# ON (T, S)-INTUITIONISTIC FUZZY BI-IDEALS IN NEAR-RINGS

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**Abstract:** The aim of this paper is to discuss the concept of $(T, S)$-intuitionistic fuzzy bi-ideals of near-ring using $t$-norm and $t$-co-norm and to investigate some of their properties.

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## 1.Introduction

In 1965 Zadeh [15] introduced the concept of fuzzy subsets and studied their properties on the lines parallel to set theory. In 1971, Rosenfeld [13] defined a fuzzy subgroup and gave some of its properties. The notions of fuzzy subnear-ring and ideal were first introduced by Abou-Zaid [1]in 1991. The idea of fuzzy ideals of near-rings was first proposed by Kim and Jun [10] and they defined the concept of fuzzy R-subgroups of near-rings. The concept of bi-ideals was applied to near-rings in [14]. Moreover, Manikantan [12] introduced the notion of fuzzy bi-ideals of near-rings and discussed some of its properties. Dheena, Mohanraj [7] and Akram [4] have studied several properties of T-fuzzy ideals of near-rings.

The intuitionistic fuzzy sets (IFSs) are substantial extensions of the ordinary fuzzy sets. IFSs are objects having degrees of membership and non-membership such that their sum is less than or equal to 1. The most important property of IFSs not shared by the fuzzy sets is that model-like operators can be defined over IFSs. The IFSs have essentially higher describing possibilities than fuzzy sets.

Biswas [5] introduced the notion of intuitionistic fuzzy subgroup of a group by using the notion of intuitionistic fuzzy sets. Kim and Jun [8] introduced the concept of intuitionistic fuzzy ideals of semigroups and in [9], Kim and Lee studied intuitionistic fuzzy bi-ideals of semigroups. Kim and Lee[11] gave the concept of intuitionistic $(T, S)$ normed fuzzy ideals of $\Gamma$-rings. Zhan Jianming and Ma Xueling [16], also discussed the various properties on intuitionistic fuzzy ideals of near-rings. In [6] the notion of a normal intuitionistic fuzzy N-subgroup in a near-ring is introduced and related properties are investigated.

In this paper we introduce the notion of intuitionistic fuzzy bi-ideal of a near-ring with respect to $t-$norm $T$ and $t-$ conorm $S$. Then we characterize all of them based on special kind of level sets $U(A;[t,s])$ and $(A;[t,s])$, which is a generalization of classic level subsets. At the following the behaviour of these structures under homomorphisms is investigated. In particular, by the help of the congruence relations on near-rings, we construct $(T, S)$-intuitionistic fuzzy bi-ideals of near-rings.

## 2.Preliminaries

**Definitiion 2.1**

An algebra $(N,+,.)$ is said to be a near-ring if it satisfies the following conditions:

- $(N,+)$ is a group (not necessarily abelian),
- $(N,\cdot)$ is a semi group,
- For all $x, y, z \in N, x\cdot(y+z) = x\cdot y + y\cdot z$.

**Definition2.2**

A mapping $f: N \to N'$ is called a near-ring homomorphism if

$$f(x+y) = f(x)+f(y) \; and$$
$$f(xy) = f(x)f(y)$$

for all $x, y \in N.$

**Definition 2.3**

[6]. A mapping $\mu: X \to [0,1],$ where X is an arbitary nonempty set and is called a fuzzy set in X.

**Definition 2.4**

Let X be a non-empty set. A mapping $A: X \to [0,1]^2$ is called intuitionstic fuzzy set and $A(x) = (\mu(x), \gamma(x))$ for all $x \in X$ where $\mu$ and $\gamma$ are fuzzy subsets of X such that $\mu(x)+\gamma(x) \le 1$ for all $x \in X.$

**Definition 2.5**

An intuitionistic fuzzy subset $A = (\mu, \gamma)$ in a near-ring N is said to be an intuitionistic fuzzy subnear-ring of N. if it satisfies the following conditions:

- $\mu(x-y) \ge \min\{\mu(x), \mu(y)\}$
- $\gamma(x-y) \le \max\{\gamma(x), \gamma(y)\}$
- $\mu(xy) \ge \min\{\mu(x), \mu(y)\}$ and
- $\gamma(xy) \le \max\{\gamma(x), \gamma(y)\}$ for all $x, y \in N.$

**Definition 2.6**

A mapping $S:[0,1]\times[0,1] \to [0,1]$ is called a t-conorm defined on $[0,1]\times[0,1],$ if the following conditions are satisfied:

- $s(a,0) = a \quad \forall a \in [0,1]$
- $s(a,b) = s(b,a) \quad \forall a,b \in [0,1]$
- $s(a, s(b,c)) = s(s(a,b),c) \quad \forall a,b,c \in [0,1]$
- If $a \le c, b \le d,$ then $s(a,b) \le s(c,d) \quad \forall a,b,c,d \in [0,1]$

**Definition 2.7**

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be the intuitionistic fuzzy subsets of a set X. An intuitionstic fuzzy subset $(A \cap B)$ is defined as

$$(A \cap B)(x) = \left\{min\left\{\mu_A(x), \mu_B(x)\right\}, max\left\{\gamma_A(x), \gamma_B(x)\right\}\right\}$$

**Definition 2.8**

Let $A = A(\mu_A, \gamma_A)$ and $B = B(\mu_B, \gamma_B)$ be the intuitionistic fuzzy subsets of a set x. An intuitionistic fuzzy subset $(A \wedge B)$ is defined as

$$(A \wedge B)(x) = (T(\mu_A(x), \mu_B(x)), S(\gamma_A(x), \gamma_B(x)))$$

**Definition 2.9**

An intuitionistic fuzzy set $A = (\mu, \gamma)$ of a near-ring N is said to be an intuitionistic fuzzy bi-ideal if

- $\mu(x-y) \ge min\left\{\mu(x), \mu(y)\right\}$
- $\gamma(x-y) \le max\left\{\gamma(x), \gamma(y)\right\}$
- $\mu(xyz) \ge min\left\{\mu(x), \mu(z)\right\}$ and

- $\gamma(xyz) \leq max\{\gamma(x), \gamma(z)\}$ for all $x, y, z \in N$.

**Definition 2.10**

An intuitionistic fuzzy bi-ideal $A(\mu, \gamma)$ of a near-ring $N$ is said to be normal if A(0)=(1,0) (i.e) $\mu(0) = 1$ and $\gamma(0) = 0$.

**Definition 2.11**

Let $N$ and $N'$ be two near-rings and f a function of $N$ into $N'$.

• If $A = (\mu_1, \gamma_1)$ is an intuitionistic fuzzy set of $N'$, then the pre-image of A under f is the intuitionistic fuzzy set in N defined by

$$B(x) = (\mu_2, \gamma_2)(x) = (A \circ f)(x) = A(f(x)) = (\mu_1(f(x)), \gamma_1(f(x)) \text{ for each } x \in N$$

• If $A = (\mu_1, \gamma_1)$ is an intuitionistic fuzzy set of N, then the image of A under f is the intuitionistic fuzzy set in $N'$ defined by

$$f(A)(y) = \begin{cases} (\sup\limits_{x \in f^{-1}(y)} \mu_1(x), \inf\limits_{x \in f^{-1}(y)} \gamma_1(x)) \ if \ f^{-1}(y) \neq 0 \\ 0 \ otherwise \ for \ each \ y \in N^1 \end{cases}$$

**Definition 2.12**

An intuitionistic fuzzy subset A of a near-ring N is called $(T, S)$-intuitionistic fuzzy bi-ideal if

- $\mu(x - y) \geq T(\mu(x), \mu(y))$
- $\gamma(x - y) \leq S(\gamma(x), \gamma(y))$
- $\mu(xyz) \geq T(\mu(x), \mu(z))$
- $\gamma(xyz) \leq S(\gamma(x), \gamma(z))$ for all $x, y, z \in N$.

**Example 2.13**

Consider a nearring $N = \{0, a, b, c\}$ with the following Cayley's tables:

| + | 0 | a | b | c |
|---|---|---|---|---|
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | c | b | a | 0 |
| d | d | c | a | b |

| . | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | b | 0 | b |
| b | 0 | 0 | 0 | 0 |
| c | 0 | b | 0 | b |

We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ and by $\mu_A(a) > \mu_A(b) > \mu_A(d) = \mu_A(c)$ and $\nu_A(a) < \nu_A(b) < \nu_A(d) = \nu_A(c)$ . Let $T:[0,1] \times [0,1] \to [0,1]$ be a function defined by $T(x, y) = max(x + y - 1, 0)$ which is a $t$-norm for all $x, y \in [0,1]$ and let $S:[0,1] \times [0,1] \to [0,1]$ be a function defined by $S(x, y) = min(x + y, 1)$ which is a $t$-conorm for all $x, y \in [0,1]$. By routine calculations, it is easy to check that $A$ is a $(T, S)$- intuitionistic fuzzy bi-ideal of $N$.

**Theorem 2.14**

Every intuitionistic fuzzy bi-ideal of a near-ring $N$ is a $(T, S)$-intuitionistic fuzzy bi-ideal of N.

**Proof:** Let $A=(\mu,\gamma)$ be any intuitionistic fuzzy bi-ideal of $N$.

Let $x,y,z\in N.$ Then

$$\mu(x-y) \geq min\{\mu(x),\mu(y)\}$$
$$\geq T(\mu(x),\mu(y)) \rightarrow (1)$$
$$and\ \mu(xyz) \geq min\{\mu(x),\mu(z)\}$$
$$\geq T(\mu(x),\mu(z))$$
$$\Rightarrow \mu(xyz) \geq T(\mu(x),\mu(z)) \rightarrow (2)$$
$$similarly,$$
$$\gamma(x-y) \leq max\{\gamma(x),\gamma(y)\}$$
$$\leq S(\gamma(x),\gamma(y))$$
$$therefore, \gamma(x-y) \leq S(\gamma(x),\gamma(y)) \rightarrow (3)$$
$$and\ \gamma(xyz) \leq max\{\gamma(x),\gamma(z)\}$$
$$\leq S(\gamma(x),\gamma(z))$$
$$therefore\ \gamma(xyz) \leq S(\gamma(x),\gamma(z)) \rightarrow (4)$$

There fore, from (1),(2),(3) and (4), $A=A(\mu,\gamma)$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of N.

**Theorem 2.15**

If $A=(\mu_1,\gamma_1)$ and $B=(\mu_2,\gamma_2)$ are $(T,S)$-intuitionistic fuzzy bi-ideals of a near-ring N, then $A\wedge B$ is a(T,S)-intuitionistic fuzzy bi-ideal of a near-ring N.

**Proof:** Let $A=(\mu_1,\lambda_1)$ and $B=(\mu_2,\lambda_2)$ be $(T,S)$-intuitionistic fuzzy bi-ideal of a near-ring N.

For,Let $x,y,z\in N$ and

$$(1)(A\wedge B)(x-y) = T[\mu_1(x-y),\mu_2(x-y)]$$
$$\geq T[T(\mu_1(x),\mu_1(y)),T(\mu_2(x),\mu_2(y))]$$
$$\geq T[T(\mu_1(x),\mu_1(y)),T(\mu_2(x),\mu_2(y))]$$
$$= T[T(T(\mu_1(x),\mu_1(y),\mu_2(x)),\mu_2(y))]$$
$$= T(T(\mu_2(x),T(\mu_1(x),\mu_1(y)),\mu_2(y))$$
$$= T(T(T\mu_2(x),\mu_1(x)),\mu_1(y)),\mu_2(y))$$
$$= T(T(\mu_1(x),\mu_2(x)),T(\mu_1(y),\mu_2(y))$$
$$= T((\mu_1\wedge\mu_2)(x),(\mu_1\wedge\mu_2)\wedge(y))$$
$$Therefore\ (\mu_1\wedge\mu_2)(x-y) \geq T\left((\mu_1\wedge\mu_2)(x),(\mu_1\wedge\mu_2)\wedge(y)\right) \rightarrow (1)$$
$$Similarly\ we\ can\ prove\ that$$
$$(\lambda_1\wedge\lambda_2)(x-y) \leq S\left((\lambda_1\wedge\lambda_2)(x),(\mu_1\wedge\mu_2)(y)\right) \rightarrow (2)$$

$$(2)\ (A \wedge B)(xyz) = T\left(\mu_1(xyz), \mu_2(xyz)\right)$$
$$\geq T\left(T(\mu_1(x), \mu_1(z)), T(\mu_2(x), \mu_2(z))\right)$$
$$= T\left\{T[T(\mu_1(x), \mu_1(z)), \mu_2(x)], \mu_2(z)\right\}$$
$$= T\left\{T[\mu_2(x), T(\mu_1(x), \mu_1(z))], \mu_2(z)\right\}$$
$$= T\left\{T[T(\mu_2(x)\mu_1(x)), \mu_1(z)], \mu_2(z)\right\}$$
$$= T\left\{T(\mu_2(x), \mu_1(x)), T(\mu_1(z), \mu_2(z))\right\}$$
$$= T\left((\mu_1 \wedge \mu_2)(x), (\mu_1 \wedge \mu_2)(z)\right)$$
$$(\mu_1 \wedge \mu_2)(xyz) \geq T\left((\mu_1 \wedge \mu_2)(x), (\mu_1 \wedge \mu_2)(z)\right) \to (3)$$

*Similarly,*

$$(\lambda_1 \vee \lambda_2)(xyz) \leq S\left((\lambda_1 \wedge \lambda_2)(x), (\lambda_1 \wedge \lambda_2)(z)\right) \to (4)$$

Combaining (1),(2),(3) and (4), we get

(A $\wedge$ B) is a $(T,S)$-intuitionistic fuzzy bi-ideal of $N$.

**Theorem 2.16**

Every $(T,S)$-intuitionistic fuzzy bi-ideal of a regular near-ring $N$ is a $(T,S)$-intuitionistic fuzzy subnear-ring of $N$.

**Proof:**

Let $A = (\mu, \gamma)$ be any (T,S)-intuitionistic fuzzy bi-ideal of a near-ring N.

Let $a, b \in N$. then

$$\mu(a-b) \geq T(\mu(a), \mu(b)) \text{ and}$$
$$\gamma(a-b) \leq S(\gamma(a), \gamma(b))$$

*It is enough to prove that* $\mu(ab) \geq T(\mu(a), \mu(b))$ *and*

$$\gamma(ab) \leq S(\gamma(a), \gamma(b)).$$

*since* $N$ *is regular, there exists* $x \in N$ *such that* $a = axa$.

$$\text{Now } \mu(ab) = \mu((axa)b) = \mu(a(xa)b)$$
$$\geq T(\mu(a), \mu(b)) \quad (5)$$

*Similarly,*

$$\gamma(ab) = \gamma((axa)b) = \gamma(a(xa)b)$$
$$\geq S(\mu(a), \mu(b)) \quad (6)$$

There fore From (5) and (6), it is clear that $A = A(\mu, \gamma)$ is a $(T,S)$-intuitionistic fuzzy subnear-ring of $N$.

**Theorem 2.17**

An intuitionistic fuzzy set $A$ in a near-ring $N$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of $N$ iff the level set

$\cup(A;\alpha,\beta) = \{x \in N / \mu(x) \geq \alpha, \gamma(x) \leq \beta\}$ is a bi-ideal of $N$ when it is non-empty.

**Proof:**

Let $A = A(\mu, \alpha)$ be an intuitionistic fuzzy bi-ideal of $N$

Let $x, y \in \cup(A;\alpha,\beta)$.

Then $\mu(x) \geq \alpha$ and $\mu(y) \geq \alpha$ also $\gamma(x) \leq \beta$ and $\gamma(y) \leq \beta$.

Now, consider $\mu(x-y)$, then

$$\mu(x-y) \geq T(\mu(x),\mu(y))$$
$$\geq \alpha$$
$$\Rightarrow \mu(x-y) \geq \alpha \rightarrow \quad (1)$$
$$similarly,$$
$$\gamma(x-y) \leq S(\gamma(x),\gamma(y))$$
$$\leq \beta$$
$$\Rightarrow \gamma(x-y) \leq \beta \rightarrow \quad (2)$$
$$Therefore\ From\ (1)\ and\ (2),\ we\ get\ (x-y) \in \cup(A;\alpha,\beta)$$
$$\Rightarrow \cup(A;\alpha,\beta)\ is\ a\ subgroub\ of\ N.$$
$$Let\ x,z \in \cup(A;\alpha,\beta)\ and\ y \in N.$$
$$\Rightarrow \mu(x) \geq \alpha \quad and \quad \mu(z) \geq \alpha$$
$$\gamma(x) \leq \beta \quad and \quad \gamma(z) \leq \beta$$
$$\mu(xyz) \geq T(\mu(x),\mu(z))$$
$$\geq \alpha$$
$$Also\ \gamma(xyz) \leq S(\gamma(x),\gamma(z))$$
$$\leq \beta$$
$$\Rightarrow xyz \in \cup(A;\alpha,\beta)$$
$$Therefore \cup(A;\alpha,\beta)\ is\ a\ bi-ideal$$
$$Conversely,\ suppose\ that\ xy \in N$$
$$and\ \mu(x-y) < T(\mu(x),\mu(y))$$
$$and\ \gamma(x-y) > S(\gamma(x)\gamma(y))$$
$$Choose\ \alpha\ and\ \beta\ such\ that\ \mu(x-y) < \alpha < T(\mu(x),\mu(y))$$
$$and\ \gamma(x-y) > \beta > S(\gamma(x),\gamma(y))$$
$$(i.e)\ S(\gamma(x),\gamma(y)) < \beta < \gamma(x-y)$$
$$so\ that,\ x,y \in \cup(A;\alpha,\beta)\ but\ x-y \notin \cup(A;\alpha,\beta)$$
$$Which\ is\ a\ contradiction\ for \cup(A;\alpha,\beta)\ is\ a\ bi-ideal.$$
$$Therefore\ \mu(x-y) \geq T(\mu(x),\mu(y))$$
$$and\ \gamma(x-y) \leq S(\gamma(x),\gamma(y))$$
$$Similarly,\ we\ can\ prove\ that\ \mu(xyz) \geq T(\mu(x),\mu(z))$$
$$and\ \gamma(xyz) \leq S(\gamma(x),\gamma(z)).$$

Therefore $A=(\mu,\gamma)$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of $N$.

**Theorem 2.18**

Let $f: N \rightarrow N'$ be an onto homomorphism of near-rings.If $A=(\mu,\gamma)$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of $N$, then $f(A)$ is a $(T,S)$-intuitionistic fuzzy bi-ideal in $N'$.

**Proof:**

Let A be a $(T,S)$-intuitionistic fuzzy bi-ideal of $N$.

Then $x/x \in f^{-1}(y_1-y_2) \supseteq \{(x_1-x_2)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$$\begin{aligned}
(1)\ f(\mu)(y_1-y_2) &= \sup\{\mu(x)/x\in f^{-1}(y_1-y_2)\}\\
&\geq \sup\{\mu(x_1-x_2)/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}\\
&\geq \sup\{T(\mu(x_1),\mu(x_2))/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}.\\
&= T\{\sup\mu(x_1)/x_1\in f^{-1}(y_1), sup\mu(x_2)/x_2\in f^{-1}(y_2)\}\\
&= T(f(\mu)(y_1), f(\mu)(y_2))\\
f(\mu)(y_1-y_2) &\geq T(f(\mu)(y_1), f(\mu)(y_2)).
\end{aligned}$$

$$\begin{aligned}
(2)\ f(\mu)(y_1y_2y_3) &= \sup\{\mu(x)/x\in f^{-1}(y_1y_2y_3)\}\\
&\geq \sup\{\mu(x_1x_2x_3)/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2), x_3\in f^{-1}(y_3)\}\\
&\geq \sup\{T(\mu(x_1),\mu(x_3))/x_1\in f^{-1}(y_1), x_3\in f^{-1}(y_3)\}\\
f(\mu)(y_1y_2y_3) &\geq T(f(\mu)(y_1), f(\mu)(y_3))
\end{aligned}$$

$$\begin{aligned}
(3)\ f(\gamma)(y_1-y_2) &= \inf\{\gamma(x)/x\in f^{-1}(y_1-y_2)\}\\
&\leq \inf\{\gamma(x_1-x_2)/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}\\
&\leq \inf\{S(\gamma(x_1),\gamma(x_2))/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}\\
&= S(\inf\gamma(x_1)/x_1\in f^{-1}(y_1), \inf\gamma(x_2)/x_2\in f^{-1}(y_2))\\
&= S(f(\gamma)(y_1), f(\gamma)(y_2))\\
f(\gamma)(y_1-y_2) &\leq S(f(\gamma)(y_1), f(\gamma)(y_2))
\end{aligned}$$

$$\begin{aligned}
(4)\ f(\gamma)(y_1y_2y_3) &= \inf\{\gamma(x)/x\in f^{-1}(y_1y_2y_3)\}\\
&\leq \inf\{\gamma(x_1x_2x_3)/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2), x_3\in f^{-1}(y_3)\}\\
&\leq \inf\{S(\gamma(x_1),\gamma(x_3))/x_1\in f^{-1}(y_1), x_3\in f^{-1}(y_3)\}\\
&= S(inf\{\gamma(x_1)/x_1\in f^{-1}(y_1), inf\,\gamma(x_3)/x_3\in f^{-1}(y_3)\})\\
&= S(f(\gamma)(y_1), f(\gamma)(y_2))\\
f(\gamma)(y_1-y_2) &\leq S(f(\gamma)(y_1), f(\gamma)(y_2))
\end{aligned}$$

Hence combining all the results obtained above, we get
$f(A)=f(\mu,\gamma)$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of $N'$.

**Theorem 2.19**

Let $A=(\mu,\gamma)$ be a $(T,S)$-intuitionistic fuzzy bi-ideal of a near-ring $N$ and $A^*=(\mu^*,\gamma^*)$ be a intuitionistic fuzzy set in $N$ defined by $\mu^*(x)=\mu(x)+1-\mu(0)$ and $\gamma^*(x)=\gamma(x)-\gamma(0)$ for all $x\in N$. Then $A^*$ is a normal $(T,S)$-intuitionistic fuzzy bi-ideal of $N$ containing $A$.

**Proof:**

Let $A=A(\mu,\gamma)$ be a $(T,S)$-intuitionistic fuzzy bi-ideal of a near-ring N. For any $x,y\in N.$

$$
\begin{aligned}
\mu^*(x-y) &= \mu(x-y)+1-\mu(0) \\
&\geq T(\mu(x),\mu(y))+1-\mu(0) \\
&= T((\mu(x)+1-\mu(0)),(\mu(y)+1-\mu(0))) \\
&= T(\mu^*(x),\mu^*(y\mu)) \\
\mu^*(x-y) &\geq T(\mu^*(x),\mu^*(y)) \\
and\ \gamma^*(x-y) &= \gamma(x-y)-\gamma(0) \\
&\leq S(\gamma(x),\gamma(y)-\gamma(0)) \\
&= S(\gamma^*(x),\gamma^*(y)) \\
\gamma^*(x-y) &\leq S(\gamma^*(x),\gamma^*(y)) \\
For\ any\ x,y,z \in N\mu^*(xyz) &= \mu(xyz)+1-\mu(0) \\
&\geq T(\mu(x),\mu(z))+1-\mu(0) \\
&= T((\mu(x)+1-\mu(0)),(\mu(z)+1-\mu(0))) \\
&= T(\mu^*(x),\mu^*(z)) \\
\mu^*(xyz) &\geq T(\mu^*(x),\mu^*(z)) \\
and\ \gamma^*(xyz) &= \gamma(xyz)-\gamma(0) \\
&\leq S(\gamma(x),\gamma(z))-\gamma(0) \\
&= S((\gamma(x-\gamma(0)),(\gamma(z)-\gamma(0))) \\
&= S(\gamma^*(x),\gamma^*(z)) \\
\gamma^*(xyz) &\leq S(\gamma^*(x),\gamma^*(z))
\end{aligned}
$$

$A^*$ is a $(T,S)$-intuitionistic fuzzy bi-ideal of a near ring $N$.

clearly $A^*(0)=(\mu^*(0),\gamma^*(0))=(1,0)$ and $A \subseteq A^*$

Hence Proved.

**Conclusion:** In this article, $(T,S)$-intuitionistic fuzzy bi-ideals has been studied and some related properties are investigated.

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