$D_M(\tau_i,\tau_j)$-σ_k-continuous Maps and *MC* - *bi* -continuous Maps

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Abstract

The aim of this paper is to introduce and investigate the concept of $D_M(\tau_i,\tau_j)$-σ_k -continous maps which are introduced in a bitopological space in analogy with M - continous maps in topological spaces. Also, we have introduced the concept of M - bi -continuity, M - s - bi -continuity and pairwise M -irresolute in bitopological spaces and study some of the properties.

Keywords and phrases: M - bi -continuity, M - s - bi -continuity, pairwise M -irresolute

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1. Introduction and Preliminaries

Levine in 1963 initiated a new types of open set called semiopen set[8]. A subset A of a space (X,τ) is called regular open (resp., regular closed) [11]if $A=int(cl(A))$ (resp., $A=cl(int(A))$. The delta interior [3] of a subset A of (X,τ) is the union of all regular open sets of X contained in A and is denoted by $\delta int(A)$. A subset A of a space (X,τ) is called δ -open [9] if $A=\delta int(A)$. The complement of δ -open set is called δ -closed. Alternatively, a set A of (X,τ) is called δ -closed [3] if $A=\delta cl(A)$, where $\delta cl(A)=\{x\in X: A\cap int(cl(U))\neq\phi,\ U\in\tau$ and $x\in U\}$. A subset A of a space X is called θ-open [1] if $A=\theta int(A)$, where $\theta int(A)=\bigcup\{int(U): U\subseteq A,\ U\in\tau^c\}$, and a subset A is called θ -semiopen [2] (resp., δ - preopen [9] , e -open [4] and M -open[5]) if $A\subseteq cl(\theta int(A))$ (resp., $A\subseteq int(\delta cl(A))$, $A\subseteq cl(\delta int(A))\cup int(\delta cl(A))$ and $A\subseteq cl(\theta int(A))\cup int(\delta cl(A)))$, where $int()$, $cl()$, $\theta int()$, $\delta int()$ and $\delta cl()$ are the interior, closure, θ-interior, δ -interior and δ -closure operations, respectively. The notion of bitopological spaces (in short, Bts's) was first introduced by Kelly [6].

Throughout this paper, let (X,τ_1,τ_2) or simply X be a Bts and $i,j\in\{1,2\}$. A subset S of a Bts X is said to be $\tau_{1,2}$-open [7] if $S=A\cup B$ where $A\in\tau_1$ and $B\in\tau_2$. A subset S of X is said to be $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. and $\tau_{1,2}$ -clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. For a subset A of X , the interior (resp., closure) of A with respect to τ_i will be denoted by $int_i(A)$ (resp., $cl_i(A)$) for $i=1,2$. In this paper, we introduce and investigate the concept of $D_m(\tau_i,\tau_j)$-σ_k-continous maps in a bitopological space. Also, we have introduced the concept of M - bi -continuity, M - s - bi

-continuity and pairwise M -irresolute in bitopological spaces and study some of the properties. In addition, several properties of these notions and connections to several other known ones are provided.

Let (X,τ_1,τ_2) be a Bts. A subset A of X is called $\tau_i\tau_j$-M -open [10] (briefly, $\tau_i\tau_j$-M -o) if $A\subseteq cl_j(\theta int_i(A))\cup int_i(\delta cl_j(A))$ and A is $\tau_i\tau_j$-M closed (in short, $\tau_i\tau_j$-M -c) if $X\setminus A$ is $\tau_i\tau_j$-M -o. A is pairwise M -open if it is both $\tau_i\tau_j$-M -o and $\tau_j\tau_i$-M -o. A subset A of X is called $\tau_i\tau_j$-θ-semiopen [10] (briefly, $\tau_i\tau_j$-θ-so) if $A\subseteq cl_j(\theta int_i(A))$, $\tau_i\tau_j$-δ -preopen (briefly, $\tau_i\tau_j$ - δ -po) if $A\subseteq int_i(\delta cl_j(A))$, $\tau_i\tau_j$ - e -open if $A\subseteq cl_j(\delta int_i(A))\cup int_i(\delta cl_j(A))$. Clearly A is $\tau_i\tau_j$ - M -c if and only if $int_j(\theta cl_i(A))\cap cl_i(\delta int_j(A))\subseteq A$. We denote the family of all (i,j)-M -c (resp., (i,j)-M -o) sets in a Bts (X,τ_1,τ_2) by $D_{MC}(\tau_i,\tau_j)$ (resp., $D_{MO}(\tau_i,\tau_j)$). The intersection of all $\tau_i\tau_j$-M -c sets containing A is called the $\tau_i\tau_j$-M closure of A, denoted by $\tau_i\tau_j$-M cl(A). i.e., τ_i,τ_j-$Mcl(A)=\bigcap\{U:A\subseteq U,\ U\in D_{MC}(\tau_i,\tau_j)\}$. The union of all $\tau_i\tau_j$-M -o sets contained in A is called the $\tau_i\tau_j$ - M interior of A, denoted by $\tau_i\tau_j$ - $Mint(A)$. i.e., τ_i,τ_j - $Mint(A)=\bigcup\{U:U\subseteq A,\ U\in D_{MO}(\tau_i,\tau_j)\}$.

2. $D_{MC}(\tau_i,\tau_j)$ - σ_k - continuous Maps and MC - bi - continuous Maps

Definition 2.1 *A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called $D_{MC}(\tau_i,\tau_j)$ - σ_k -continuous (in short, $D_{MC}(\tau_i,\tau_j)$-σ_k-cts) if the inverse image of every σ_k-c set is an $\tau_i\tau_j$-M -c set in (X,τ_1,τ_2).*

Remark 2.1 *If $\tau_1=\tau_2=\tau$ and $\sigma_1=\sigma_2=\sigma$ in Definition* ***Error! Reference source not found.****, then the $D_{MC}(\tau_i\tau_j)$-σ_k-continuity of maps coincides with M -continuity of maps in topological spaces.*

Theorem 2.1 *If a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is an*

(i) $\tau_i\tau_j$-σ_k-θ-scts then it is a $\tau_i\tau_j$-σ_k-M -cts.

(ii) $\tau_i\tau_j$-σ_k-δ -pcts then it is a $\tau_i\tau_j$-σ_k-M -cts.

(iii) τ_i-σ_k-θ-cts then it is a $\tau_i\tau_j$-σ_k-θ-scts.

(iv) τ_i-σ_k-θ-cts then it is a τ_i-σ_k-cts.

(v) τ_i-σ_k-cts then it is a $\tau_i\tau_j$-δ-pcts.

(vi) a $\tau_i\tau_j$-σ_k-M -cts then it is a $\tau_i\tau_j$-e-cts.

Proof. (i) Let V be a σ_k-c set. Since f is $\tau_i\tau_j$-σ_k-θ-scts, $f^{-1}(V)$ is $\tau_i\tau_j$-θ-sc. By Proposition 2.1 in [10] $f^{-1}(V)$ is $\tau_i\tau_j$-M -c in (X,τ_1,τ_2). Therefore f is $\tau_i\tau_j$-σ_k-M -cts.

The proof of (ii) to (vi) are similar.

Example 2.1 *Let $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a\}\}$ and $\sigma_2=\{\phi,Y,\{a\},\{b\},\{a,b\}\}$. Then the identity map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $\tau_1\tau_2$-M -cts but is not $\tau_1\tau_2$-θ-scts, since for the σ_1-c set*

$\{b,c,d\}$, $f^{-1}(\{b,c,d\})=\{b,c,d\}$ *which is not* $\tau_1\tau_2$*-*θ*-sc set.*

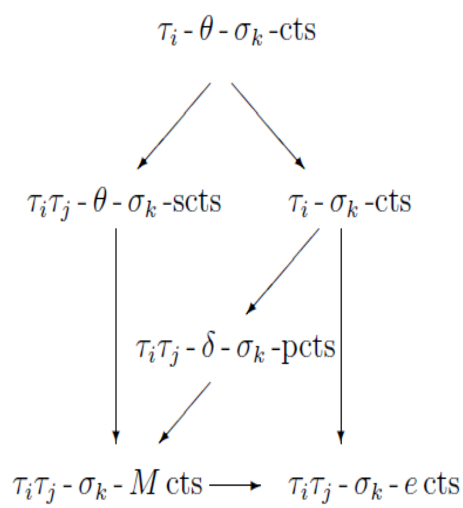
Example 2.2 *Let* $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a\},\{a,d\}\}$ *and* $\sigma_2=\{\phi,Y,\{a\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_1\tau_2$*-*M*-cts but is not* $\tau_1\tau_2$*-*δ*-pcts, since for the* σ_1*-c set* $\{b,c\}$, $f^{-1}(\{b,c\})=\{b,c\}$ *which is not* $\tau_1\tau_2$*-*δ*-pc set.*

Example 2.3 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{b,c,d\}\}$ *and* $\sigma_2=\{\phi,Y,\{a\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_1\tau_2$*-*θ*-scts but is not* τ_1*-*θ*-cts, since for the* σ_1*-c set* $\{a\}$, $f^{-1}(\{a\})=\{a\}$ *which is not* $\tau_1-\theta$*-c set.*

Example 2.4 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{b,c\},\{a,b,c\}\}$ *and* $\sigma_2=\{\phi,Y,\{a\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* τ_1*-cts but is not* τ_1*-*θ*-cts, since for the* σ_1*-c set* $\{a,d\}$, $f^{-1}(\{a,d\})=\{a,d\}$ *which is not* τ_1*-*θ*-c set.*

Example 2.5 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a,b\},\{a,b,d\}\}$ *and* $\sigma_2=\{\phi,Y,\{a\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_1\tau_2$*-*δ*-pcts but is not* τ_1*-cts, since for the* σ_1 *-c set* $\{c,d\}$, $f^{-1}(\{c,d\})=\{c,d\}$ *which is not* τ_1*-c set.*

Example 2.6 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\},\{a,b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a,c\}\}$ *and* $\sigma_2=\{\phi,Y,\{a\}\}$. *Then the identity map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $\tau_1\tau_2$*-*e*-cts but is not* $\tau_1\tau_2$*-*M*-cts, since for the* σ_1*-c set* $\{b,d\}$, $f^{-1}(\{b,d\})=\{b,d\}$ *which is not* $\tau_1\tau_2$*-*M*-c set.*



Not t not conversely.

Theorem 2.2 *A map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is* $D_M(\tau_i,\tau_j)$ - σ_k *-cts. iff the inverse image of every* σ_k*-p set in* Y *is* (τ_i,τ_j)*-*M *-p in X.*

Proof. Let G be a σ_k-p set in Y. Then G^c is σ_k-c set in Y. Since f is $D_M(\tau_i,\tau_j)$ - σ_k -cts, $f^{-1}(G^c)$ is (τ_i,τ_j)-M -c in X. That is $f^{-1}(G^c)=(f^{-1}(G))^c$ and so $f^{-1}(G)$ is (τ_i,τ_j)-M -p in (X,τ_1,τ_2).

Conversely, let F be a σ_k-c set in Y. Then F^c is σ_k-p set in Y. By hypothesis, $f^{-1}(F^c)$ is (τ_i,τ_j)-M -p in X . That is $f^{-1}(F^c)=(f^{-1}(F))^c$ and so $f^{-1}(F)$ is (τ_i,τ_j)-M

-c in (X,τ_1,τ_2). Therefore f is $D_M(\tau_i,\tau_j)$-σ_k-cts.

Theorem 2.3 *If a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $D_{MC}(\tau_i,\tau_j)$ - σ_k-cts, then $f((\tau_i\tau_j)\text{-}M\text{-}cl(A))\subset\sigma_k\text{-}cl(f(A))$ holds for every subset A of X.*

Proof. Let A be any subset of X. Then $f(A)\subseteq\sigma_k\text{-}cl(f(A))$ and $\sigma_k\text{-}cl(f(A))$ is σ_k-c set in Y. Also $f^{-1}(f(A))\subseteq f^{-1}(\sigma_k\text{-}cl(f(A)))$. That is $A\subseteq f^{-1}(\sigma_k\text{-}cl(f(A)))$. Since f is $D_{MC}(\tau_i,\tau_j)$-σ_k-cts, $f^{-1}(\sigma_k\text{-}cl(f(A)))$ is $(\tau_i\tau_j)$-M-c in (X,τ_1,τ_2). By Theorem 2.7 in [10] $\tau_i\tau_j$ - M - $cl(A)\subseteq f^{-1}(\sigma_k$ - $cl(f(A)))$. Therefore $f((\tau_i\tau_j$ - M - $cl(A))\subseteq f(f^{-1}(\sigma_k$ - $cl(f(A)))\subseteq\sigma_k\text{-}cl(f(A))$. Hence $f((\tau_i\tau_j\text{-}M\text{-}cl(A))\subseteq\sigma_k\text{-}cl(f(A))$ for every subset A of (X,τ_1,τ_2).

Converse of the above Theorem 2.3 is not true as seen from the following Example.

Example 2.7 *Let $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau_2=\{\phi,X,\{a\},\{b,c\},\ \{a,b,c\}\}$ and $Y=\{p,q\}$, $\sigma_1=\{\phi,Y,\{p\}\}$ and $\sigma_2=\{Y,\phi\}$. Then $D_M(2,1)=\{X,\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,c\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\}\}$. Define a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=f(c)=f(d)=p$ and $f(b)=q$. Then $f((2,1)$-M-$cl(A))\subseteq\sigma_1\text{-}cl(f(A))$ for every subset A of X. But f is not $D_{MC}(2,1)$-σ_1-cts, since for the σ_1-c set $\{q\}$, $f^{-1}(\{q\})=\{b\}$ which is not $(2,1)$-M-c set in (X,τ_1,τ_2).*

Theorem 2.4 *If a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $D_{MC}(\tau_i,\tau_j)$ - σ_k-cts and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is σ_k-η_n-cts, then gof if $D_{MC}(\tau_i,\tau_j)$-η_n-cts.*

Proof. Let F be η_n-c set in (Z,η_1,η_2). Since g is σ_k-η_n-cts, $g^{-1}(F)$ is σ_k-c set in (Y,σ_1,σ_2). Since f is $D_{MC}(\tau_i\tau_j)$-σ_k-cts, $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is (τ_i,τ_j)-M-c set in (X,τ_1,τ_2) and hence gof in $D_{MC}(\tau_i,\tau_j)$-η_n-cts.

Definition 2.2 *A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called*

(i) M-bi-cts if f is $D_{MC}(1,2)$-σ_2-cts and is $D_{MC}(2,1)$-σ_1-cts.

(ii) M-strongly-bi-cts (briefly M-s-bi-cts) if f is M-bi-cts, $D_{MC}(2,1)$-σ_2-cts and $D_{MC}(1,2)$-σ_1-cts.

(iii) pairwise M-irresolute if $f^{-1}(A)\in D_M(\tau_i,\tau_j)$ in (X,τ_1,τ_2) for every $A\in D_M(k,e)$ in (Y,σ_1,σ_2).

Remark 2.2 *If $\tau_1=\tau_2$ and $\sigma_1=\sigma_2$ simultaneously, then f becomes a M-irresolute map.*

Theorem 2.5 *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a map.*

(i) If f is bi-cts then f is M-bi-cts.

(ii) If f is s-bi-cts then f is M-s-bi-cts.

(iii) If f is θ-s-bi-cts then f is M-bi-cts.

(iv)If f is δ-pcts then f is M-s-bi-cts.

(v) If f is M-bi-cts then f is e-bi-cts.

(vi) If f is M - s - bi -cts then f is e - s - bi -cts.

Proof. (i) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a bi -cts map. Then f is τ_1-σ_1-cts and τ_2-σ_2-cts and so by Theorem 2.1, f is $D_{MC}(1,2)$-σ_2-cts and $D_{MC}(2,1)$-σ_1-cts. Thus f is M - bi -cts.

The proof of (ii) to (vi) are similar.

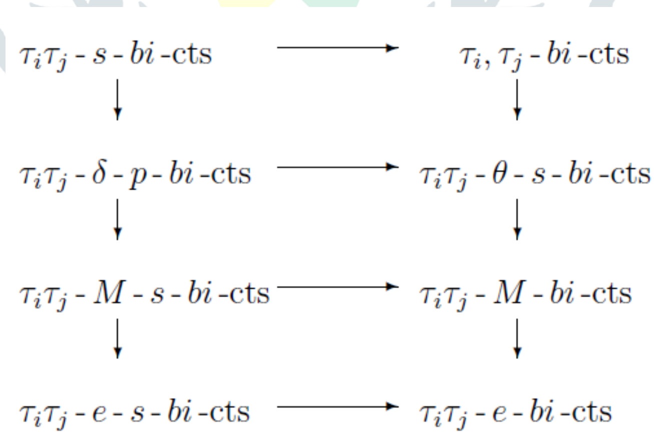
The converse of this Theorem 2.5 need not be true in general as seen from the following Examples.

Example 2.8 *Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{q\},\{b\},\{a,b\}\}$ and $\tau_2=\{X,\phi,\{a\},\{b,c\},\ \{a,b,c\}\}$ and $Y=\{p,q\}$, $\sigma_1=\{Y,\phi\}$ and $\sigma_2=\{Y,\phi,\{p\}\}$. Define a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=f(b)=f(c)=q$ and $f(d)=p$. Then f is M - s - bi -cts but not s - bi -cts. This map is also M - bi -cts but not bi -cts.*

Example 2.9 *Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\tau_2=\{X,\phi,\{a\},\{b,c\},\ \{a,b,c\}\}$ and $Y=\{p,q\}$, $\sigma_1=\{Y,\phi\}$ and $\sigma_2=\{Y,\phi,\{p\}\}$. Define a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=f(b)=f(c)=q$ and $f(d)=p$. Then this function f is M - bi -cts but not θ-s-bi -cts. This map is also M - s - bi -cts but not δ -p-bi -cts.*

Example 2..10 *Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\tau_2=\{X,\phi,\{a\},\{b,c\},\ \{a,b,c\}\}$ and $Y=\{p,q\}$, $\sigma_1=\{Y,\phi\}$ and $\sigma_2=\{Y,\phi,\{p\}\}$. Define a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=f(b)=p$ and $f(c)=f(d)=q$. Then this function f is e - bi -cts but it is not M - bi -cts. This map is also e - s - bi -cts but not M - bi -cts.*

Remark 2.3 *The following diagram summarizes the above discussions.*



Note: $A\to B$ denotes A implies B, but not conversely.

Theorem 2.6 *If a map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise M -irresolute, then f is $D_M(\tau_i,\tau_j)$-σ_e-cts.*

Proof. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be pairwise M -irresolute and F be a σ_e-c set in (Y,σ_1,σ_2). Then F is (k,e) - M -c in (Y,σ_1,σ_2) by Proposition 2.1 in **Error! Reference source not found.**. By hypothesis, $f^{-1}(F)$ is (τ_i,τ_j)-M -c set in (X,τ_1,τ_2).

Therefore f is $D_M(\tau_i,\tau_j)$-σ_e-cts.

The converse of this Theorem [10] is not true in general as seen from the following Example.

Example 2.11 *Let* $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ *and* $\tau_2=\{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$ *and* $Y=\{p,q\}$, $\sigma_1=\{Y,\phi\}$ *and* $\sigma_2=\{Y,\phi,\{p\}\}$. *Define a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *by* $f(b)=f(c)=p$ *and* $f(a)=f(d)=q$. *Then* f *is* (τ_1,τ_2)-M-σ_2*-cts but it is not pairwise* M*-irresolute, since for the* (τ_1,τ_2)-M*-c set* $\{p\}$ *in* (Y,σ_1,σ_2), $f^{-1}(\{p\})=\{b,c\}$ *which is not* (τ_1,τ_2)-M*-c set in* (X,τ_1,τ_2).

Theorem 2.7 *A map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is pairwise* M*-irresolute iff the inverse image of every* (k,e)-M*-o set in* (Y,σ_1,σ_2) *is* (τ_i,τ_j)-M*-o set in* (X,τ_1,τ_2).

Proof. Proof is similar to that of Theorem 2.2

Theorem 2.8 *If* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *and* $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ *are two pairwise* M*-irresolute maps, then their composition* gof *is also pairwise* M*-irresolute.*

Proof. Let $A\in D_M(m,n)$ in (Z,η_1,η_2). Since g is pairwise M-irresolute, $g^{-1}(A)\in D_M(k,e)$ in (Y,σ_1,σ_2). Since f is pairwise M-irresolute $f^{-1}(g^{-1}(A))=(gof)^{-1}(A)\in D_M(\tau_i,\tau_j)$. Hence gof is pairwise M-irresolute.

Theorem 2.9 *If a map* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is pairwise* M*-irresolute and* $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ *is* $D_M(k,e)$-η_n*-cts, then* $gof:(X,\tau_1,\tau_2)\to(Z,\eta_1,\eta_2)$ *is* $D_M(\tau_i,\tau_j)$-η_n*-cts.*

Proof. Let F be a η_n-c set in (Z,η_1,η_2). Since g is $D_M(k,e)$-η_n-cts, $g^{-1}(F)\in D_M(k,e)$ in (Y,σ_1,σ_2). Since f is pairwise M-irresolute, $f^{-1}(g^{-1}(A))=(gof)^{-1}(A)\in D_M(\tau_i,\tau_j)$ in (X,τ_1,τ_2) and hence gof is $D_M(\tau_i,\tau_j)$-η_n-cts.

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