$\tau_i\tau_j$-M^*-σ_k -continuous Maps

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Abstract

The aim of this paper is to introduce and investigate the concept of $\tau_i\tau_j$-M^*-σ_k-continuous maps in a bitopological space . Moreover we investigate the relationship between $\tau_i\tau_j$-δ-σ_k-continuous, $\tau_i\tau_j$-δs-σ_k- continuous, $\tau_i\tau_j$-a-σ_k- continuous, $\tau_i\tau_j$-e^*-σ_k- continuous and respective some other closed mappings..

Keywords and phrases: $\tau_i\tau_j$-M^*-σ_k-continuity, $\tau_i\tau_j$-δ-σ_k- continuous, $\tau_i\tau_j$-δs-σ_k-continuous, $\tau_i\tau_j$-a-σ_k- continuous, $\tau_i\tau_j$-e^*-σ_k- continuous.
AMS (2000) subject classification: 54E55.

Introduction and Preliminaries

Levine in 1963 initiated a new types of open set called semiopen set [10]. A subset A of a space (X,τ) is called regular open (resp., regular closed) [15] if $A = int(cl(A))$ (resp., $A = cl(int(A))$. The delta interior [4] of a subset A of (X,τ) is the union of all regular open sets of X contained in A and is denoted by $\delta int(A)$. A subset A of a space (X,τ) is called δ-open [12] if $A = \delta int(A)$. The complement of δ-open set is called δ-closed. Alternatively, a set A of (X,τ) is called δ -closed [4] if $A = \delta cl(A)$, where $\delta cl(A) = \{x \in X : A \cap int(cl(U)) \neq \phi,\ U \in \tau$ and $x \in U\}$. A subset A of a space X is called θ-open [1] if $A = \theta int(A)$, where $\theta int(A) = \bigcup\{int(U) : U \subseteq A,\ U \in \tau^c\}$, and a subset A is called θ-semiopen [2] (resp., δ- preopen [12] , e-open[5], M -open [6], M^*-open[3], δ -semiopen [11] , δ -open[15], e^* -open [5] and a -open[5]) if $A \subseteq cl(\theta int(A))$ (resp., $A \subseteq int(\delta cl(A))$, $A \subseteq cl(\delta int(A)) \cup int(\delta cl(A))$ and $A \subseteq cl(\theta int(A)) \cup int(\delta cl(A)))$, $A \subseteq int(cl(\theta int(A)))$, $A \subseteq cl(\delta int(A))$, $A = \delta int(A)$, $A \subseteq cl(int(\delta cl(A)))$ and $A \subseteq int(cl(A)(\delta int(A))$, where $int()$, $cl()$, $\theta int()$, $\delta int()$ and $\delta cl()$ are the interior, closure, θ -interior, δ -interior and δ -closure operations, respectively. The notion of bitopological spaces (in short, Bts's) was first introduced by Kelly[8].

Through out this paper, let (X,τ_1,τ_2) or simply X be a Bts and $i, j \in \{1,2\}$. A subset S of a Bts X is said to be $\tau_{1,2}$-open [9] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is said to be $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. and $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. For a subset A of X, the interior (resp., closure) of A with respect to τ_i will be denoted by $int_i(A)$ (resp., $cl_i(A)$) for $i = 1,2$. In this paper, we introduce and investigate the concept of $\tau_i\tau_j$-M^*-σ_k-continuous maps in a bitopological spaces. In addition, several properties of these notions and connections to several other known ones are provided.

Let (X,τ_1,τ_2) be a Bts. A subset A of X is called $\tau_i\tau_j$-M-open [13] (briefly, $\tau_i\tau_j$-M-o) if $A\subseteq cl_j(\theta int_i(A))\cup int_i(\delta cl_j(A))$ and A is $\tau_i\tau_j$-M closed (in short, $\tau_i\tau_j$-M-c) if $X\setminus A$ is $\tau_i\tau_j$-M-o. A is pairwise M-open if it is both $\tau_i\tau_j$-M-o and $\tau_j\tau_i$-M-o. A subset A of X is called $\tau_i\tau_j$-M^*-open [13] (briefly, $\tau_i\tau_j$-M^*-o) if $A\subseteq int_i(cl_j(\theta int_i(A)))$ and A is $\tau_i\tau_j$-M^*- closed (briefly, $\tau_i\tau_j$-M^*-c) if X/A is $\tau_i\tau_j$-M^*-o. A is pairwise M^*-o if it is both $\tau_1\tau_2$-M^*-o and $\tau_2\tau_1$-M^*-o.

Clearly A is $\tau_i\tau_j$-M^*-c iff $A\supseteq cl_j(int_i(\theta cl_j(A)))$. We denote the family of all $\tau_i\tau_j$-M^*-c sets in a Bts (X,τ_1,τ_2) by $D_{M^*C}(\tau_i,\tau_j)$. A subset A of X is called $\tau_i\tau_j$-θ-semiopen [13] (briefly, $\tau_i\tau_j$ - θ -so) if $A\subseteq cl_j(\theta int_i(A))$, $\tau_i\tau_j$ - δ -preopen [13] (briefly, $\tau_i\tau_j$ - δ -po) if $A\subseteq int_i(\delta cl_j(A))$, $\tau_i\tau_j$-e-open if $A\subseteq cl_j(\delta int_i(A))\cup int_i(\delta cl_j(A))$, $\tau_i\tau_j$-δ-semi open [13] (briefly, $\tau_i\tau_j$ - δ - so) if $A\subseteq cl_j(\delta int_i(A))$, $\tau_i\tau_j$ - δ -open [13] (briefly, $\tau_i\tau_j$ - δ - o) if $A=\delta int_i(A)$, $\tau_i\tau_j$-e^*-open [13] (briefly, $\tau_i\tau_j$-e^*-o) if $A\subseteq cl_j(int_i(\delta cl_j(A)))$, $\tau_i\tau_j$-a-open [13] (briefly, $\tau_i\tau_j$-a-o) if $A\subseteq int_i(cl_j(A)(\delta int_i(A))$.

Definition 1.1 *A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called [14] $\tau_i\tau_j$ - θ - σ_k -continous (briefly, $\tau_i\tau_j$-θ-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-θ-c set in (X,τ_1,τ_2); $\tau_i\tau_j$-θs-σ_k-continuous (briefly, $\tau_i\tau_j$-θs-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-θs-c set in (X,τ_1,τ_2); $\tau_i\tau_j$-M -σ_k-continuous (briefly, $\tau_i\tau_j$-M -σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-M -c set in (X,τ_1,τ_2); $\tau_i\tau_j$-e-σ_k-continuous (briefly, $\tau_i\tau_j$-e-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-e-c set in (X,τ_1,τ_2); $\tau_i\tau_j$-δp-σ_k-continuous (briefly, $\tau_i\tau_j$-δp-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-δp-c set in (X,τ_1,τ_2) and τ_i-σ_k-continuous* ***Error! Reference source not found.*** *(briefly, τ_i-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-τ_i-c set in (X,τ_1,τ_2).*

2. $\tau_i\tau_j$-M^*-σ_k-continuous Maps

Definition 2.1 *A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called*

(1) $\tau_i\tau_j$-M^*-σ_k-continuous (briefly, $\tau_i\tau_j$-M^*-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-M^*-c set in (X,τ_1,τ_2).

(2) $\tau_i\tau_j$-δ-σ_k-continuous (briefly, $\tau_i\tau_j$-δ-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-δ-c set in (X,τ_1,τ_2).

(3) $\tau_i\tau_j$-δs-σ_k-continuous (briefly, $\tau_i\tau_j$-δs-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-δs-c set in (X,τ_1,τ_2).

(4) $\tau_i\tau_j$-a-σ_k-continuous (briefly, $\tau_i\tau_j$-a-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-a-c set in (X,τ_1,τ_2).

(5) $\tau_i\tau_j$-e^*-σ_k-continuous (briefly, $\tau_i\tau_j$-e^*-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-e^*-c set in (X,τ_1,τ_2).

Theorem 2.1 *If a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is a*

(1) τ_i-σ_k-cts then it is a $\tau_i\tau_j$-M^*-σ_k-cts

(2) $\tau_i\tau_j$-θ-σ_k-cts then it is a $\tau_i\tau_j$-M^*-σ_k-cts

(3) $\tau_i\tau_j$-θs-σ_k-cts then it is a $\tau_i\tau_j$-M^*-σ_k-cts

(4) $\tau_i\tau_j$-θ-σ_k-cts then it is a $\tau_i\tau_j$-θs-σ_k-cts

(5) $\tau_i\tau_j$-M^*-σ_k-cts then it is a $\tau_i\tau_j$-M-σ_k-cts cts

(6) $\tau_i\tau_j$-M-σ_k-cts then it is a $\tau_i\tau_j$-e-σ_k-cts

(7) $\tau_i\tau_j$-δp-σ_k-cts then it is a $\tau_i\tau_j$-e-σ_k-cts

(8) $\tau_i\tau_j$-θ-σ_k-cts then it is a $\tau_i\tau_j$-δ-σ_k-cts

(9) $\tau_i\tau_j$-θs-σ_k-cts then it is a $\tau_i\tau_j$-δs-σ_k-cts

(10) τ_i-σ_k-cts then it is a $\tau_i\tau_j$-M^*-σ_k-cts

(11) $\tau_i\tau_j$-δ-σ_k-cts then it is a $\tau_i\tau_j$-a-σ_k-cts

(12) $\tau_i\tau_j$-M^*-σ_k-cts then it is a $\tau_i\tau_j$-θs-σ_k-cts

(13) $\tau_i\tau_j$-δs-σ_k-cts then it is a $\tau_i\tau_j$-e-σ_k-cts

(14) $\tau_i\tau_j$-δp-σ_k-cts then it is a $\tau_i\tau_j$-M-σ_k-cts

(15) $\tau_i\tau_j$-a-σ_k-cts then it is a $\tau_i\tau_j$-δp-σ_k-cts

(16) $\tau_i\tau_j$-e-σ_k-cts then it is a $\tau_i\tau_j$-e^*-σ_k-cts

(17) $\tau_i\tau_j$-a-σ_k-cts then it is a $\tau_i\tau_j$-δs-σ_k-cts

Proof. (1) Let V be an σ_k-c set. Since f is τ_i-σ_k-cts. $f^{-1}(v)$ is $\tau_i\tau_j$-σ_i-c. By Lemma 2.1 in[13], $f^{-1}(v)$ is $\tau_i\tau_j$-M^*-c in (X,τ_1,τ_2). Therefore f is $\tau_i\tau_j$-M^*-σ_k-cts.
The proof of (2) to (17) are similar as in (1).

Example 2.1 *Let* $X=Y=\{a,b,c,d\},\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\},\{d,c\},\{a,d,c\},\{b,c,d\}\}$ *,* $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$ *,* $\sigma_1=\{Y,\phi,\{b,c\},\{b,d\}\}$ *and* $\sigma_2=\{Y,\phi,\{b\},\{b,d\},\{a,b,c\}\}$*. Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is a*

(1) $\tau_1\tau_2$-M^*-σ_1-cts but it is not τ_1-σ_1-cts, since for the σ_1-c set $\{a,d\}$, $f^{-1}(\{a,d\})=\{a,d\}$ which is not τ_1-c set.

(2) $\tau_1\tau_2$-M^*-σ_1-cts but it is not $\tau_1\tau_2$-θ-σ_1-cts, since for the σ_1-c set $\{a,c\}$, $f^{-1}(\{a,c\})=\{a,c\}$ which is not $\tau_1\tau_2$-θ-c set.

Example 2.2 *Let* $X=Y=\{a,b,c,d\},\tau_1$ *and* τ_2 *are defined as in Example2.1,* $\sigma_1=\{Y,\phi,\{d\},\{a,d\}\}$ *and* $\sigma_2=\{Y,\phi,\{d\},\{a,b,c\},\{a,b,d\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-M^*-σ_1*-cts but it is not* $\tau_1\tau_2$-θs-σ_1*-cts. Since for the* σ_1 *-c set* $\{b,c\}$, $f^{-1}(\{b,c\})=\{b,c\}$ *which is not* $\tau_1\tau_2$-θ*-sc set.*

Example 2.3 *Let* $X=Y=\{a,b,c,d\},\tau_1$ *and* τ_2 *are defined in Example2.1,* $\sigma_1=\{Y,\phi,\{b,c\},\{a,b,c\}\}$ *and* $\sigma_2=\{Y,\phi,\{b\},\{a,b,c\},\{a,b,d\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-θs-σ_1*-cts but it is not* $\tau_1\tau_2$-θ-σ_1*-cts. Since for the* σ_1*-c set* $\{d\}$, $f^{-1}(\{d\})=\{d\}$ *which is not* $\tau_1\tau_2$-θ*-c set.*

Example 2.4 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.1,* $\sigma_2 = \{Y, \phi, \{d\}, \{a,d\}, \{a,b,c\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-M-σ_1*-cts but it is not* $\tau_1\tau_2$-M^*-σ_1*-cts. Since for the* σ_1*-c set* $\{a,b,c\}, f^{-1}(\{a,b,c\}) = \{a,b,c\}$ *which is not* $\tau_1\tau_2$-M^**-c set.*

Example 2.5 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example 2.1,* $\sigma_1 = \{Y, \phi, \{c\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{c\}, \{a,c\}, \{a,c,d\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-e-σ_1*-cts but it is not* $\tau_1\tau_2$-M-σ_1*-cts. Since for the* σ_1*-c set* $\{b,d\}, f^{-1}(\{b,d\}) = \{b,d\}$ *which is not* $\tau_1\tau_2$-M*-c set.*

Example 2.6 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.1,* $\sigma_1 = \{Y, \phi, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,c\}, \{a,c,d\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-e-σ_1*-cts but it is not* $\tau_1\tau_2$-δp-σ_1*-cts. Since for the* σ_1*-c set* $\{b,d\}, f^{-1}(\{b,d\}) = \{b,d\}$ *which is not* $\tau_1\tau_2$-δ*-pc set.*

Example 2.7 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.1,* $\sigma_1 = \{Y, \phi, \{c\}, \{b,d\}\}$ *and* $\sigma_2 = \{Y, \phi, \{b\}, \{b,d\}, \{a,b,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-δ-σ_1*-cts but it is not* $\tau_1\tau_2$-θ-σ_1*-cts. Since for the* σ_1*-c set* $\{a,b,d\}, f^{-1}(\{a,b,d\}) = \{a,b,d\}$ *which is not* $\tau_1\tau_2$-θ*-c set.*

Example 2.8 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.1,* $\sigma_1 = \{Y, \phi, \{c\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{b\}, \{a,c\}, \{a,b,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-δs-σ_1*-cts but it is not* $\tau_1\tau_2$-θs-σ_1*-cts. Since for the* σ_1*-c set* $\{b,d\}, f^{-1}(\{b,d\}) = \{b,d\}$ *which is not* $\tau_1\tau_2$-θ*-sc set.*

Example 2.9 *Let* $X = Y = \{a,b,c,d\}, \tau_1 = \{\phi, X, \{a,b\}\}$*,* $\tau_2 = \{\phi, X, \{c\}\}\sigma_1 = \{Y, \phi, \{a,b\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,b\}, \{a,b,c\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-M^*-σ_1*-cts but it is not* τ_1-σ_1-σ_1*-cts. Since for the* σ_1*-c set* $\{c,d\}, f^{-1}(\{c,d\}) = \{c,d\}$ *which is not* τ_1*-c set.*

Example 2.10 *Let* $X = Y = \{a,b,c,d\}, \tau_1 = \{\phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}$ *,* $\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a,c\}, \{a,b,d\}\}$*,* $\sigma_1 = \{Y, \phi, \{b\}, \{a,b\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,b\}, \{a,b,d\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-a-σ_1*-cts but it is not* $\tau_1\tau_2$-δ-σ_1*-cts. Since for the* σ_1*-c set* $\{a,c,d\}, f^{-1}(\{a,c,d\}) = \{a,c,d\}$ *which is not* $\tau_1\tau_2$-δ*-c set.*

Example 2.11 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.10,* $\sigma_1 = \{Y, \phi, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,c\}, \{a,b,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-θs-σ_1*-cts but it is not* $\tau_1\tau_2$-M^*-σ_1*-cts. Since for the* σ_1*-c set* $\{b,d\}, f^{-1}(\{b,d\}) = \{b,d\}$ *which is not* $\tau_1\tau_2$-M^**-c set.*

Example 2.12 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.10,* $\sigma_1 = \{Y, \phi, \{a,b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,b\}, \{a,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-e-σ_1*-cts but it is not* $\tau_1\tau_2$-δs-σ_1*-cts. Since for the* σ_1*-c set* $\{c,d\}, f^{-1}(\{c,d\}) = \{c,d\}$ *which is not* $\tau_1\tau_2$-δ*-sc set.*

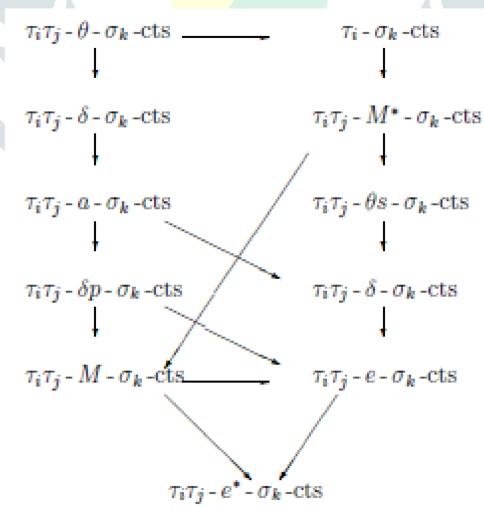
Example 2.13 *Let* $X = Y = \{a,b,c,d\}, \tau_1$ *and* τ_2 *are defined in Example2.10,* $\sigma_1 = \{Y, \phi, \{b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,b\}, \{a,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-M-σ_1*-cts but it is not* $\tau_1\tau_2$-δp-σ_1*-cts. Since for the* σ_1 *-c set* $\{a,c,d\}, f^{-1}(\{a,c,d\}) = \{a,c,d\}$ *which is not* $\tau_1\tau_2$-δ*-pc set.*

Example 2.14 *Let* $X = Y = \{a,b,c\}, \tau_1 = \{\phi, X, \{a\}, \{b\}, \{a,b,\}\}$ *,* $\tau_2 = \{\phi, X, \{a\}, \{a,b\}\}$ *,* $\sigma_1 = \{Y, \phi, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{a\}, \{a,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a* $\tau_1\tau_2$-δp-σ_1*-cts but it is not* $\tau_1\tau_2$-a-σ_1*-cts. Since for the* σ_1*-c set* $\{b,d\}, f^{-1}(\{b,d\}) = \{b,d\}$ *which is not* $\tau_1\tau_2$-a*-c set.*

Example 2.15 *Let* $X = Y = \{a,b,c,\}, \tau_1$ *and* τ_2 *are defined in Example 2.14,* $\sigma_1 = \{Y, \phi, \{b,c\}\}$ *and* $\sigma_2 = \{Y, \phi, \{b\}, \{b,c\}\}$ *. Then the identity map* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is a*

(1) $\tau_1\tau_2$-e^*-σ_1-cts but it is not $\tau_1\tau_2$-e-σ_1-cts, since for the σ_1-c set $\{a\}, f^{-1}(\{a\}) = \{a\}$ which is not $\tau_1\tau_2$-e-c set.

(2) $\tau_1\tau_2$-δp-σ_1-cts but it is not $\tau_1\tau_2$-a-σ_1-cts, since for the σ_1-c set $\{a\}, f^{-1}(\{a\}) = \{a\}$ which is not $\tau_1\tau_2$-a-c set.



Note: $A \to B$ denotes A implies B, but not conversely.

Theorem 2.2 *A map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_i\tau_j$-M^*-σ_k*-cts. iff the inverse image of every* σ_k*-o set in* Y *is* $\tau_i\tau_j$-M^**-o in* X.

Proof. Let G be a σ_k-o set in Y. Then G^c is σ_k-c set in Y. Since f is $\tau_i\tau_j$-M^*-σ_k-cts, $f^{-1}(G^c)$ is $\tau_i\tau_j$-M^*-c in X. That is $f^{-1}(G^c)=(f^{-1}(G))^c$ and so $f^{-1}(G)$ is $\tau_i\tau_j$-M^*-o in (X,τ_1,τ_2).

Conversely, let F be a σ_k-c set in Y. Then F^c is σ_k-p set in Y. By hypothesis, $f^{-1}(F^c)$ is $\tau_i\tau_j$ - M^*-o in X. That is $f^{-1}(F^c)=(f^{-1}(F))^c$ and so $f^{-1}(F)$ is $\tau_i\tau_j$ - M^*-c in (X,τ_1,τ_2). Therefore f is $\tau_i\tau_j$-M^*-σ_k-cts.

Theorem 2.3 *If a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_i\tau_j$ - M^* - σ_k *-cts, then* $f(\tau_i\tau_j$-$M^*cl(A))\subset\sigma_k cl(f(A))$ *holds for every subset* A *of* X.

Proof. Let A be any subset of X. Then $f(A)\subseteq\sigma_k cl(f(A))$ and $\sigma_k cl(f(A))$ is σ_k-c set in Y. Also $f^{-1}(f(A))\subseteq f^{-1}(\sigma_k cl(f(A)))$. That is $A\subseteq f^{-1}(\sigma_k cl(f(A)))$. Since f is $\tau_i\tau_j$-M^*-σ_k- cts, $f^{-1}(\sigma_k cl(f(A)))$ is $\tau_i\tau_j$-M^*-c in (X,τ_1,τ_2). By Theorem 2.7 in [13] $\tau_i\tau_j$-$M^*cl(A)\subseteq f^{-1}(\sigma_k cl(f(A)))$. Therefore $f(\tau_i\tau_j$-$M^*cl(A)\subseteq f(f^{-1}(\sigma_k cl(f(A)))\subseteq\sigma_k cl(f(A))$. Hence $f(\tau_i\tau_j$-$M^*cl(A)\subseteq\sigma_k cl(f(A))$ for every subset A of (X,τ_1,τ_2).

Converse of the above Theorem **Error! Reference source not found.** is not true as seen from the following Example.

Example 2.16 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ *and* $\tau_2=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$ *and* $Y=\{p,q\}$, $\sigma_1=\{\phi,Y,\{p\}\}$ *and* $\sigma_2=\{Y,\phi\}$. *Then* $\tau_2\tau_1$ - $M^*=\{X,\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,c\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\}\}$. *Define a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *by* $f(a)=f(c)=f(d)=p$ *and* $f(b)=q$. *Then* $f((2,1)$ - $M^*cl(A))\subseteq\sigma_1 cl(f(A))$ *for every subset* A *of* X. *But* f *is not* $\tau_2\tau_1$-M^*-σ_1*-cts, since for the* σ_1*-c set* $\{q\}$, $f^{-1}(\{q\})=\{b\}$ *which is not* $(2,1)$-M^**-c set in* (X,τ_1,τ_2).

Theorm 2.4 *If a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_i\tau_j$ - M^* - σ_k *-cts and* $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ *is* η_n-σ_k*-cts, then* gof *if* $\tau_i\tau_j$-M^*-η_n*-cts.*

Proof. Let F be η_n-c set in (Z,η_1,η_2). Since g is η_n-σ_k-cts, $g^{-1}(F)$ is σ_k-c set in (Y,σ_1,σ_2). Since f is $\tau_i\tau_j$ - M^* - σ_k -cts, $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is $\tau_i\tau_j$ - M^*-c set in (X,τ_1,τ_2) and hence gof in $\tau_i\tau_j$-M^*-η_n-cts.

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