(i,j)-$I_{r\omega}$ CLOSED SETS IN BITOPOLOGICAL SPACES

Mohanarao Navuluri and A. Vadivel

[1]Department of Mathematics, Govt. College of Engg. Bodinayakkanur-2; Mathematical Section (FEAT), Annamalai University, AnnamalaiNagar-608002, Tamilnadu.

[2]Post Graduate and Research Department of Mathematics, Government Arts College (Autonomous), Karur - 639 005, Tamilnadu; Department of Mathematics, Annamalai University, Annamalai Nagar- 608 002,Tamilnadu.

Abstract

The aim of this paper is to introduce the concepts of (i,j)-regular weakly closed sets, (i,j)-regular weakly open sets and study their basic properties in ideal bitopological spaces. In particular, it is proved that (i,j)-I_{rw} closed sets are closed under finite unions. Also some relations are given and we establish that some relations are not reversible, which are justified with suitable examples. Further the necessary and sufficient condition for a subset A of an ideal bitopological space (X,τ_1,τ_2,I) to be an (i,j)-I_{rw}-open set is established.

Key words and phrases: (i,j)-I_{rw}-closed sets, (i,j)-I_{rw}-open sets.

AMS (2000) subject classification: 54C10.

1 .Introduction and Preliminaries

The concept of bitopological space was introduced by J. C. Kelly [5]. On the other hand S. S. Benchalli and R. S. Wali [2] introduced the concepts of regular weakly closed sets and regular weakly open sets in a topological space. R. S. Wali [10] introduced the concepts of regular weakly closed sets and regular weakly open sets in bitopological spaces.

Recently many authors are introducing the topological concepts with respect to an ideal I. For instant, Palaniappan and Alagar [7] introduced regular generalized closed sets and regular generalized locally closed sets with respect to an ideal. Alagar and Thenmozhi [1] introduced regular generalized star closed sets with respect to an ideal. T. Noiri and N. Rajesh [6] introduced (i,j)-I_g closed sets in bitopological spaces.

In this paper, we introduce the concepts of (i,j)-regular weakly closed sets with respect to an ideal $\{(i,j)$-I_{rw}-closed sets$\}$ and (i,j)-regular weakly open sets with resepect to an ideal $\{(i,j)$-I_{rw}-open sets$\}$ and study their basic properties in ideal bitopological spaces.

2.Preliminaries

Let (X,τ_1,τ_2,I) or simply X denote an ideal bitopological space. The intersection (resp. union) of all τ_i-semi closed sets containing A (resp. τ_i-semi open sets contained in A) is called the τ_i-semi closure (resp. τ_i-semi interior) of A, denoted by τ_i-$scl(A)$ {resp. τ_i-$sint(A)$ }. For any subset $A \subseteq X$, τ_i-$int(A)$ and τ_i-$cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i respectively. The closure and interior of B relative to A with respect to the topology τ_i are written as τ_i - $cl_A(B)$ and τ_i - $int_B(A)$ respectively. The set of all τ_i-regular closed sets in X is denoted by τ_i-$R.C(X,\tau_1,\tau_2)$. The set of all τ_j-regular open sets in X is denoted by τ_j-$R.O(X,\tau_1,\tau_2)$. A^C denotes the complement of A in X unless explicitly stated.

We shall require the following known definitions :

Definition 2.1 *A subset A of a space (X,τ) is called*

(i) regular open [8] if $A = int(cl(A))$ and regular closed [8] if $A = cl(int(A))$,

(ii) regular semiopen [3] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$,

(iii) rw-closed [2] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen,

(iv) rw-open [2] if $X - A$ is rw-closed.

Definition 2.2 *A nonempty collection of subsets I of a set X is called an ideal if*

(a) If $A \in I$ and $B \subseteq A$ implies $B \in I$ (heridity)

(b) If $A \in I$ and $B \in I$ implies $A \cup B \in I$ (additivity).

Definition 2.3*A subset A of a space (X,τ,I) is said to be a regular weakly closed set with respect to the ideal (I_{rw}-closed) [9] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen.*

Definition 2.4 *Let $i, j \in \{1,2\}$ be fixed integers. In a bitopological space (X,τ_1,τ_2), a subset A of X is said to (i, j)-rw-closed [10] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen in τ_i.*

Definition 2.5 *Let $i, j \in \{1,2\}$ be fixed integers. In a bitopological space (X,τ_1,τ_2), a subset A of X is called*

(i) (i, j)-generalized closed with respect to an ideal $\{(i, j)$-I_g closed} [6] in X if and only if τ_j-$cl(A) - U \in I$ whenever $A \subseteq U$ and U is τ_i-open in X,

(ii) (i, j)-regular generalized star closed with respect to an ideal $\{(i, j)$-I^*_{rg} closed} [4] in X if and only if τ_j-$cl(A) - U \in I$ whenever $A \subseteq U$ and U is $\tau_i\tau_j$-regular open in X.

3. (i, j)-I_{rw} closed **sets**

Definition 3.1A subset A of an ideal topological space (X,τ_1,τ_2,I) is called an (i, j)-regular weakly closed set with respect to an ideal $I\{(i, j)$-I_{rw} closed} in X if and only if τ_j-$cl(A) - U \in I$ whenever $A \subseteq U$ and U is τ_i-regular semiopen in X, $i, j = 1,2$ and $i \neq j$.

Example 3.1 *Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\}, X\}$, $\tau_2 = \{\phi,\{a\},\{b,c\},\{a,b,c\}, X\}$, $I = \{\phi,\{a\},\{d\},\{a,d\}\}$. Then $\phi, X,\{a\},\{d\}$, $\{a,b\},\{a,d\},\{b,c\},\{a,b,c\},\{a,b,d\},\{b,c,d\}$ are $(1,2)$-I_{rw} closed sets in (X,τ_1,τ_2,I).*

Theorem 3.1 *Let (X,τ_1,τ_2,I) be an ideal bitopological space. Every (i, j)-rw closed set is (i, j)-I_{rw} closed in X, $i, j = 1,2$ and $i \neq j$.*

Proof. Let A be (i, j)-rw closed. Let $A \subseteq U$ and U is τ_i-regular semiopen in X, $i, j = 1,2$ and $i \neq j$. Then τ_j-$cl(A) \subseteq U$. Hence τ_j-$cl(A) - U = \phi \in I$. Therefore, A is (i, j)-I_{rw} closed.

Remark 3.1 *The converse of the above theorem is not true in general as can be seen from the following example.*

Example 3.2 *In Example 3.1 $\{a\}$ is $(1,2)$-I_{rw} closed, but not $(1,2)$-rw closed in (X,τ_1,τ_2,I).*

Remark 3.2 *$(1,2)$-I_{rw} closed sets, $(1,2)$-I_{rg^*} closed sets and $(1,2)$-I_g closed sets are independent in general as can be seen from the following example.*

Example 3.3 *In Example 3.1 $(1,2)$-I_{rw} closed sets are $\phi,\{a\},\{d\}$, $\{a,b\},\{a,d\},\{b,c\},\{a,b,c\},\{a,b,d\},\{b,c,d\}$, $(1,2)$-I_g closed sets are $P(x) - \{\{b\},\{d\},\{a,b\},\{a,d\}\}$ and $(1,2)$-I_{rg^*} closed sets are $P(x) - \{\{b\},\{d\}, \{a,d\}\}$. Clearly* *these sets are independent to each other.*

Theorem 3.2 *Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,I). If A is (i,j)-I_{rw} closed then τ_j-$cl(A)-A$ does not contain τ_i-regular semiclosed sets such that $F\notin I$, $i,j=1,2$ and $i\neq j$.*

Proof. Suppose that A is (i,j)-I_{rw} closed, $i,j=1,2$ and $i\neq j$. Let F be an τ_i-regular semiclosed set such that $F\subseteq\tau_j$-$cl(A)-A$. Since $F\subseteq\tau_j$-$cl(A)-A$, we have $F\subseteq[\tau_j$-$cl(A)]\cap -A^C$. Consequently $F\subseteq A^C$ and $F\subseteq\tau_j$-$cl(A)$. Since $F\subseteq A^C$, we have $A\subseteq F^C$. Since F is τ_i-regular semiclosed sets, we have F^C is τ_i-regular semiopen. Sinec A is (i,j)-I_{rw} closed, we have τ_j-$cl(A)-F^C=\tau_j$-$cl(A)\cap F=F\in I$. Thus, τ_j-$cl(A)-A$ does not contain τ_i-regular semiclosed sets such that $F\notin I$.

Theorem 3.3 *If A and B are (i,j)-I_{rw} closed sets then $A\cup B$ is (i,j)-I_{rw} closed, $i,j=1,2$ and $i\neq j$.*

Proof. Suppose that A and B are (i,j)-I_{rw} closed sets, $i,j=1,2$ and $i\neq j$. We shall show that $A\cup B$ is (i,j)-I_{rw} closed. Let $A\cup B\subseteq U$ and U is τ_i-regular semiopen. Since $A\cup B\subseteq U$, we have $A\subseteq U$ and $B\subseteq U$. Since $A\subseteq U$ and U is τ_i-regular semiopen, we have τ_j-$cl(A)-U\in I$. {since A is (i,j)-I_{rw} closed}. Since $B\subseteq U$ and U is τ_i-regular semiopen, we have τ_j-$cl(B)-U\in I$. {since B is (i,j)-I_{rw} closed}. Therefore, τ_j-$cl(A\cup B)-U=\{\tau_j$-$cl(A)-U\}\cup\{\tau_j$-$cl(B)-U\}\in I$. Hence $A\cup B$ is (i,j)-I_{rw} closed.

Remark3.3 *The intersection of two (i,j)-I_{rw} closed sets is not an (i,j)-I_{rw} closed set in general as can be seen from the following example.*

Example 3.4 *In Example 3.1, $A=\{a,b\}$, $B=\{a,c\}$ are $(1,2)$-I_{rw} closed sets, but $A\cap B=\{b\}$ is not an $(1,2)$-I_{rw} closed set in X.*

Lemma 3.1 *Let A be an τ_i-open set in (X,τ_1,τ_2) and let U be τ_i-regular semiopen in A. Then $U=A\cap W$ for some τ_i-regular semiopen set W in X, $i,j=1,2$ and $i\neq j$.*

Lemma 3.2 *If A is $\tau_i\tau_j$-open and U is τ_i-regular semiopen in X then $U\cap A$ is τ_i-regular semiopen in A, $i,j=1,2$ and $i\neq j$.*

Lemma 3.3 *If A $\tau_i\tau_j$-open in (X,τ_1,τ_2), then τ_j-$cl_A(B)\subseteq A\cap\tau_j$-$cl(B)$ for any subset B of A, $i,j=1,2$ and $i\neq j$.*

Theorem 3.4 *Let I be an ideal in X. Let $B\subseteq A$ where A is τ_i-regular semiopen, τ_j-regular semiopen and (i,j)-I_{rw} closed. Then B is (i,j)-I_{rw} closed relative to A with respect to an ideal $I_A=\{F\subseteq A|F\in I\}$ if B is (i,j)-I_{rw} closed in X, $i,j=1,2$ and $i\neq j$.*

Proof. Suppose that B is (i,j)-I_{rw} closed in X, $i,j=1,2$ and $i\neq j$. We shall show that B is (i,j)-I_{rw} closed relative to A. Let $B\subseteq U$ and U is τ_i-regular semiopen in A. Since A is τ_i-open in X and U is τ_i-regular semiopen in A, we have $U=A\cap W$ for some τ_i-regular semiopen set W in X { By Lemma 0}. Since A is $\tau_i\tau_j$-open in X and W is τ_i-regular semiopen in X, we have $U=A\cap W$ is τ_i-regular semiopen set in X {by Lemma 0}. Hence $B\subseteq U$ and U is τ_i-regular semiopen set in X. Since B is (i,j) - I_{rw} closed in X, we have τ_j - $cl(B)-U\in I$. Therefore, τ_j - $cl(B)\cap U^C\in I$. Consequently, τ_j - $cl(B)\cap A\cap U^C\in I_A$. Since A is $\tau_i\tau_j$-open in X, we have τ_j - $cl(B)\cap A=\tau_j$-$cl_A(B)$ { by Lemma 0 }. Hence τ_j-$cl_A(B)-U\in I_A$. Therefore, B is (i,j)-

I_{rw} closed relative to A.

Theorem 3.5 *Let A and B be subsets such that $A \subseteq B \subseteq \tau_j\text{-}cl(A)$. If A is $(i,j)\text{-}I_{rw}$ closed, then B is $(i,j)\text{-}I_{rw}$ closed, $i,j=1,2$ and $i \neq j$.*

Proof. Let A and B be subsets such that $A \subseteq B \subseteq \tau_j\text{-}cl(A)$. Suppose that A is $(i,j)\text{-}I_{rw}$ closed, $i,j=1,2$ and $i \neq j$. Let $B \subseteq U$ and U is τ_i-regular semiopen in X. Since $A \subseteq B$ and $B \subseteq U$, we have $A \subseteq U$. Hence $A \subseteq U$ and U is τ_i-regular semiopen in X. Since A is $(i,j)\text{-}I_{rw}$ closed, we have $\tau_j\text{-}cl(A) - U \in I$. Since $B \subseteq \tau_j\text{-}cl(A)$, we have $\tau_j\text{-}cl(B) \subseteq \tau_j\text{-}cl(A)$. Hence $\tau_j\text{-}cl(B) - U \subseteq \tau_j\text{-}cl(A) - U \in I$. Therefore, B is $(i,j)\text{-}I_{rw}$ closed.

Theorem 3.6 *Suppose that $\tau_j\text{-}R.S.O(X,\tau_1,\tau_2) \subseteq \tau_j\text{-}R.S.C(X,\tau_1,\tau_2)$, then every subset of X is $(i,j)\text{-}I_{rw}$-closed, $i,j=1,2$ and $i \neq j$.*

Proof. Suppose that $\tau_j\text{-}R.S.O(X,\tau_1,\tau_2) \subseteq \tau_j\text{-}R.S.C(X,\tau_1,\tau_2)$, $i,j=1,2$ and $i \neq j$. Let A be a subset of X. Let $A \subseteq U$ and U is τ_i-regular semiopen in X. Since $\tau_j\text{-}R.S.O(X,\tau_1,\tau_2) \subseteq \tau_j\text{-}R.S.C(X,\tau_1,\tau_2)$, we have U is τ_j-regular closed in X. Then $\tau_j\text{-}cl(U) = U$. Since $A \subseteq U$, we have $\tau_j\text{-}cl(A) \subseteq \tau_j\text{-}cl(U) = U$. Therefore, $\tau_j\text{-}cl(A) \subseteq U$. Consequently, $\tau_j\text{-}cl(A) - U = \phi \in I$. Hence A is $(i,j)\text{-}I_{rw}$-closed.

$(i,j)\text{-}I_{rw}$ open sets

Definition 4.1 *A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is called (i,j)-regular weakly open with respect to an ideal {$(i,j)\text{-}I_{rw}$ open} in X if and only if its complement is (i,j)-regular weakly closed with respect to an ideal ($(i,j)\text{-}I_{rw}$ closed) in X, $i,j=1,2$ and $i \neq j$.*

Example 4.1 *In Example 3.1, $\phi, X, \{a\}, \{c\}, \{d\}, \{a,d\}, \{b,c\}, \{c,d\}$, $\{a,b,c\}, \{b,c,d\}$ are $(1,2)\text{-}I_{rw}$ open sets in (X,τ_1,τ_2,I).*

Theorem 4.1 *A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is $(i,j)\text{-}I_{rw}$ open if and only if $F - \tau_j\text{-}int(A) \in I$ whenever $F \subseteq A$ and F is τ_i-regular semiclosed in X, $i,j=1,2$ and $i \neq j$.*

Proof. Suppose that A is $(i,j)\text{-}I_{rw}$ open, $i,j=1,2$ and $i \neq j$. We shall show that $F - \tau_j\text{-}int(A) \in I$ whenever $F \subseteq A$ and F is τ_i-regular semiclosed in X. Let $A \subseteq F$ and F is τ_i-regular semiclosed in X. Then $A^C \subseteq F^C$ and F^C is τ_i-regular semiopen in X. Since A is $(i,j)\text{-}I_{rw}$ open, we have A^C is $(i,j)\text{-}I_{rw}$ closed. Hence $\tau_j\text{-}cl(A^C) - F^C \in I$. Consequently, $[\tau_j\text{-}int(A)]^C \cap F = F - \tau_j\text{-}int(A) \in I$.

Conversely, suppose that $F - \tau_j\text{-}int(A) \in I$ whenever $F \subseteq A$ and F is τ_i-regular semiclosed in X. Let $A^C \subseteq U$ and U is τ_i-regular semiopen in X. Then, $U^C \subseteq A$ and U^C is τ_i-regular semiclosed in X. By our assumption, we have $U^C - \tau_j\text{-}int(A) \in I$. Hence $[\tau_i\text{-}int(A)]^C - U = \tau_j\text{-}cl(A^C) - U \in I$. Consequently, A^C is $(i,j)\text{-}I_{rw}$-closed. Hence A is $(i,j)\text{-}I_{rw}$ open.

Remark 4.1 *$(1,2)\text{-}I_{rw}$ open sets, $(1,2)\text{-}I_{rg}$ open sets and $(1,2)\text{-}I_{g}$ open sets are independent in general as can be seen from the following example.*

Example 4.2 *In Example 3.1,* $(1,2)$ - I_{rw} *open sets are* $\phi,\{a\},\{c\},\{d\}$, $\{a,d\},\{c,d\},\{b,c\},\{a,b,c\},\{b,c,d\}$ *,* $(1,2)$ - I_g *open sets are* $P(x)-\{\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\}\}$ *and* $(1,2)$ - I_{rg^*} *closed sets are* $P(x)-\{\{b,c\},\{a,c,d\},\{a,b,c\}\}$. *Clearly these sets are independent to each other.*

Theorem 4.2 *Let* A *and* B *be subsets such that* $\tau_j\text{-}int(A)\subseteq B\subseteq A$. *If* A *is* $(i,j)\text{-}I_{rw}$ *open, then* B *is* $(i,j)\text{-}I_{rw}$ *open,* $i,j=1,2$ *and* $i\neq j$.

Proof. Suppose that A and B are subsets such that $\tau_j\text{-}int(A)\subseteq B\subseteq A$. Let A be $(i,j)\text{-}I_{rw}$ open, $i,j=1,2$ and $i\neq j$. Let $F\subseteq B$ and F is τ_i-regular semiclosed in X. Since $F\subseteq B$ and $B\subseteq A$, we have $F\subseteq A$. Since A is $(i,j)\text{-}I_{rw}$ open, we have $F-\tau_j\text{-}int(A)\in I$. Since $\tau_j\text{-}int(A)\subseteq B$, we have $\tau_j\text{-}int(A)\subseteq\tau_j\text{-}int(B)$. Therefore, $F-\tau_j\text{-}int(B)\subseteq F-\tau_j\text{-}int(A)\in I$. Consequently, B is $(i,j)\text{-}I_{rw}$ open.

Theorem 4.3 *If a subset* A *is* $(i,j)\text{-}I_{rw}$ *closed, then* $\tau_j\text{-}cl(A)-A$ *is* $(i,j)\text{-}I_{rw}$ *open,* $i,j=1,2$ *and* $i\neq j$.

Proof. Suppose that A is $(i,j)\text{-}I_{rw}$ closed, $i,j=1,2$ and $i\neq j$. Let $F\subseteq\tau_j\text{-}cl(A)-A$ and F is τ_i-regular semiclosed . Since A is $(i,j)\text{-}I_{rw}$ closed, we have $\tau_j\text{-}cl(A)-A$ does not contain τ_i-regular semiclosed such that $F\notin I$ { by Theorem 0 }. Hence, $F\in I$. Therefore, $F-\tau_j\text{-}int[\tau_j\text{-}cl(A)-A]\in I$. Consequently, $\tau_j\text{-}cl(A)-A$ is $(i,j)\text{-}I_{rw}$ open.

Theorem 4.4 *If* A *and* B *are* $(i,j)\text{-}I_{rw}$ *open sets then* $A\cap B$ *is* $(i,j)\text{-}I_{rw}$ *open,* $i,j=1,2$ *and* $i\neq j$.

Proof. Suppose that A and B are $(i,j)\text{-}I_{rw}$ open sets, $i,j=1,2$ and $i\neq j$. Let $F\subseteq A\cap B$ and F is τ_i-regular semiclosed. Since $F\subseteq A\cap B$, we have $F\subseteq A$ and $F\subseteq B$. Since $F\subseteq A$ and F is τ_i-regular semiclosed, we have $F-\tau_j\text{-}int(A)\in I$. {since A is $(i,j)\text{-}I_{rw}$ open}. Since $F\subseteq B$ and F is τ_i-regular semiclosed, we have $F-\tau_j\text{-}int(B)\in I$ { since B is $(i,j)\text{-}I_{rw}$ open } . Therefore, $F-\tau_j\text{-}int(A\cap B)=\{F-\tau_j\text{-}int(A)\}\cap\{F-\tau_j\text{-}int(B)\}\in I$. Hence $A\cap B$ is $(i,j)\text{-}I_{rw}$ open.

Remark 4.2 *The union of two* $(i,j)\text{-}I_{rw}$ *open sets is not an* $(i,j)\text{-}I_{rw}$ *open set in general as can be seen from the following example.*

Example 4.3 *In Example 3.1,* $A=\{a\}$, $B=\{c\}$ *are* $(1,2)\text{-}I_{rw}$ *open sets, but* $A\cup B=\{a,c\}$ *is not an* $(1,2)\text{-}I_{rw}$ *open set in* X.

References

[1] R. Alagar, R. Thenmozhi, Regular generalized star closed sets with respect to an ideal, Int. J. of Math. Sci. and Engg. Appls., 2007, I(2), 183-191.

[2] S. S. Benchalli and R. S. Wali, On rw-closed sets in topological spaces, Bulletin of the Malaysian Mathematical Sciences and Society, (2) 30 (2) (2007), 99-110.

[3] D. E. Cameron, Properties of s-closed spaces, Proc. Amer. Math. Soc., 72 (1978), 581-586.

[4] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao and M. S. Srinivasan, $\tau_1\tau_2$-I_{rg^*} closed sets in ideal bitopological spaces, Journal of Advanecd Research in Pure Mathematics, Vol. 2, Issue. 4, 2010, 56-62.

[5] J. C. Kelly, Bitopological spaces, Proc. London Math. Society, 1963, 13, 71-89.

[6] T. Noiri, N. Rajesh, Generalized closed sets with respect to an ideal in bitopological spaces, Acta Math. Hungar., 2009, 125(1-2), 17-20.

[7] N. Palaniappan, R. Alagar, Regular generalized locally closed sets with respect to an ideal, Antarctica J. Math, 2006, 3(1), 1-6.

[8] M. H. Stone, Application of the theory of boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937), 374-481.

[9] A. Vadivel and Mohanarao Navuluri, Regular weakly closed sets in ideal topological spaces, Submitted.

[10] R. S. Wali, Some Topics in General and Fuzzy Topological Spaces, Ph. D., Thesis, Karnatak University, Karnataka(2006).