I_{\tau_1}(\bar{1}-\theta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \wedge C_{\tau_1}(\bar{1}-\delta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \le \bar{1}-f^{-1}(I_{\tau_2}(\mu,\iota)).$$
$$\left[\bar{1}-C_{\tau_1}(\theta I_{\tau_1}(f^{-1}(\mu),\iota),\iota)\right] \wedge \left[\bar{1}-I_{\tau_1}(\delta C_{\tau_1}(f^{-1}(\mu),\iota),\iota)\right] \le \bar{1}-f^{-1}(I_{\tau_2}(\mu,\iota)).$$
$$C_{\tau_1}(\theta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \vee I_{\tau_1}(\delta C_{\tau_1}(f^{-1}(\mu),\iota),\iota) \ge f^{-1}(I_{\tau_2}(\mu,\iota)).$$

Thus (iv) is proved.

(iv)$\Rightarrow$(i): Let μ be ι-fuzzy open set of Y. Then $\mu = I_\eta(\mu,\iota)$. By (iv), $f^{-1}(\mu) \le C_{\tau_1}(\theta I_{\tau_1}(f^{-1}(\mu),\iota),\iota) \vee I_{\tau_1}(\delta C_{\tau_1}(f^{-1}(\mu),\iota),\iota)$. That is, $f^{-1}(\mu)$ is ι-fMo set.

Definition 2.2 *A fuzzy set λ in a fts (X,τ) is called ι-fuzzy dense if there exists no ι-fc set μ in (X,τ) such that $\lambda < \mu < 1$.*

Definition 2.3 *A fuzzy set λ in a fts (X,τ) is called ι-fuzzy nowhere dense if there exists no non-zero ι-fo set μ in (X,τ) such that $\mu < C_\tau(\lambda,\iota)$. That is., $I_\tau(C_\tau(\lambda,\iota),\iota) = \bar{0}$, in (X,τ).*

Lemma 2.1 *For a sfts (X,τ), every ι-fuzzy dense set is ι-fpo.*

Proposition 2.1 *Let (X,τ_1) and (Y,τ_2) be fts's and $f: X \to Y$ be a mapping. An fM-cts mapping f is fδ p-cts if for any fuzzy subset λ of X is ι-fuzzy nowhere dense.*

Proof. Let $\mu \in \tau_2$. Since f is an fM-cts mapping, then $f^{-1}(\mu)$ is an ι-fM o set in (X,τ_1). Put $f^{-1}(\mu) = \lambda$ is ι-fM o set in X. Hence

$\lambda \le C_\tau(\theta I_\tau(\lambda,\iota),\iota) \vee I_\tau(\delta C_\tau(\lambda,\iota),\iota).$

But $\theta I_\tau(\lambda,\iota) \le I_\tau(\lambda,\iota) \le C_\tau(\lambda,\iota),$ then $\theta I_\tau(\lambda,\iota) \le I_\tau(C_\tau(\lambda,\iota),\iota).$ Since λ is ι -fuzzy nowhere dense and Lemma **Error! Reference source not found.**, we have $\theta I_\tau(\lambda,\iota) = \bar{0}.$ Therefore f is fδ p-cts.

Definition 2.4 *Let (X,τ_1) and (Y,τ_2) be sfts's and $f:(X,\tau_1)\to(Y,\tau_2)$ be a mapping. Then f is called fuzzy θ-open map if the image of every ι-fuzzy open set of $X,\tau_1)$ is ι -fuzzy θ-open set in (Y,τ_2).*

Definition 2.5 *Let (X,τ_1) and (Y,τ_2) be sfts's and $f:(X,\tau_1)\to(Y,\tau_2)$ be a mapping. Then f is called fuzzy θ-bicontinuous if f is fuzzy θ-open map and θ-continuous map.*

Theorem 2.3 *Let (X,τ_1) and (Y,τ_2) be sfts's and $f:(X,\tau_1)\to(Y,\tau_2)$ be a fuzzy θ -bicontinuous mapping. Then the inverse image of each ι-fM o set in (Y,τ_2) under f is ι-f M o set in (X,τ_1).*

Proof. Let f be a fuzzy θ-bicontinuous mapping and μ be a ι-fM o set in $(Y\,\tau_2)$. Then

$\mu \le C_{\tau_2}(\theta I_{\tau_2}(\mu,\iota),\iota) \vee I_{\tau_2}(\delta C_{\tau_2}(\mu,\iota),\iota).$

$f^{-1}(\mu) \le f^{-1}(C_{\tau_2}(\theta I_{\tau_2}(\mu,\iota),\iota)) \vee f^{-1}(I_{\tau_2}(\delta C_{\tau_2}(\mu,\iota),\iota)).$

$\le C_{\tau_2}(f^{-1}(\theta I_{\tau_2}(\mu,\iota)),\iota) \vee f^{-1}(I_{\tau_2}(\delta C_{\tau_2}(\mu,\iota),\iota)).$

Since f is an fuzzy θ-bicontinuous mapping, then f is fuzzy θ-open map and θ-continuous map. Then f is fθs-cts map and fθp-cts map. Hence

$f^{-1}(\mu) \le C_{\tau_2}(\theta I_{\tau_2}(f^{-1}(\theta I_{\tau_2}(\mu,\iota)),\iota),\iota) \vee I_{\tau_2}(\delta C_{\tau_2}(f^{-1}(I_{\tau_2}(\delta C_{\tau_2}(\mu,\iota),\iota)),\iota),\iota).$

$\le C_{\tau_2}\theta I_{\tau_2}(f^{-1}(\theta I_{\tau_2}(\mu,\iota)),\iota),\iota) \vee I_{\tau_2}(\delta C_{\tau_2}(f^{-1}(\delta C_{\tau_2}(\mu,\iota)),\iota),\iota).$

$\le C_{\tau_2}\theta I_{\tau_2}(f^{-1}(\mu),\iota),\iota) \vee I_{\tau_2}\delta C_{\tau_2}(f^{-1}(\mu),\iota),\iota).$

This shows that $f^{-1}(\mu)$ is ι-fM o set in (X,τ_1).

Remark 2.2 *Let (X,τ_1) and (Y,τ_2) be sfts's and $f:(X,\tau_1)\to(Y,\tau_2)$ be a fuzzy θ -bicontinuous mapping. Then the inverse image of each ι-fδ po (resp. ι-fθso) set in Y under f is ι-fM o set in X.*

Remark 2.3 *Let (X,τ_1) and (Y,τ_2) be fts's and $f:X\to Y$ be a mapping. The composition of two fM -cts mappings need not be fM -cts as shown by the following example.*

Example 2.8 *Let λ, ω and μ be fuzzy subsets of $X=Y=Z=\{a,b,c\}$ defined as follows*

$\lambda(x)=0.4,\quad \lambda(y)=0.5,\quad \lambda(z)=0.2;$

$\omega(x)=0.7,\quad \omega(y)=\bar{1},\quad \omega(z)=0.5.$

$\mu(x)=0.5,\quad \mu(y)=0.4,\quad \mu(z)=0.7;$

Then τ_1,τ_2 *and* τ_3 : $I^X \to I$ defined as

$$\tau_1(\lambda)=\{1,$$

= 0 *or* 1 , $\frac{1}{2}$, if = , 0, otherwise, $\tau_2(\omega) = \{1,$ = 0 *or* 1 , $\frac{1}{2}$, if = , 0, otherwise,

$\tau_2(\omega) = \{1,$

= 0 *or* 1 , $\frac{1}{2}$, if = , 0, otherwise, $\tau_3(\mu) = \{1,$ = 0 *or* 1 , $\frac{1}{2}$, if = , 0, otherwise, are fuzzy topologies on X, Y and Z. Consider the identity mapping $f:(X,\tau_1)\to(Y,\tau_2)$ and $g:(Y,\tau_2)\to(Z,\tau_3)$. Take $\iota = \frac{1}{2}$. For any $\frac{1}{2}$-fo set ω in (Y,τ_2), $f^{-1}(\omega)=\omega$ is $\frac{1}{2}$- fMo set in (X,τ_1). Also, for any $\frac{1}{2}$-fuzzy open set μ in (Z,τ_3), $g^{-1}(\mu)=\mu$ is $\frac{1}{2}$- fMo in (Y,τ_2). Thus f is fM -cts and g is fM -cts. But $g\circ f$ is not fM -cts, as μ is $\frac{1}{2}$-fo set in (Z,τ_3), $(g\circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu)) = \mu$ is not $\frac{1}{2}$- fMo in (X,τ_1).

Example 2.9 *Let* (X,τ_1), (Y,τ_2) *and* (Z,τ_3) *be sfts's. If* $f:(X,\tau_1)\to(Y,\tau_2)$ *and* $g:(Y,\tau_2)\to(Z,\tau_3)$ *are mappings, then*

(i) $g\circ f$ is fM -cts mapping if f is fM -cts and g is f-cts.

(ii) $g\circ f$ is fM -cts mapping if f is fuzzy θ-bicontinuous and g is fM -cts mapping.

Proof. (i) Let $\mu\in\tau_3$. Since g is f-cts, then $g^{-1}(\mu)$ is an ι-fo set in (Y,τ_2). Since f is f M -cts, then $f^{-1}(g^{-1}(\mu)) = (g\circ f)^{-1}(\mu)$ is ι-fM o set in τ_1. Hence $g\circ f$ is fM -cts.

(ii) Let $\mu\in\tau_3$. Since g is fM -cts, then $g^{-1}(\mu)$ is an ι-fM o set in (Y,τ_2). Since f is fuzzy θ-bicontinuous, by Theorem **Error! Reference source not found.**, $(g\circ f)^{-1}(\mu)$ is ι-f M o set in τ_1. Hence $g\circ f$ is fM -cts.

Conclusion: In this paper, fM -cts, fθ-cts and fθs-cts in sfts's. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between f-cts, fθs-cts, fθ-cts, fδs- cts, fδp-cts, fa-cts, fM -cts, fe-cts and fe^*-cts mappings.

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