Design of Central Planetary Gear Train

Sameer Todkar[#1], Swanand Vaidya[#2], Rupesh Shewale[#3]N.P Sherje[#4]

[#]*Mechanical Department, Savitribai Phule Pune University*

Abstract— **Planetary Gear Trains are effectively used for power transmission and speed variation. Planetary Gear trains as speed reducers have compact gear dimensions as compared to other gear trains. This paper presents the methodology or design of a planetary gear train. The different design parameters are to be selected based on the running conditions, manufacturing processes and other dynamic conditions. The paper also presents the salient applications of planetary gear trains.**

Keywords **— Epicyclic gear train, helical gears, reduction ratio, input-output configuration of planetary gear train.**

I. INTRODUCTION

Epicyclic gear trains provide high load capacity and compactness to gear drives. It has a wide scope for a range of applications with varieties of combinations of Planetary gear arrangements with ranges of gear ratios owing to the high reduction ratios in compacts volumes, high torque to weight ratios high reliability and high efficiency. In planetary gear trains, if the input power is equally shared among planet gears, the torque on each planet gear will be significantly reduced. This results in a longer service life for gears and a less radial support requirement for bearings. Due to inevitable manufacturing and installation errors as well as the deformation of components and other factors, it results in uneven load sharing among the planet gears. Potential areas of applications include different aerospace drives such as flap actuators, robotic mechanisms, tandem bike, helicopters, noise critical medical equipment, printing machines, unconventional energy applications like wind turbine power plant, automatic structures etc.

II. NOMENCLATURE

Z_s	No. of teeth on pinion	τ	Shear Stress
Z_p	No. of teeth on gear	σ_h	Contact Stress
d_s	Pitch Diameter of pinion	σ_{sf}	Contact Fatigue Stress
d_p	Pitch Diameter of gear	σ_{sf}'	Surface fatigue strength
d_{sa}	Addendum Diameter pinion	J	Geometry Factor
d_{sh}	Dedendum Diameter of pinion	K_L	Life Factor
d_{pa}	Addendum Diameter of gear	K_h	Hardness Factor
d_{sb}	Base Diameter of pinion	K_s	Service Factor
d_{pb}	Base Diameter of gear	K_m	Miscellaneous Factor
Φ_n	Normal Pressure Angle	K_o	Overload factor
ψ	Helix Angle	K_v	Velocity Factor
Φ	Transverse pressure Angle	K_R	Reliability factor
m_n	Normal Module	K_M	Load Distribution Factor
m_t	Tangential Module	C_p	Elastic coefficient
p	Circular Pitch	CR	Contact Ratio
p_a	Axial Pitch	F_T	Tangential Force
b	Face Width	F_R	Radial Force
T	Torque	F_a	Axial Force
M	Moment	F_n	Normal Force
ω	Angular Velocity	FOS	Factor of Safety
S_{ut}	Ultimate Tensile Strength		
S_{yt}	Yield Tensile Strength		
σ_b	Bending Stress		
σ_{ut}	Ultimate Tensile Stress		
σ_e' & σ_e	Endurance Limit Stresses		

III. DESIGN

Calculate the input torque (Ti), from the specified power rating and speed, calculate the input torque (Ti). Choose the appropriate material by considering the fatigue performance and strength requirements. Select the standard material properties from any Standard Machine Design and Manufacturing Handbook. Assume suitable factor of safety considering all design constraints

When designing the planet gears, the following conditions must satisfy

- [(Number of teeth of internal gear + number of teeth of sun gear)/Number of planets] should be an integer
  - Maximum number of planets (N_{pmax}) = int [π/sin-1((No. of teeth on planets + addendum)/ (addition of teeth on planet and sun))]

In order to select, the input-output configuration, let us assume number of teeth on sun as 20, planets as 30 and ring gear as 80. The gear ratio can be calculated as follows:

INPUT	OUTPUT	STATIONARY	FORMULA	GEAR RATIO (i)
SUN(S)	CARRIER(C)	RING(R)	1+R/S	5
CARRIER(C)	RING(R)	SUN(S)	1/(1+S/R)	0.8
SUN(S)	RING(R)	CARRIER(C)	-R/S	-4

TABLE I: Gear Configurations

Maximum reduction ratio for the respective number of teeth can be obtained in the first case, when the ring gear stationary. Hence, input is given to sun gear, output is taken from the planet carrier and the ring gear is fixed for the following gearbox design.

Helical gear train is selected over spur for the following advantages:

- High contact ratio
- High load carrying capacity
- High reduction ratio
- Noiseless operation
- Compact dimensions

For a 5:1 reduction ratio, assume the number of teeth on sun (Zs) as 20.
Assume a standard normal module(m_n) and a suitable Factor of Safety (FOS) considering the material selections, cost and weight constraints.
Number of teeth on ring gear (Z_r) = (i-1) *Zs
Number of teeth on planet gear (Zp) = (Zr- Zs)/2
Assume suitable helix angle.
Equivalent number of teeth for sun= $Zs/cos^3\alpha$
Equivalent number of teeth for sun= $Z_P/cos^3\alpha$

Factors necessary for gear design calculations:

Evaluate the J factor and its multiplier equivalent for helix angle (25^0) from graphs shown below.

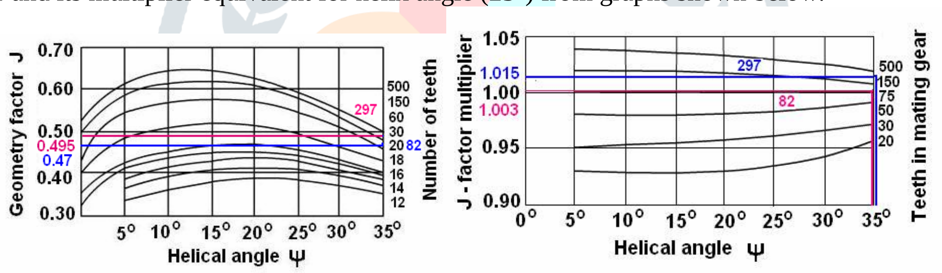


Fig.1. J Factor and J Multiplier

Geometry Factor (J) = J factor*J multiplier

Overload factor (K_o)

Power Source	Uniform	Moderate shock	Heavy shock
Uniform	1	1.25	1.75
Light shock	1.25	1.5	2
Heavy shock	1.5	1.75	2.25

TABLEII: Overload Factor

Reliability Factor (K_r)

Reliability	0.5	0.9	0.95	0.99	0.999	.9999
Factor (K_r)	1	.897	.8688	0.814	0.753	0.702

TABLE III: Reliability Factor

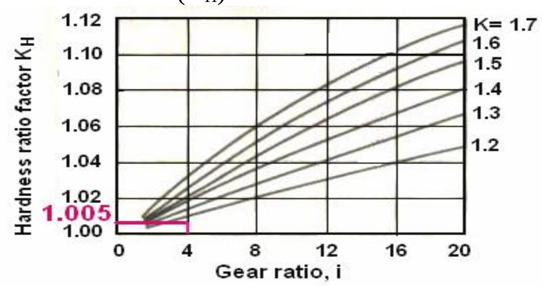
Load Distribution Factor (K_M):

	Face Width			
Support Characteristics	0-50	150	225	400<
Accurate Mountings, Precision Gears	1.2	1.3	1.4	1.7
Less Precision Gears	1.5	1.6	1.7	2

Accuracy and mounting such that less than full-face contact exists	>2	>2	>2	>2

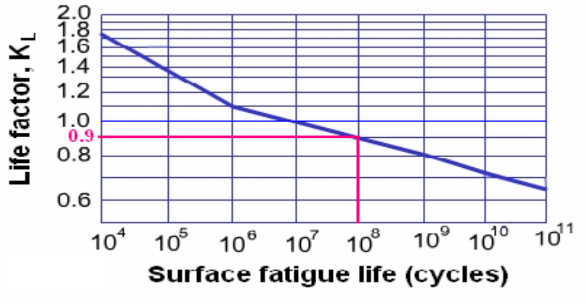
TABLE IV: Load Distribution Factor

Hardness Factor (K_H):

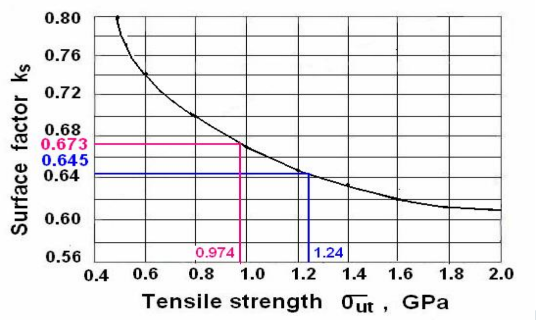


Graph 3: Hardness Factor

Life Factor (K_L):

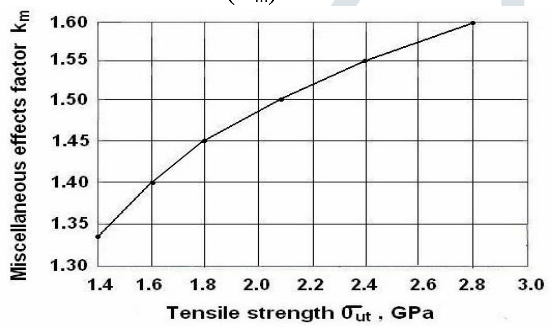


Service Factor (K_S):



Graph 1: Service Factor

Miscellaneous Factor (K_m):



Graph 2: Miscellaneous Factor

Graph 4: Life Factor

Surface Fatigue Strength (σ_{sf} 10^7 cycles 99% Reliability):

Material	σ_{sf}' (MPa)
Steel	2·8 (Bhn) – 69 MPa
Nodular Iron	0.95 [2·8 (Bhn) – 69 MPa]
Cast Iron, grade 20	379
Cast Iron, grade 30	482
Cast Iron, grade 40	551
Tin Bronze, AGMA 2C (11% Sn)	207
Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)	448

TABLEV Surface Fatigue Strength

Elastic Coefficient (C_P):

Pinion Material (μ = 0.3 in all cases)	Gear Material			
	Steel	Cast Iron	Al Bronze	Tin Bronze
Steel, E = 207 GPa	191	166	162	158
Cast Iron, E = 131 GPa	166	149	149	145
Al Bronze, E=121 GPa	162	149	145	141
Tin Bronze, E=110GPa	158	145	141	137

TABLE VI: Elastic Coefficient

Design Calculations:

1) Angular Velocity (ω)= $2\pi N/60$
2) Circular Pitch (p)= $\pi m_n/\cos\psi$
3) Axial Pitch (p_a)= $p/\tan\psi$
4) Face Width(b)= $15m_n$
5) Pitch Diameter of Sun(d_s)= mZ_s
6) Pitch Diameter of Planet(d_p)= mZ_p
7) Addendum Diameter Sun (d_{sa})= $d_s+(2*m_n)$
8) Dedendum Diameter of Sun(d_{sh})= $d_s-(2.5*m_n)$
9) Addendum Diameter of Planet(d_{pa})= $d_p+(2*m_n)$
10) Dedendum Diameter of Planet(d_P)= $d_p-(2.5*m_n)$
11) Base Diameter of Sun(d_{sb})= $d_s(\cos\Phi)$
12) Base Diameter of Planet(d_{pb})= $d_p(\cos\Phi)$
13) PCD of Ring= $2(d_p)+d_s$
14) Ring Addendum= pcd-$2*m_n$
15) Ring Dedendum= pcd+$2.5*m_n$
16) Centre Distance (C.D) = $0.5(d_s+d_p)$
17) Transmitted Load (F_t)= $(2*T_I)/(d_s*N_{pmax})$
18) Pitch Line Velocity(V) =0.5*10-3*ω*ds

The transmitted load acts as the crucial factor in determining the design safety.

SAFETY CHECK:

1.) $\sigma_{b\ sun} = (F_t*K_v*K_o*0.93*K_m)/(b*m_n*J_{sun})$
2.) $\sigma_{b\ planet} = (Ft*Kv*Ko*0.93*Km)/(b*m_n*J_{planet})$

Corrected bending Fatigue Strength of the Pinion:

3.) $\sigma_e' = 0.5*\sigma_{ut}$
4.) $\sigma_e = \sigma e'*K_L K_V K_S K_R K_T K_F K_M$
5.) $FOS_{bending} = \sigma_e/\sigma_{b\ sun}$
6.) $FOS_{bending} = \sigma_e/\sigma_{b\ planet}$

Contact Stresses on Gears:

7.) $I = (\sin(\Phi)*\cos(\Phi)*i)\ (i+1)$
8.) Contact Ratio $(CR) = (((r_1+a)^2-r_{b1}^2)^{0.5} + ((r_2+a)^2-r_{b2}^2)^{0.5} - (r_1+r_2)\sin\Phi)/\pi m\cos\psi$
9.) $K_v = ((78+(200V)^{0.5})/78)^{0.5}$
10.) $\sigma_h = C_P((F_t/bdI)*(\cos\psi/0.95CR)*K_V*K_{0*}0.93K_m)^{0.5}$

Contact Fatigue Strength of Pinion:

11) $\sigma_{sf}' = 2.8(BHN)-69$ (For Steels)
12) $\sigma_{sf} = \sigma sf'*K_L K_H K_R K_T$
13) $FOS = \sigma_{sf}/\sigma_h$

If the calculated Factor of Safety is less than the assumed FOS, the design fails. In order to ensure design safety, increase the normal module, or Face width or Helix angle depending on the cost, manufacturing and space constraints.

FORCE CALCULATIONS:

1.) Transmitted Load $(F_t) = (2*T_I)/(d_s*N_{pmax})$
2.) Radial Load $(F_R) = F_T\tan\Phi$
3.) Axial Load $(F_{aS}) = F_T*\tan\psi$

IV. MODEL

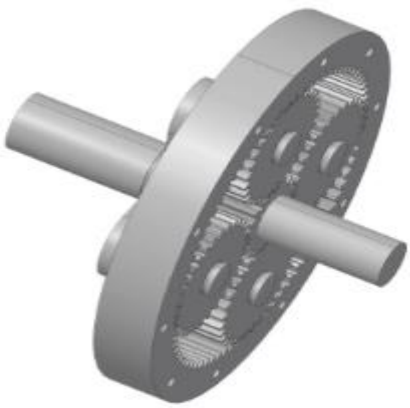


Figure 1. Planetary Gear Train

V. CONCLUSION AND FUTURE SCOPE

The future scope of Planetary Gear Train for variable speeds is very bright as it facilitates a high magnitude of reduction with easily compact dimensions of the overall assembly. The fatigue performance of the Planetary gearbox dominates the other types of gearbox with high values of Elastic coefficient, Brinell Hardness, Pitch-line velocity, Overload factor; and optimum values of face width, helix angles and pitch circle diameters of gears. The design can further be optimised for shafts, gears and casing dimensions. A further study can also be improved by modifying the gear orientations or backup mechanisms using other robust gear drives such as worm and worm gear configurations while satisfying the dimensional constraints. A Central Planetary Gearbox has its salient applications in Industrial Mixers, Stair lifts, Automatic Transmission Gearbox in Automobiles, Conveyer Systems, Wind Turbine Power Transmission Systems, Helicopter Reducers etc.

VI. REFERENCES

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