Fuzzy rw -Connectedness and Fuzzy rw -Disconnectedness in Fuzzy Bitopological Spaces

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In this paper, two weaker forms of connectedness namely, pairwise fuzzy super rw-disconnectedness and pairwise fuzzy strong rw-connectedness are introduced and studied in fuzzy bitopological spaces. Also two stronger forms of rw-disconnectedness namely, pairwise fuzzy total rw-disconnectedness and pairwise fuzzy extremally rw-disconnectedness in fuzzy bitopological spaces are defined and studied. Some characterizations and properties of these spaces are also established.

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1. Introduction and Preliminaries

Fuzzy relations were first introduced by Lofti A. Zadeh in 1971 [24]. Fuzzy sets have application in applied fields such as information [18], control [19, 20] and pattern reognition [13, 17]. This concept has been applied by many authors to several branches of mathematics particuarly in the fields such as fuzzy numbers[5], fuzzy groups[21], fuzzy topological groups[7], L-fuzzy sets[9], fuzzy linear spaces[10], fuzzy algebra [12] fuzzy vector spaces [15] and fuzzy proximity space[16]. One of the applications was the study of fuzzy topological spaces [4] introduced and studied by chang in 1968. Ever since the introducting of fuzzy topological spaces notions like fuzzy regular open [1] and fuzzy regular semiopen [23] were extended from general topological structures.

The concept of rw-open sets and fuzzy rw-open sets was introduced and studied by Wali[23], fuzzy rw-connectedness and fuzzy rw-disconnectedness was introduced and studied by Vadivel [22] motivated us to study on fuzzy rw-connectedness in fuzzy bitopological space [14] (in short fbts). In this paper, two weaker forms of connectedness namely, pairwise fuzzy super rw-disconnectedness and pairwise fuzzy strong rw-connectedness are introduced and studied in fuzzy bitopological spaces. Also two stronger forms of rw-disconnectedness namely, pairwise fuzzy total rw-disconnectedness and pairwise fuzzy extremally rw-disconnectedness in fuzzy bitopological spaces are defined and studied. Some characterizations and properties of these spaces are also established.

Throughtout the paper I will denote the unit interval $[0,1]$ of the real line R. X, Y, Z will be nonempty sets. The symbols $\lambda, \mu, \gamma, \eta \ldots$ are used to denote fuzzy sets and all other symbols have their usual meaning unless explicity stated.

Definition 1.1 *A fuzzy set λ in a fts (X, τ) is said to be fuzzy regular open set [1] if $int(cl\lambda) = \lambda$ and a fuzzy regular closed set if $cl(int(\lambda)) = \lambda$,*

(iii) fuzzy regular semi open [25] if there exists fuzzy regular open set σ in X such that $\sigma \le \alpha \le cl(\sigma)$.

Definition 1.2 *A fts (X,T) is said to be fuzzy connected [11] if it has no proper fuzzy clopen set. Otherwise it is called fuzzy disconnected.*

Definition 1.3 *A fuzzy set λ in fts (X,T) is proper if $\lambda \neq 0$ and $\lambda \neq 1$.*

Definition 1.4 *A fts (X,T) is called fuzzy super connected [6] if it has no proper fuzzy regular open set.*

Definition 1.5 *A fts (X,T) is called fuzzy strongly connected [2] if it has no non-zero fuzzy closed sets λ and μ such that $\lambda+\mu \leq 1$.*

Definition 1.6 *A fts (X,T) is said to be fuzzy extremally disconnected [2] if $\lambda \in T$ implies $Cl(\lambda) \in T$.*

Definition 1.7 *A fts (X,T) is said to be fuzzy totally disconnected [2] if and only if for every pair of fuzzy points p,q with $p \neq q$ in (X,T) there exists non-zero fuzzy open sets λ, μ such that $\lambda+\mu=1$, λ contains p and μ contains q. Suppose $A \subset X$. A is said to be a fuzzy totally disconnected subset of X if A as a fuzzy subspace of (X,T) is fuzzy totally disconnected.*

Definition 1.8 *Let (X,T) be fts and Y be an ordinary subset of X. Then $T/Y=\{\lambda/Y : \lambda \in T\}$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology. The pair $(Y,T/Y)$ is called a fuzzy subspace [8] of (X,T). $(Y,T/Y)$ is called fuzzy open (fuzzy closed) subspace if the characteristic function of Y viz., χ_Y is fuzzy open (fuzzy closed).*

Definition 1.9 *A fuzzy set λ in a fts (X,τ) is said to be a fuzzy regular weakly closed set (briefly, frw-closed) [23] if $cl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy regular semi-open in X.*

From this definition it is clear that λ is fuzzy rw-open $\Leftrightarrow 1-\lambda$ is fuzzy rw-closed. Also we define T_i-fuzzy rw-interior and T_i-fuzzy rw-closure of λ, denoted by $rw\text{-}Int_{T_i}(\lambda)$ and $rw\text{-}Cl_{T_i}(\lambda)$ respectively, for $i=1,2,$ as follows:

(i) $rw\text{-}Int_{T_i}(\lambda) = \vee\{\mu / \mu$ is T_i- fuzzy rw-open set and $\mu \leq \lambda\}$

(ii) $rw\text{-}Cl_{T_i}(\lambda) = \wedge\{\mu / \mu$ is T_i- fuzzy rw-closed set and $\mu \geq \lambda\}$

For any fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) we have the following results. (i) $1-rwCl_{T_i}(\lambda) = rwInt_{T_i}(1-\lambda)$ and (ii) $1-rwInt_{T_i}(\lambda) = rwC_{T_i}(1-\lambda)$ for $i=1,2$.

Definition 1.10 *A fuzzy set λ in fbts (X,T_1,T_2) is said to be (T_1,T_2)-fuzzy rwclopen if it is T_1-fuzzy rw-closed as well as T_2-fuzzy rw-open.*

A fuzzy set λ in a fbts (X,T_1,T_2) is T_i-fuzzy rw-open iff $\lambda = rwInt_{T_i}(\lambda)$ and T_i-fuzzy rw-closed iff $\lambda = rwCl_{T_i}(\lambda)$ for $i=1,2$.

We shall denote the class of all fuzzy rw-open sets in T_i and fuzzy rw-closed sets in T_i of the fuzzy bitopological space (X,T_1,T_2) by $FRWO_{T_i}$ and $FRWC_{T_i}$ for $i=1,2$.

Pairwise Fuzzy rw-Connected Spaces

Definition 2.1 *A fbts* (X,T_1,T_2) *is said to be pairwise fuzzy* rw*-connected* $\Leftrightarrow X$ *has no proper fuzzy sets* λ_1 *and* λ_2 *which are* T_1*-fuzzy* rw*-open and* T_2*-fuzzy* rw*-open respectively such that* $\lambda_1+\lambda_2=1$. *A fbts* (X,T_1,T_2) *is pairwise fuzzy* rw*-disconnected if it is not pairwise fuzzy* rw*-connected.*

Remark 2.1 *The pairwise fuzzy* rw*-connectedness of* (X,T_1,T_2) *is not governed by the fuzzy* rw*-connectedness of the spaces* (X,T_1) *and* (X,T_2) *as the following example shows:*

Example 2.1 *Let* $X=\{a,b\},T_1$ *be the discrete fuzzy topology,* T_2 *be the indiscrete fuzzy topology,* $T_3=\{0,1,\lambda\}$ *where* $\lambda:X\to[0,1]$ *is such that* $\lambda(x)=0$ *if* $x=b,\lambda(x)=1$ *if* $x=a$ *and* $T_4=\{0,1,\mu\}$ *where* $\mu:X\to[0,1]$ *is such that* $\mu(x)=0$ *if* $x=a$ *and* $\mu(x)=1$ *if* $x=b$. *Then* (X,T_1,T_2) *is pairwise fuzzy* rw*-connected but* (X,T_1) *is not fuzzy* rw*-connected and* (X,T_2) *is fuzzy* rw*-connected[22]. Also* (X,T_3,T_4) *is not pairwise fuzzy* rw*-connected but* (X,T_3) *and* (X,T_4) *are both fuzzy* rw*-connected.*

The following proposition gives some characterizations of pairwise fuzzy rw-connected fbts.

Proposition 2.1 *The following statements are equivalent for a fbts* (X,T_1,T_2).

(i) (X,T_1,T_2) is pairwise fuzzy rw-connected.

(ii) There exists no T_1-fuzzy rw-open set $\lambda_1\neq 0$ and a T_2-fuzzy rw-open set $\lambda_2\neq 0$ such that $\lambda_1+\lambda_2=1$.

(iii) There exists no T_1-fuzzy rw-closed set $\lambda_1\neq 1$ and a T_2-fuzzy rw-closed set $\lambda_2\neq 1$ such that $\lambda_1+\lambda_2=1$.

(iv) X contains no fuzzy set $\lambda\neq 0,1$ and it is both T_1-fuzzy rw-open and T_2-fuzzy rw-closed or both T_2-fuzzy rw-open and T_1-fuzzy rw-closed.

Proof. (i) $\Rightarrow$ (ii). Assume that (i) is true. Then (ii) follows from the definition 2.1

(ii) $\Rightarrow$ (iii). Assume that (ii) is true. Let us suppose that there exists a T_1-fuzzy rw-closed set $\lambda_1\neq 1$ and a T_2-fuzzy rw-closed set $\lambda_2\neq 1$ such that $\lambda_1+\lambda_2=1$. Then $(1-\lambda_1)\neq 1-1\neq 0$ is a non-zero T_1-fuzzy rw-open set. Similarly we get $1-\lambda_2$ is a non-zero T_2-fuzzy rw-open set.

Now

$$(1-\lambda_1)+(1-\lambda_2)=2-(\lambda_1+\lambda_2)=2-1=1.$$

This is a contradiction to (ii). Hence (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (iv). Assume that (iii) is true. Suppose that X contains a fuzzy set $\lambda\neq 0,1$ which is both T_1-fuzzy rw-open and T_2-fuzzy rw-closed. Then $(1-\lambda)$ is a proper T_1-fuzzy rw-closed set. Also by assumption λ is T_2-fuzzy rw-closed. Now $(1-\lambda)+\lambda=1$. This is a contradiction to (iii). Hence (iii) $\Rightarrow$ (iv).

(iv) $\Rightarrow$ (i). Consider (iv) is true. Let us suppose that (X,T_1,T_2) is not pairwise fuzzy rw-connected. Then X has proper fuzzy sets λ_1 and λ_2 where λ_1 is T_1-fuzzy rw-open set and λ_2 is T_2-fuzzy rw-open set respectively, such that $\lambda_1+\lambda_2=1$.

Now $\lambda_1+\lambda_2=1$ implies $\lambda_1=1-\lambda_2$. This implies that λ_1 is both T_2- fuzzy rw-closed and $\lambda_1\neq 1$ as λ_2 is a non-zero T_2- fuzzy rw-open set. Clearly $\lambda_1\neq 0,1$ is in X. This is a contradiction to (iv). Hence (iv) $\Rightarrow$ (i).

3. Pairwise Fuzzy Super rw-Connected Spaces

Definition 3.1 *Let* (X,T_1,T_2) *be any fbts and let* λ *be any fuzzy set in* X. *Then*

(i) λ is called $(1,2)$ fuzzy regular rw-open if $rwInt_{T_1}[rwCl_{T_2}(\lambda)]=\lambda$

(ii) λ is called $(2,1)$ fuzzy regular rw-open if $rwInt_{T_2}[rwCl_{T_1}(\lambda)]=\lambda$ and

(iii) λ is called pairwise fuzzy regular rw-open if it is both $(1,2)$ fuzzy regular rw-open and $(2,1)$ fuzzy regular rw-open.

Definition 3.2 *Let* (X,T_1,T_2) *be a fbts. Then* (X,T_1,T_2) *is called pairwise fuzzy super* rw*-connected if it has no proper* $(\neq 0,1)$ *pairwise fuzzy regular* rw*-open set.*

Example 3.1 *Let* $X=\{a,b\}, T_1=\{0,1,\lambda\}$ *and* $T_2=\{0,1,\mu\}$ *where* $\lambda: X\to[0,1]$ *is such that* $\lambda(x)=\frac{1}{4}$ *if* $x=a, \lambda(x)=0$ *if* $x=b$ *and* $\mu: X\to[0,1]$ *is such that* $\mu(x)=\frac{3}{4}$ *if* $x=a, \mu(x)=1$ *if* $x=b$. *Then the fbts* (X,T_1,T_2) *is clearly pairwise fuzzy super* rw*-connected space eventhough* (X,T_1,T_2) *is pairwise fuzzy* rw*-disconnected space.*

Proposition 3.1 *If* (X,T_1,T_2) *is any fbts, then (i)* $\Rightarrow$ *(ii) and (ii)* $\Rightarrow$ *(iii) where*

(i) (X,T_1,T_2) is pairwise fuzzy super rw-connected.

(ii)_ The T_2-rw-closure (or T_1-rw-closure) of a pariwise fuzzy regular rw-open set which is different from zero is 1.

(iii) The T_2-rw-interior (or T_1-rw-interior) of a pairwise fuzzy regular rw-closed set which is different from 1 is zero.

Proof. (i) $\Rightarrow$ (ii). Assume (i) is true. Suppose that there exists a pairwise fuzzy regular rw-open set $\lambda\neq 0$ such that $rwCl_{T_2}(\lambda)\neq 1$. Then

$$rwInt_{T_1}[rwCl_{T_2}(\lambda)]\neq 1.(I)$$

But since λ is pairwise fuzzy regular open set ,

$$rwInt_{T_1}[rwCl_{T_2}(\lambda)]=\lambda.(II)$$

From (I) and (II) we get $\lambda\neq 1$. Thus we find that (X,T_1,T_2) has a proper pairwise fuzzy regular rw-open set λ .This is a contradiction to (i). Hence (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (iii). Assume (ii) is true. Suppose that there exists a pairwise fuzzy regular rw-closed set $\lambda\neq 1$, such that $rwInt_{T_2}(\lambda)\neq 0$.

Now $\mu=(1-\lambda)\neq 0$ and μ is a non-zero pairwise fuzzy regular rw-open set. Then

$$rwCl_{T_2}(\mu)=1-rwInt_{T_2}(1-\mu)$$
$$=1-rwInt_{T_2}(\lambda)$$
$$\neq 1(\text{ since } rwInt_{T_2}(\lambda)\neq 0)$$

which is a contradiction to our assumption (ii). Hence (ii) $\Rightarrow$ (iii). This completes the proof of the proposition.

Remark 3.1 *The following examples gives the relation between pairwise fuzzy super rw-connectedness , pairwise fuzzy rw-connectedness and pairwise fuzzy rw-disconnectedness.*

Example 3.1 *Let $X=\{a,b\}, T_1=\{0,1,\lambda\}$ and $T_2=\{0,1,\mu\}$ where λ and μ are respectively defined as $\lambda : X \to [0,1]$ such that $\lambda(x)=1$ if $x=a, \lambda(x)=0$ if $x=b$ and $\mu : X \to [0,1]$ such that $\mu(x)=1$ if $x=b, \mu(x)=0$ if $x=a$. Then fbts it is pairwise fuzzy super rw-connected but (X,T_1,T_2) is not pairwise fuzzy rw-connected space.*

Example 3.2 *Let $X=\{a,b\}, T_1=\{0,1,\lambda\}$ and $T_2=\{0,1,\mu\}$ where λ and μ are respectively defined as $\lambda : X \to [0,1]$ such that $\lambda(x)=\frac{1}{4}$ if $x=a, \lambda(x)=0$ if $x=b$ and $\mu : X \to [0,1]$ such that $\mu(x)=\frac{3}{4}$ if $x=b, \mu(x)=0$ if $x=a$. Then fbts (X,T_1,T_2) is pairwise fuzzy super rw-disconnected but it is not pairwise fuzzy rw-disconnected space.*

4.Pairwise Fuzzy Strongly rw-Connected Spaces

Definition 4.1 *A fbts (X,T_1,T_2) is said to be pairwise fuzzy strongly rw-connected if it has no proper fuzzy sets $\lambda_1, \lambda_2 \in FRWC_{T_1} \cup FRWC_{T_2}$ such that $\lambda_1+\lambda_2 \le 1$. If (X,T_1,T_2) is not pairwise fuzzy strongly rw-connected then it will be called pairwise fuzzy weakly rw-connected*

Proposition 4.1 *A fbts (X,T_1,T_2) is pairwise fuzzy strongly rw-connected $\Leftrightarrow$ it has no proper fuzzy rw-open sets $\lambda, \mu \in FRWO_{T_1} \cup FRWO_{T_2}$ such that $\lambda+\mu \ge 1$.*

Proof. (X,T_1,T_2) is pairwise fuzzy weakly rw-connected $\Leftrightarrow$ it has proper fuzzy sets $f,k \in FRWC_{T_1} \cup FRWC_{T_2}$ such that $f+k \le 1$ $\Leftrightarrow$ it has proper fuzzy sets $\lambda, \mu \in FRWO_{T_1} \cup FRWO_{T_2}$ where $\lambda = 1-f, \mu = 1-k$ such that $\lambda+\mu \ge 1$.

Remark 4.1 *Pairwise fuzzy strong rw-connectedness implies pairwise fuzzy rw-connectedness. However the converse is not true.*

Example 4.1 *Let $X=[0,1]$. Define $\lambda : X \to I$ such that $\lambda(x)=\frac{2}{3}$ for all $x \in X$ and $\mu : X \to [0,1]$ is such that $\mu(x)=\frac{1}{2}$ for all $x \in X$. Let $T_1=\{0,1,\lambda\}, T_2=\{0,1,\mu\}$. Then fbts (X,T_1,T_2) is pairwise fuzzy rw-connected but not pairwise fuzzy strongly rw-connected.*

Proposition 4.2 *Let (X,T_1,T_2) be any fbts and $A \subset X$ be any subset. Then the following statements are equivalent.*

(i) $(A, T_1/A, T_2/A)$ is pairwise fuzzy strongly rw-connected subspace of (X,T_1,T_2).

(ii) For any proper fuzzy sets $\lambda_1, \lambda_2 \in T_1 \cup T_2, 1_A \le \lambda_1/A + \lambda_2/A \le \lambda_1+\lambda_2$ implies either $1_A = \lambda_1/A$ or $1_A = \lambda_2/A$.

Proof. (ii) $\Rightarrow$ (i). Suppose A is not pairwise fuzzy strongly rw-connected subset of X.

Then there exists proper fuzzy sets $f,k \in FRWC(T_1/A) \cup FRWC(T_2/A)$ such that $f+k \le 1_A$. Therefore we can find proper fuzzy sets $\lambda_1,\lambda_2 \in T_1 \cup T_2$ such that $\lambda_1/A = 1_A - f, \lambda_2/A = 1_A - k$. Then

$$\lambda_1/A + \lambda_2/A = (1_A - f) + (1_A - k)$$
$$= 2 - (f+k)$$

$$(ie) \qquad \lambda_1/A + \lambda_2/A \ge 1_A \{\text{since} f+k \le 1_A\} (I)$$

Since

$$0 < \lambda_1 < 1_A (II)$$

and

$$0 < \lambda_2 < 1_A, (III)$$

we have from (I), (II) and (III) we get $1_A \ne \lambda_1/A$ and $1_A \ne \lambda_2/A$. This proves (ii) $\Rightarrow$ (i).

Now we shall prove (i) $\Rightarrow$ (ii). Suppose there exits proper fuzzy sets $\lambda_1,\lambda_2 \in FRWO_{T_1} \cup FRWO_{T_2}$ such that $1_A \le \lambda_1/A + \lambda_2/A$ but both $1_A \ne \lambda_1/A$ and $1_A \ne \lambda_2/A$. This shows, by Proposition 4.1 , A is not pairwise fuzzy strongly rw-connected. Thus we have shown (i) $\Rightarrow$ (ii). Hence the proposition.

Proposition 4.3 *Let $F \subset (X,T_1,T_2)$ be such that $\chi_F \in FRWC_{T_1} \cup FRWC_{T_2}$. Then (X,T_1,T_2) is pairwise fuzzy strongly rw-connected implies $(F,T_1/F,T_2/F)$ is pairwise fuzzy strongly rw-connected.*

Proof. We shall assume that (X,T_1,T_2) is pairwise fuzzy strongly rw-connected. Let $F \subset (X,T_1,T_2)$ be such that $\chi_F \in FRWC_{T_1} \cup FRWC_{T_2}$. We want to show that $(F,T_1/F,T_2/F)$ is pairwise fuzzy strongly rw-connected. Suppose $(F,T_1/F,T_2/F)$ is not pairwise fuzzy strongly rw-connected. This means there exists proper fuzzy sets $f,k \in FRWC(T_1/F) \cup FRWC(T_2/F)$ such that

$$f + k \le 1_F.(I)$$

Hence we can find proper fuzzy sets $\lambda_1,\lambda_2 \in FRWC_{T_1} \cup FRWC_{T_2}$ such that $f = \lambda_1/F, k = \lambda_2/F$. Now consider $\lambda_1 \wedge \chi_F + \lambda_2 \wedge \chi_F$. Since $\chi_F \in FRWC_{T_1} \cup FRWC_{T_2}, \lambda_1 \wedge \chi_F, \lambda_2 \wedge \chi_F \in FRWC_{T_1} \cup FRWC_{T_2}$. Further from (I) we find

$$\lambda_1 \wedge \chi_F + \lambda_2 \wedge \chi_F \le 1_X.(II)$$

This shows (X,T_1,T_2) is not pairwise fuzzy strongly rw-connected, which is a contradiction. Hence the proposition.

5.Pairwise Fuzzy Extremally rw-Disconnected Spaces

Definition 5.1 *A fbts (X,T_1,T_2) is said to be pairwise fuzzy extremally rw-disconnected if T_1-fuzzy rw-closure of each T_2-fuzzy rw-open set is T_2-fuzzy rw-open and T_2-fuzzy rw-closure of each T_1-fuzzy rw-open set is T_1-fuzzy rw-open.*

Example 5.1 *Let $X = \{a,b,c,d\}$. Define $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\mu_1,\mu_2,\mu_3,\mu_4 : X \to I$ as follows:*

$$\lambda_1(x) = \begin{cases} 1 & x = b \\ 0 & x = a, c, d \end{cases}, \qquad \lambda_2(x) = \begin{cases} 1 & x = a, b \\ 0 & x = c, d \end{cases}$$

$$\lambda_3(x) = \begin{cases} 1 & x = b, d \\ 0 & x = a, c \end{cases}, \qquad \lambda_4(x) = \begin{cases} 1 & x = a, b, d \\ 0 & x = c \end{cases}$$

$$\mu_1(x) = \begin{cases} 1 & x = c \\ 0 & x = a, b, d \end{cases}, \qquad \mu_2(x) = \begin{cases} 1 & x = a, c \\ 0 & x = b, d \end{cases},$$

$$\mu_3(x) = \begin{cases} 1 & x = c, d \\ 0 & x = a, b \end{cases} \qquad \mu_4(x) = \begin{cases} 1 & x = a, c, d \\ 0 & x = b \end{cases}.$$

Clearly $T_1 = \{0,1,\lambda_1,\lambda_2,\lambda_3,\lambda_4\}$ and $T_2 = \{0,1,\mu_1,\mu_2,\mu_3,\mu_4\}$ are fuzzy topologies on X. Then we can easily see that the *fbts* (X,T_1,T_2) is pairwise fuzzy extremally rw-disconnected eventhough both (X,T_1) and (X,T_2) are fuzzy rw-connected spaces.

Proposition 5.1 *For any fbts* (X,T_1,T_2), *(i)* $\Rightarrow$ *(ii)* $\Rightarrow$ *(iii)* $\Rightarrow$ *(i) where*

(i) (X,T_1,T_2) is pairwise fuzzy extremally rw-disconnected,

(ii) Whenever λ is a T_1-fuzzy rw-closed set, $rwInt_{T_2}(\lambda)$ is a T_1-fuzzy rw-closed set. Similarly whenever μ is a T_2-fuzzy rw-closed set $rwInt_{T_1}(\mu)$ is a T_2-fuzzy rw-closed set,

(iii) Whenever λ is a T_1-fuzzy rw-open set (T_2-fuzzy rw-open set) we have

$$rwCl_{T_1}[1-rwCl_{T_2}(\lambda)] = 1-rwCl_{T_2}(\lambda)$$
$$(rwCl_{T_2}[1-rwCl_{T_1}(\lambda)] = 1-rwCl_{T_1}(\lambda)).$$

Proof. (i) $\Rightarrow$ (ii) Suppose (i) is true. Let λ be a T_1-fuzzy rw-closed set. Then $(1-\lambda)$ is a T_1-fuzzy rw-open set. Then from (i) $rwCl_{T_2}(1-\lambda)$ is T_1- fuzzy rw-open set. Clearly $1-rwCl_{T_2}(1-\lambda)$ is T_1-fuzzy rw-closed set. But $1-rwCl_{T_2}(1-\lambda) = rwInt_{T_2}(\lambda)$ and so $rwInt_{T_2}(\lambda)$ is T_1-fuzzy rw-closed set. Hence (i) $\Rightarrow$ (ii). Similar statement for a T_2-fuzzy rw-closed set also holds.

(ii) $\Rightarrow$ (iii). Assume that (ii) is true. Suppose λ is a T_1-fuzzy rw-open set. Then $(1-\lambda)$ is a T_1-fuzzy rw-closed set. Now $rwCl_{T_2}(\lambda)$ is T_1-fuzzy rw-open and therefore $1-rwCl_{T_2}(\lambda)$ is T_1-fuzzy rw-closed.

Therefore $rwCl_{T_1}[1-rwCl_{T_2}(\lambda)] = 1-rwCl_{T_2}(\lambda)$. Similarly, we can show that $rwCl_{T_2}[1-rwCl_{T_1}(\lambda)] = 1-rwCl_{T_1}(\lambda)$ when λ is T_2-fuzzy rw-open set. Hence (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (i) Assume that (iii) is true. Then λ is a T_1-fuzzy rw-open set, then

$$rwCl_{T_1}[1-rwCl_{T_2}(\lambda)] = 1-rwCl_{T_2}(\lambda).(I)$$

We claim $rwCl_{T_2}(\lambda)$ is T_1-fuzzy rw-open. From (I) we have $1-rwCl_{T_2}(\lambda)$ is a T_1-fuzzy rw-closed and therefore $rwCl_{T_2}(\lambda)$ is T_1-fuzzy rw-open which we want to prove. Similarly we can show that whenever μ is a T_2-fuzzy rw-open set then $rwCl_{T_1}(\mu)$ is T_2-fuzzy rw-open. Hence, (iii) $\Rightarrow$ (i).

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