Designand Applicability of a New Chaotic Attractor for Secure Communication

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***Abstract*—A modified three-dimensional autonomous chaotic system is designed to transmit information securely in the communication channel. The new system is similar to Lorenz like attractors in numerical simulation. Pecora and Carroll synchronization is applied andit clearly demonstrates that it can be used for secure communication. The New system is executed in MATLAB Simulink and also using Multisim.**

Keywords*—*chaotic system, chaotic attractor, synchronization, secure communication

I. Introduction

Chaos is irregular patterned with non-linear property and its complex behaviour can be used in secure communication for the transmission of information in the communication channel.Starting from Lorenz in 1963, many autonomous chaotic systemshave been designed from time to time [1,2]. Notable chaotic systems are Chen, Rossler attractor, Sprott Systems. All these systems constitute a family of Lorenz system. With the growing interest in chaotic dynamics have directed towards chaos-based applications. The designing of the circuit to produce chaotic attractors have led to real world application [3] which is being used in many chaos-based technologies [4].

In this paper modified chaotic system is introduced and synchronization is done by Drive and Response method. Then thechaotic system is numerically studied using MATLAB Simulink. The Simulink results show that the chaotic system can be used for secure communication.

Section II introduces the modified chaotic system and its numerical analysis. In Section III simulation results of synchronization of the chaotic system is presented. The Section IV showsthe MATLAB Simulink modelof the proposed system for secure communication. Finally, the results are discussed and concluded in the last section and its applicability in secure communication.

II. The Modified Chaotic System

A set of three first ordernon-linear state equations of the chaotic system is given below:

$$\frac{dx}{dt} = y - x$$

$$\frac{dy}{dt} = ay - xz \qquad (1)$$

$$\frac{dz}{dt} = xy - b$$

The system has two real parameters a and b and two quadratic non-linearities with two equilibrium points whereas the Lorenz system has three equilibrium points and the origin (0,0,0) is a point of equilibrium for all the system of Lorenz family but not an equilibrium pointsfor the new system. The circuit is less complex than the Lorenz system with few components used. The chaotic system has been model in MATLAB Simulink in Fig. 1. The system is studied by varying parametric values and phase portrait has been obtained for the parameter values a=0.5 and b=0.5with initial condition$(\dot{X},\dot{Y},\dot{Z})$=(0.001,0.001,0).

Theoretical calculation gives that the system has two equilibrium points E1 and E2:

E1$(+\sqrt{b},+\sqrt{b},a)$ and E2$(-\sqrt{b},-\sqrt{b},a)$

The Jacobian matrix of the system is:

$$J=\begin{vmatrix} -1 & 1 & 0 \\ -z & a & -x \\ y & x & 0 \end{vmatrix} \quad (2)$$

Case1: whenE1$(+\sqrt{b},+\sqrt{b},a)$ the Jacobian Matrix becomes:

$$J(E1) = \begin{vmatrix} -1 & 1 & 0 \\ -a & a & -\sqrt{b} \\ \sqrt{b} & \sqrt{b} & 0 \end{vmatrix} \quad (3)$$

Therefore, the characteristics equation of the equilibrium E1 is:

det(J(E1)-λI) =0 (4)

$$\det\left(\begin{pmatrix} -1 & 1 & 0 \\ -0.5 & 0.5 & -\sqrt{0.5} \\ \sqrt{0.5} & \sqrt{0.5} & 0 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) = 0 \quad (5)$$

$$\det\begin{pmatrix} -1-\lambda & 1 & 0 \\ -0.5 & 0.5-\lambda & -0.70711 \\ 0.70711 & 0.70711 & -\lambda \end{pmatrix} = 0 (6)$$

By solving the above determinant, we get:

$$\lambda^3 - 0.5\lambda^2 - 0.5\lambda - 1 = 0 \quad (7)$$

Solving eq. (7) we get the Eigen values:

λ1= -1, λ2= $\frac{1}{4}$ + i$\frac{\sqrt{15}}{4}$ and λ3=$\frac{1}{4}$ - i$\frac{\sqrt{15}}{4}$ which is

$(\lambda 1, \lambda 2, \lambda 3)$ =(-1,0.25+0.9682i,0.25-0.9682i) (8)

The equilibrium points are hyperbolic in nature and the trajectories diverge from the equilibrium point called a saddle which is unstable.

Case2: when E2$(-\sqrt{b},-\sqrt{b},a)$ then the Jacobian Matrix becomes:

$$J(E2) = \begin{vmatrix} -1 & 1 & 0 \\ -a & a & \sqrt{b} \\ -\sqrt{b} & -\sqrt{b} & 0 \end{vmatrix} \quad (9)$$

By solving the characteristics equation for E2 we get the same Eigen value and the real parts of the eigen valuearepositive which states that the presence of chaos and the equilibrium points are unstable. Thus, the system orbits around the two equilibrium points.

Using the MATLAB Simulink, the phase portrait the model is achieved in Fig.2, Fig.3 and Fig.4.

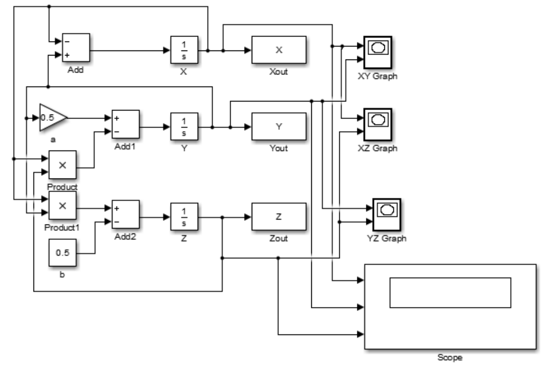

Figure 1. MATLAB Simulink model of chaotic system with a=b=0.5.

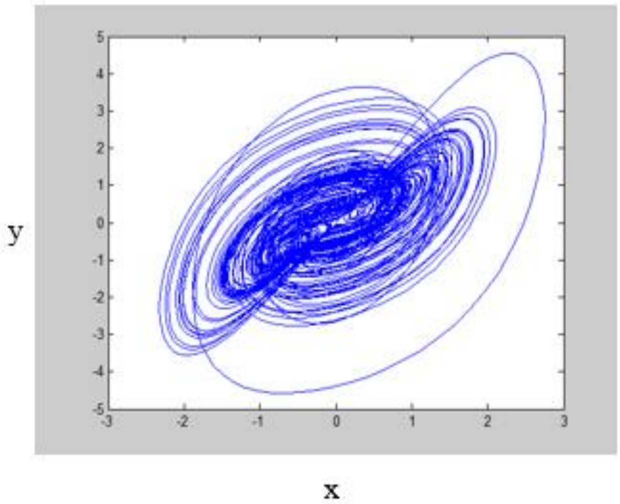

Figure 2. x-y phase portrait

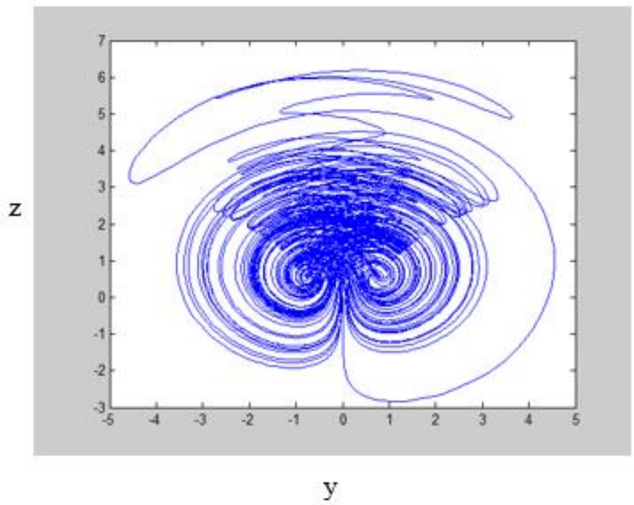

Figure 3. y-z phase portrait

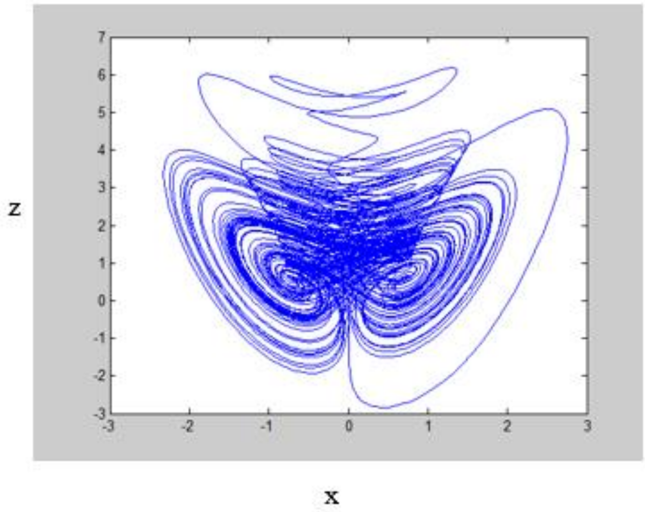

Figure 4. x-z phase portrait

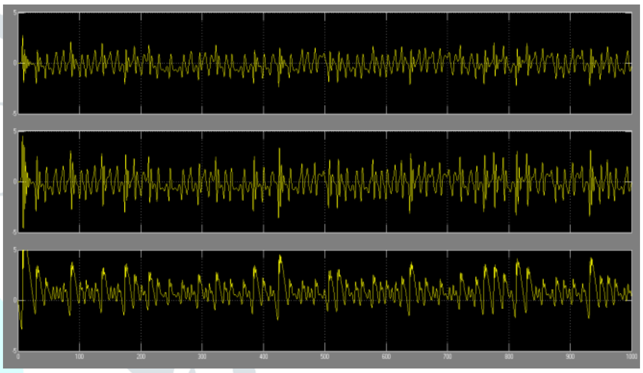

Figure 5. X, Y and Z waveform

From the Fig.5 it is evident that the waveform is irregular and random in nature. This confirms the presence of chaotic behaviour in the model.

The new system is also implemented using Multisim andthe circuit diagram is given in Fig.6.

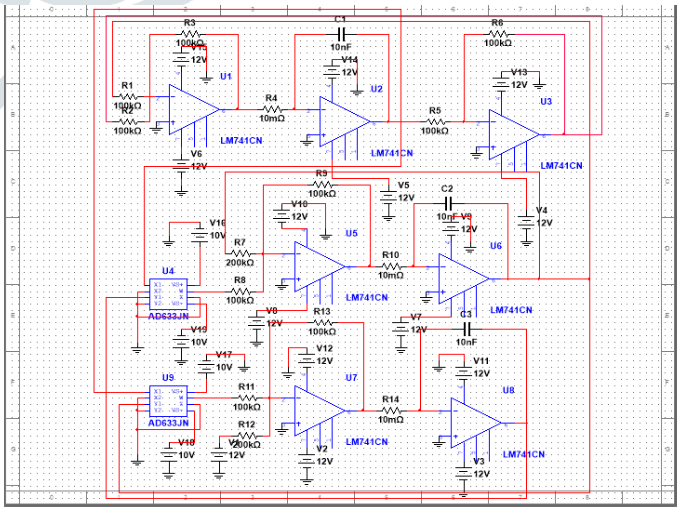

Figure 6. Circuit diagram of the new system

The given circuit is implemented in multisim using electronic components such as resistors, capacitors, operational amplifiers,and multipliers. There are 3 capacitors, 14 resistors, 7 operational amplifier,and 2 multipliers are used in the circuit. Op-amp LM741CN,AD633JN multipliers with resistors R1=R2=R3=R5=R6=R8=R9=R11=R13=100kΩ, R4=R10=R14=10mΩ, R7=R12=200Ω, C1=C2=C3=10nF, $V_N = -12V$ and V_P= 12V. Inputs to the multiplier AD633JN are -10V and +10V. The waveform obtained from the oscilloscopeis shown in Fig. 7, Fig.8 and Fig 9. Whereas the chaotic attractor in xyz plane is shown in Fig. 10, Fig.11 and Fig12.

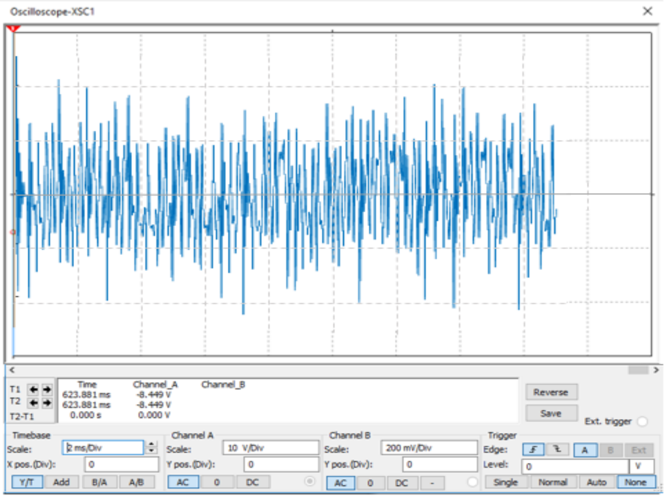

Figure 7. x waveform of the system.

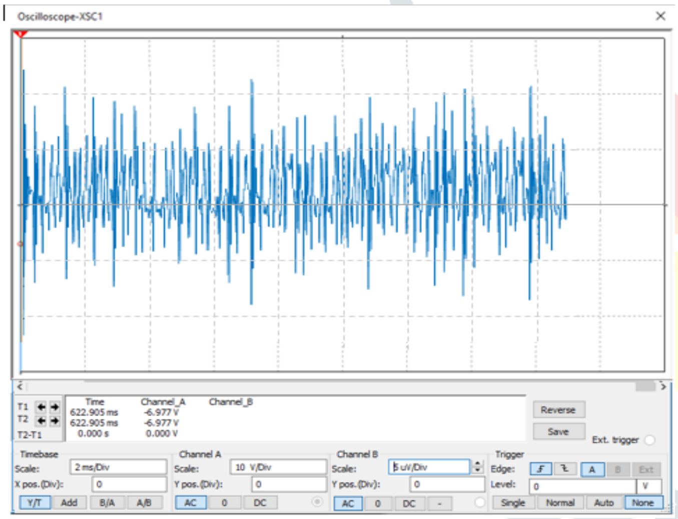

Figure 8. y waveform of the system

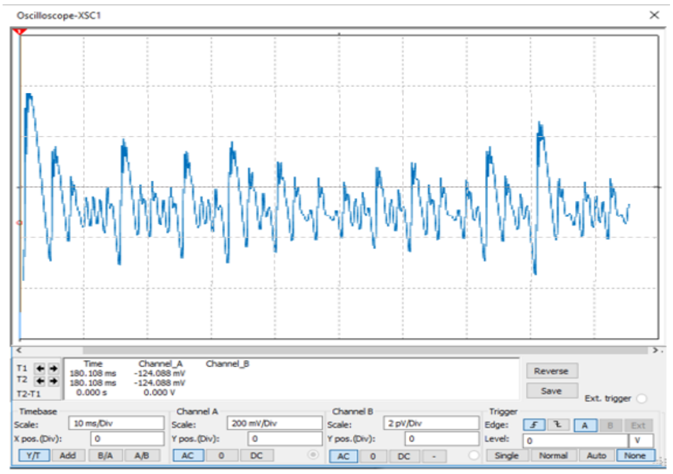

Figure 9. z waveform of the system

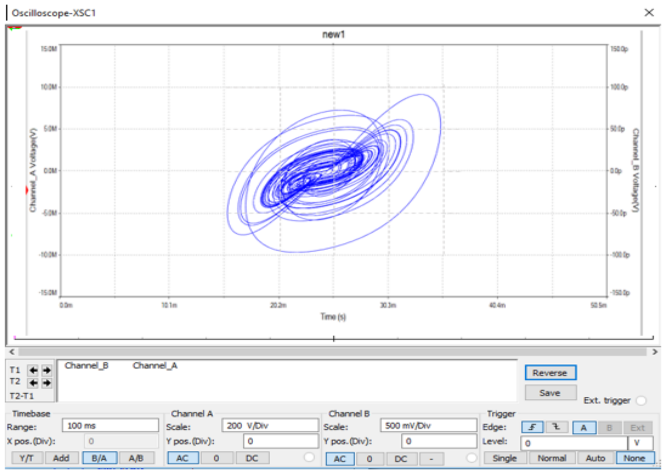

Figure 10. x-y attractor of the system.

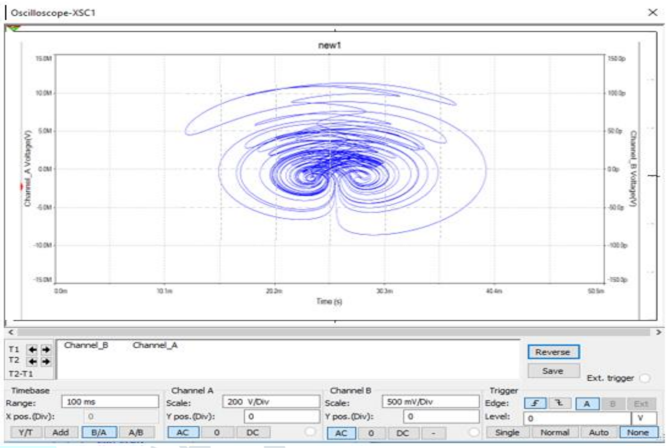

Figure 11. y-z attractor of the system

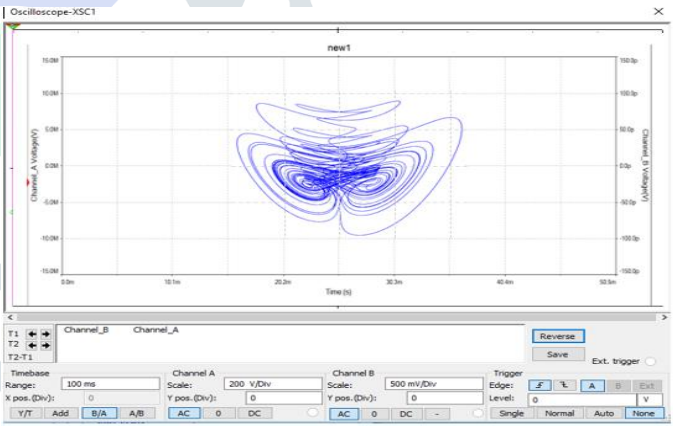

Figure 12. x-z attractor of the system

III.SYNCHRONIZATION OFTWO SIMILAR CHAOTIC SYSTEMS

The Pecora and Carroll (P-C) synchronization is applied to the two chaotic systems as shown in Fig.13.

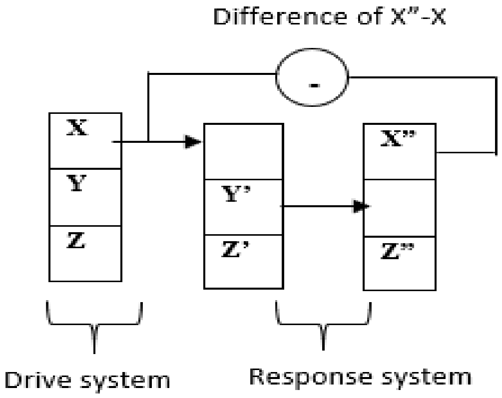

Figure 13. Block diagram of P-C Synchronization [5]

The fundamental state equations are modified for response system as follows:

Y' and Z' are first orderresponse sub-system.

$$\dot{Y} = a\dot{Y} - X\dot{Z} \quad (10)$$

$\dot{Z}$ = X$\dot{Y}$ −b

X" and Z" are the second order response sub-system.

$$\ddot{X} = \dot{Y} - \ddot{X} \quad (11)$$

$\ddot{Z}$= $X\ddot{Y} - b$

MATLAB Simulink for both drive and the response system is given in Fig.14.

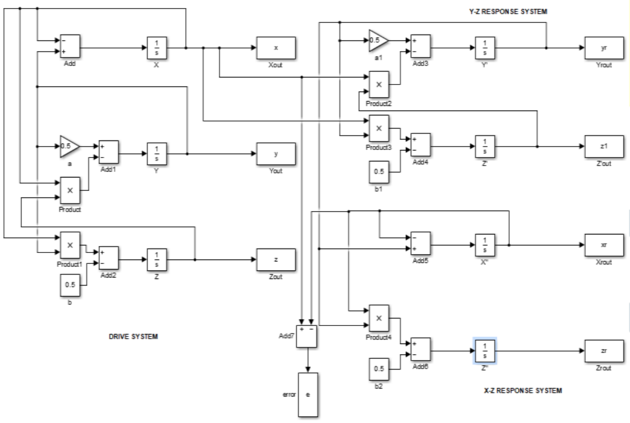

Figure 14. Simulink P-C synchronization model of Chaotic system

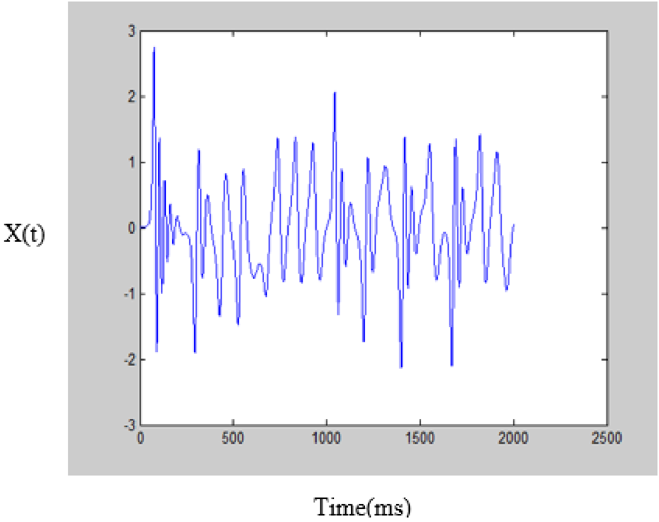

Figure 15. Graph of X signal

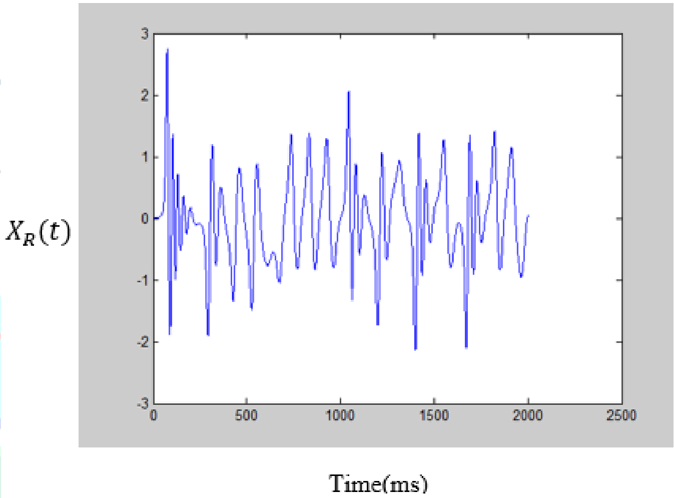

Figure 16. Graph of X_R signal

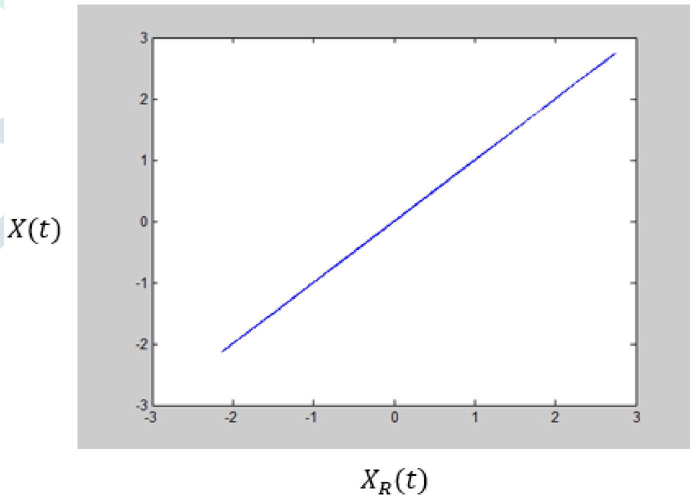

Figure 17. Synchronization between X and X_R signal

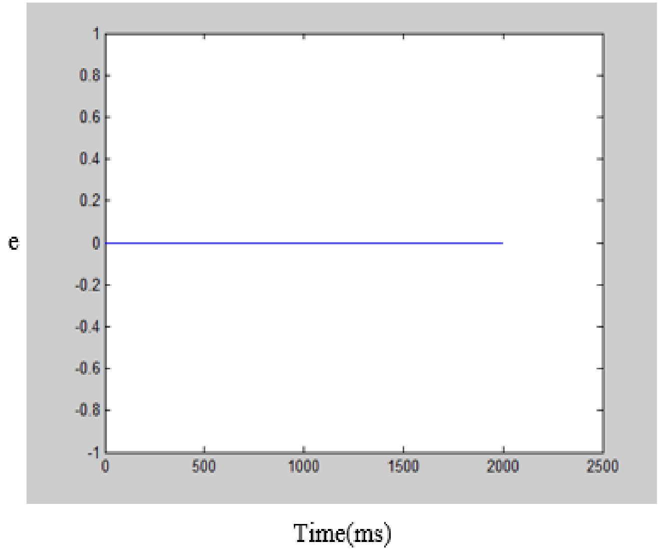

Figure 18. Synchronization error of the signal

From the Fig.15 it shows that the drive signal *X* and response signal $X_R$in Fig.16 is synchronized well as shown in Fig.17. The difference (e = error) signal is calculated by subtracting *X* from X_R as shown in Fig18. From the figure we can say that the signals are well synchronized since the error signal equals to zero.

IV.TRANSMISSION OF A SIGNAL BY SECURE COMMUNICATION

The process of hiding the information from the unauthorized user and transmitting it securely in the communication channel. The circuitry equation of the transmitter for secure communication is given:

$$\dot{X_R} = Y_R - s(t)$$

$$\dot{Y_R} = aY_R - s(t)Z_R \qquad (12)$$

$$\dot{Z_R} = s(t)Y_R - b$$

The block diagram for secure communication is given in Fig.19. The chaotic signal X(t) is added to the information signal I(t) which is a sine wave with amplitude 3 and frequency of0.65 is sent into the communication channel. The transmitted signal is S(t) which is the sum of X(t) and I(t).

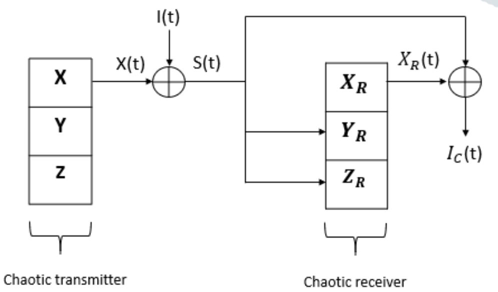

Figure 19. Principle scheme for chaotic secure communication [6]

According to the P-C synchronization method, the chaotic signal X(t) is synchronizedwith X_R(t) at the receiver side and then subtracted fromS(t) thereby, giving the signalI_c(t).The MATLAB Simulink of the model is given in Fig.20.

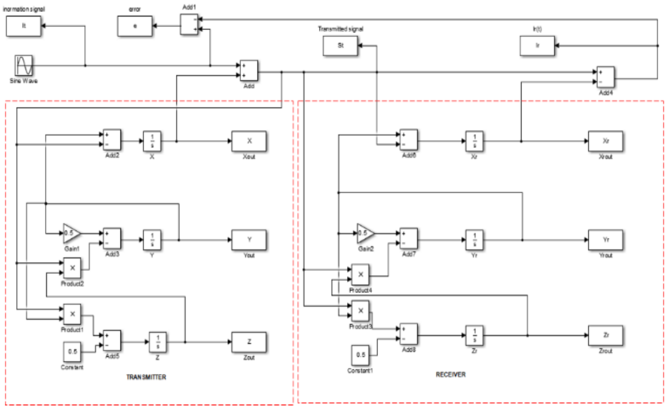

Figure 20. MATLAB Simulink for chaotic secure communication

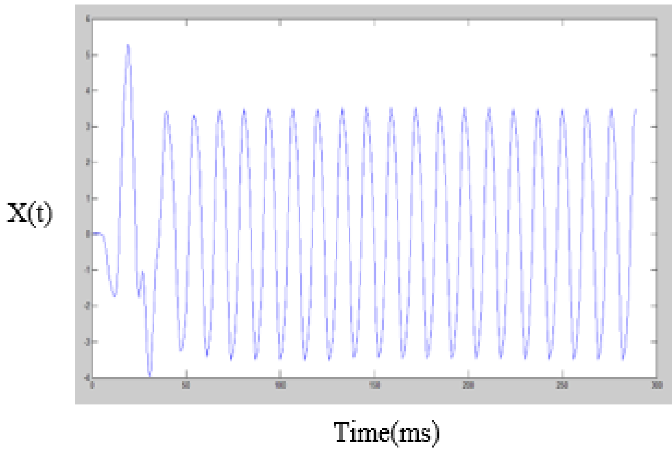

Figure 21. X(t) transmitter signal

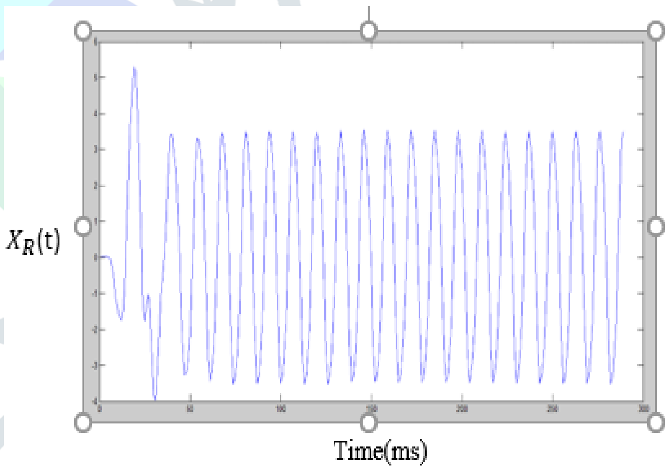

Figure 22. X_R(t) receiver signal

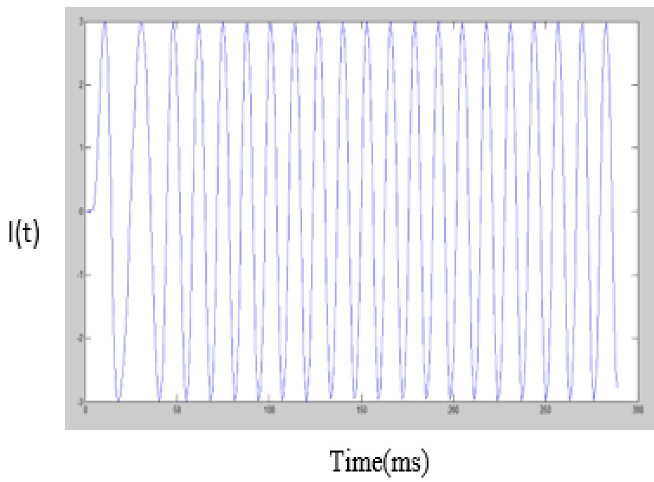

Figure 23. Information signal I(t)

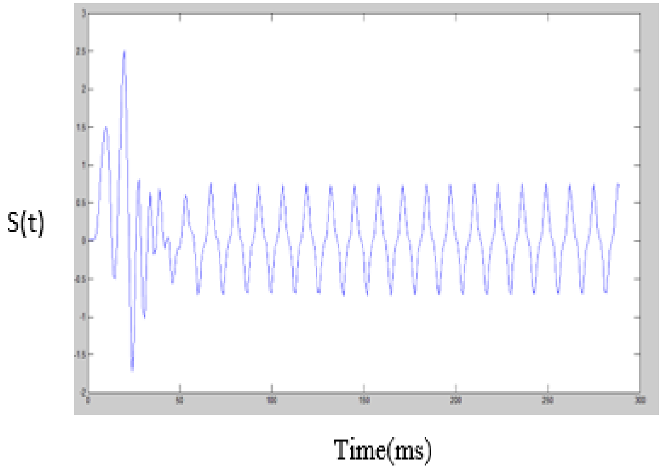

Figure 24. Transmitted signal S(t)=X(t)+I(t)

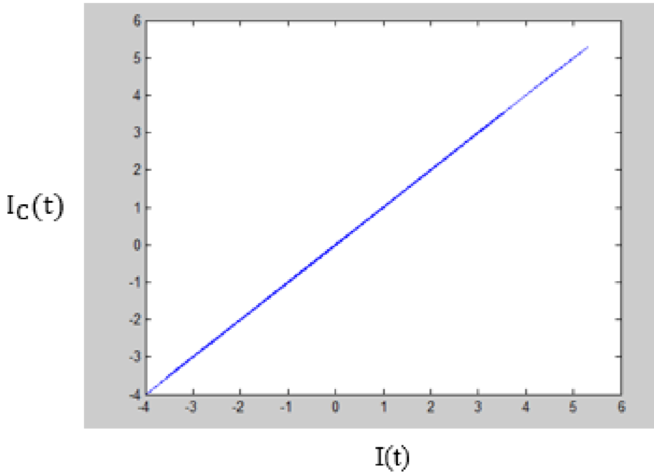

Figure 25. Synchronization between I(t) and$I_C(t)$

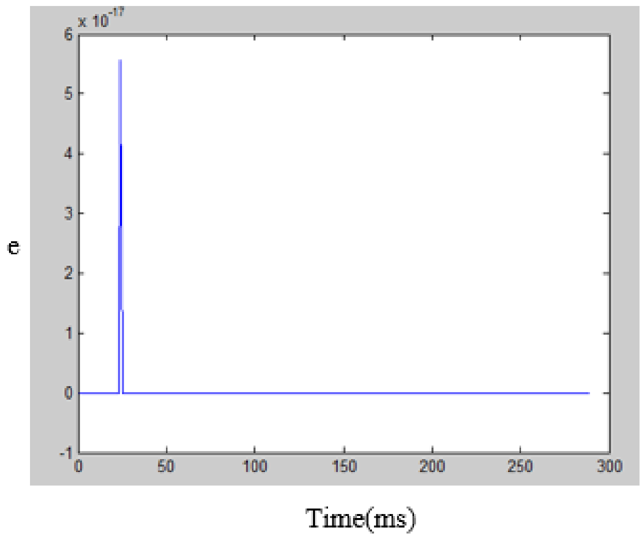

Figure 26. Synchronization error e(t) = I(t) - $I_c(t)$

The Simulink results obtained for chaotic secure communication is shown in Fig.21 to Fig.25. From the Fig.26we can see that after a few milliseconds the signal are synchronized well and the error signal equals to zero.

CONCLUSION

In this paper, a modified autonomous circuit is implemented which generates chaotic attractor. Theoretical analysis as well as the simulation in MATLAB-Simulink and Multisim is done for the given system. Then Pecora and Carroll synchronization is done for secure communication. The results show that the chaotic system can be used for secure transmission of information in the channel as the error between the transmitted and received signal is almost to zero.

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