VIBRATION CONTROL OF A SIMPLY SUPPORTED BEAM USING PIEZOELECTRIC PATCHES

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ABSTRACT- Any kind of vibration present in the machines and flexible structures are undesirable because it causes unpleasant noises, unwanted stress in structures, and malfunction or failure of the system. Now a day's Piezoelectric materials are used as sensors and actuators to control the system in terms of reducing the vibrations amplitude and frequency to improve the efficiency of the system. In the present work, a finite element model is developed for the vibration control of beams with distributed piezoelectric actuators. Smart beam considered in the analysis is modelled in simply supported configuration with piezoelectric patches bonded symmetrically on the top and bottom of the beam. The Static, Modal and Harmonic Analyses were performed using the finite element analysis software ANSYS to determine the displacements of a Piezo single layer subjected to Voltage, strain values, fundamental modal frequencies and amplitude of vibration for a simply supported beam made of different materials with patches at different locations. The optimum locations and number of actuator pairs are identified based on the amplitude of vibrations from harmonic analysis.

KEYWORDS: Piezoelectric patch, vibration control, FEM, ANSYS

1. INTRODUCTION

Piezoelectric materials exhibit the property known as a piezoelectric effect which results from the coupling between the electric and mechanical properties of a material. When a strain is applied to a piezoelectric material an electric field is produced (direct piezoelectric effect) and conversely when an electric field is applied strain occurs (converse piezoelectric effect). Due to the direct or converse piezoelectric effect, these materials can be used in the design of many devices working as sensors or actuators, respectively. For this reason, piezoelectric materials are a primary concern in the field of smart-structure technology. When a piezoelectric patch is attached to a damaged structure, the electro-elastic deformation of the piezoelectric patch induced by an applied voltage can help the structure to resist to the crack opening. This idea can be called as "active repair" in comparison with tradition bonded repair or passive repair. The piezoelectric material is used not only as an actuator to prevent crack propagation but also as a sensor to measure the crack opening displacement. Piezoelectric materials have received much attention in vibration control of structures in recent years because Piezoelectric ceramic materials have mechanical simplicity , small volume , light weight , large useful bandwidth , efficient conversion between electrical energy and mechanical energy , and easy integration with various metallic and composite structures. The active vibration control problem with a two-degrees of freedom system is given byHKaragulle*et.al.*(1).S. B. Choi*et.al.*(2)discussed about the vibration control responses of an electro rheological (ER) damper system subjected to temperature variation and time delay. C.M.A. Vasques*et.al.*(3)discussed about the one-dimensional finite element of a three-layered smart beam with two piezoelectric surface layers and metallic core is utilized. A partial layer wise theory, with three discrete layers, and a fully coupled electro-mechanical theory is considered. MelinSahin*et.al.*(4)discussed the smart structures considered in these analyses are finite and flat aluminium cantilever beam-like (called as a smart beam) and plate-like (called as smart fin) structures with surface bonded lead–zirconate–titanate patches.Zhi-chengQiu*et.al.*(5)discussed the problem of phase hysteresis and time delay caused by the non-collocated sensor/actuator pairs is considered. K Ramesh Kumar*et.al.*(6)discussed the optimal placement of collocated piezoelectric actuator–sensor pairs on flexible beams using a model-based linear quadratic regulator (LQR) controller.FayazR.Rofooei,*et.al.* (7)discussed the governing differential equation of motion for an undamped thin rectangular plate with a number of bonded piezoelectric patches on its surface and arbitrary boundary conditions is derived using Hamilton's principle.Levent Malgaca(8)discussed the direct velocity feedback (DVF) control is tested on a 3-DOF mechanical system under a step input.J. Ducarne,O.Thomas*et.al.*(9)discussed the geometry of piezoelectric patches as well as their placement on the host elastic structure. Paul murugan j *et.al.* (10) discussed the basic ideas of electrical-mechanical coupling in terms of analytical expressions with unique notations, stiffness matrix for coupled field element.Lalit R. Shendre et.al. (11)discussed about the ACV the vibration of a structure is reduced by using an opposite directional force to the structure.

2. PROBLEM STATEMENT & METHODOLOGY

The objective of the present work is to analyse a simply supported beam made up of different type of materials such as aluminium, brass, magnesium and titanium using piezoelectric patches at different positions. In this analysis different pairs of PZT patches are used to study the Active vibration control. First, the simply supported beam is modelled in the ANSYS and the piezoelectric patches are bonded symmetrically over the full length of the beam. The voltage and the loading boundary conditions are applied to the beam and patch. Static analysis is performed in order to obtain the displacements of a Piezo single layer subjected to Voltage and strain values of a simply supported beam made of different materials with patches at different locations. And then the Modal analysis is performed on the model, from this the natural frequencies are obtained and verified analytically. The Harmonic analysis is performed on the model by applying the harmonic force of 100N is applied, from this, the amplitude of vibration is obtained. And the same analysis is done for five pairs of patches, three pairs of patches at the middle and at two ends of the beam and one pair of patches at the middle of the beam. From this, we determine the best patch position for vibration control.

2.1 GEOMETRY

The dimensions of a simply supported beam and piezoelectric patch are taken as follows
simply supported beam dimensions are L= 76mm, w=26mm, h=2mm.
Where L= Length of beam in x-direction; w=width in y-direction; h=thickness in z-direction.
The piezoelectric patch dimensions are L= 76mm, w=26mm, h=1mm.

The geometry of a simply supported beam and piezoelectric Patches used for the present analysis are shown in Fig 1 and Fig 2.

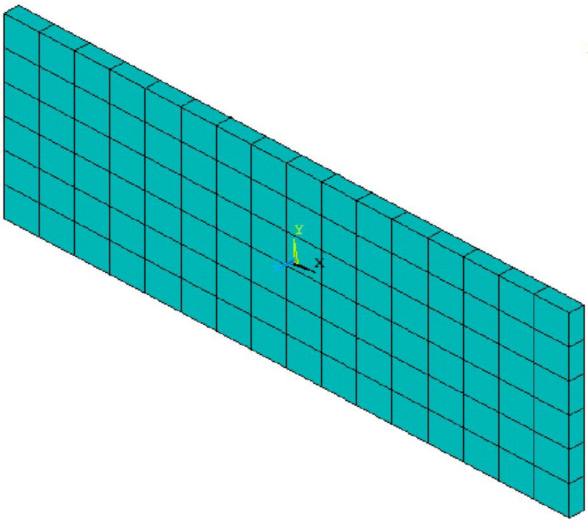

Fig 1: F.E Model

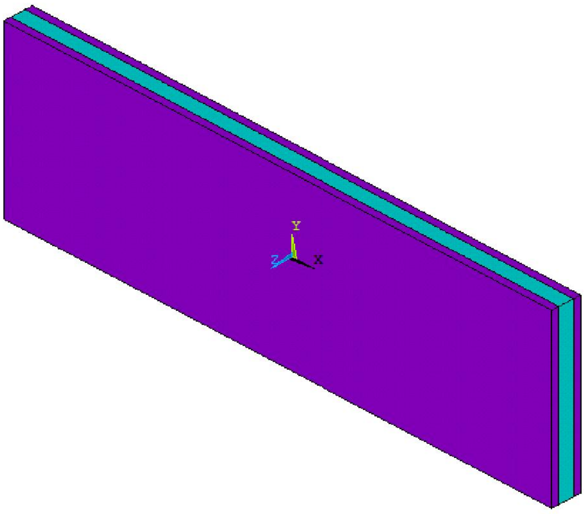

Fig 2: Beam with full patch

To investigate the effect of the number of actuators pairs on the deformation control, three sets of actuators pairs are considered: all the five pairs of actuators, three pairs of patches at the middle and at two ends of the beam and one pair of patches at the middle of the beam are as shown in Fig 3 to Fig 5. The piezoelectric patch dimensions are L= 14.4mm, w=26mm, h=1mm.

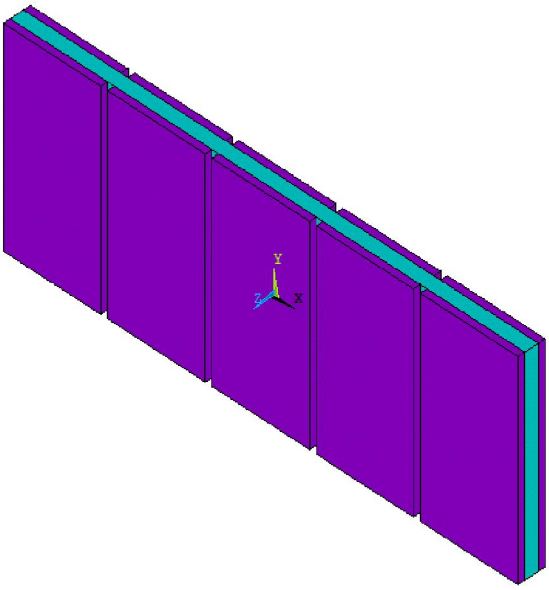

Fig 3: Beam with five pairs of patches

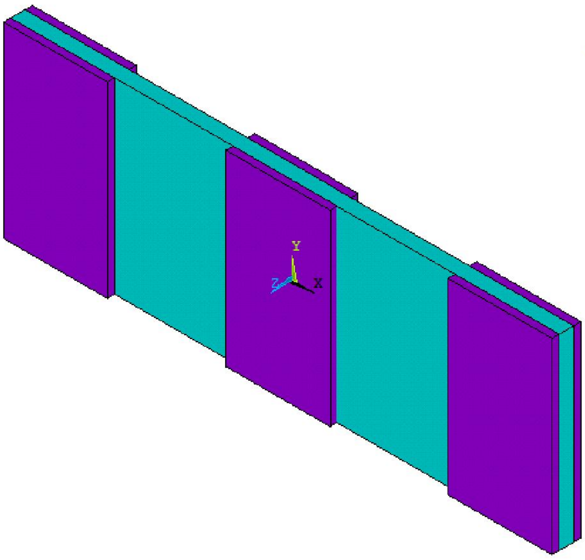

Fig 4: Beam with three pairs of patches

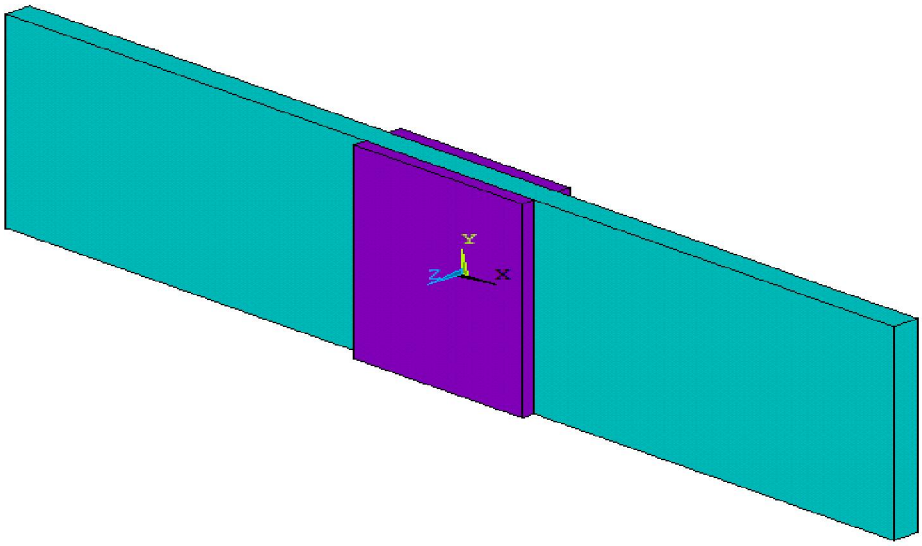

Fig 5: Beam with one pair of patches

2.2 MATERIAL PROPERTIES

The PZT(Lead-Zirconate-titanate) is the most commonly employed class of piezo-ceramic for the smart material application. These compounds are composed of PbTil-Xo_3 and $PbZrxo_3$ with x chosen to optimize electro mechanical coupling. The input material properties for PZT-5H piezoelectric ceramic is: PZT-5H has orthotropic material properties and it has 5 independent elastic constants are shown in Table-1.

Table 1. Material properties for PZT-5H patch

Parameters	Values	Unit
Density	7500	Kg/m^3
Compliance	C_{11}=C_{22} =16.6	10^{-12} m^2/N
	C_{12}=C_{21}= -4.84	
	C_{13}=C_{31}= -8.5	
	C_{33}= 20.76	
	C_{44}=C_{55}=C_{66}= 43.5	
Dielectric or Permittivity	K_{11}=K_{22}= 1.505	10^{-8} farad/meter
	K_{33}= 1.301	
Piezo-electric strain coefficient	d_{31}=d_{32}= 274	10^{-12} coulomb/N
	d_{15}=d_{24}= 741	
	d_{33}= 591	

The aluminium, brass, magnesium and titanium are used as beam material.
The material properties for simply supported beam are shown in Table-2

Table 2. Material properties for simply supported beam

Material	Young's modulus E(GPa)	Poisson's ratio, ν	Density ρ(Kg/m^3)
Aluminium	69	0.33	2700
Brass	102	0.33	8730
Magnesium	45	0.33	1738
Titanium	105	0.33	4506

2.4 FINITE ELEMENT MESHING

Solid 186 element is used for meshing Simply Supported Beam. The element contains 20 nodes with 3 DOF for each node in the nodal x,y,z directions. Solid 226 element is used for meshing the piezoelectric patch. The element has 20 nodes with up to six degrees of freedom per node.

2.5 LOADING & BOUNDARY CONDITIONS

The following types of loads and boundary conditions are applied for prediction of the response of the structure in Static, Modal and Harmonic Analysis. We considered the simply supported beam with the mechanical boundary conditions $U_x = U_z = 0$

at left end; U_z= 0 at right end and U_y= 0 at the negative y-axis. The voltage boundary Conditions are zero voltage is applied on the top and bottom surface of the patch and 100 voltage is applied on the nodes at the interface of the beam and the patch. A force of 100N is applied at the centre of the beam.

3. RESULTS AND DISCUSSION

3.1 PIEZO SINGLE LAYER SUBJECTED TO VOLTAGE

The static analysis for the PZT-5H actuator is carried out. The displacements along the length, width and depth direction's are obtained from the FEM results. The following Table-3 shows the displacement comparison between the analytical and FEM results and the error with in the acceptable limits.

The analytical formulae for calculating displacements along the length, width and depth direction's are

The displacement in length direction,$U_x = \frac{d_{31}*V*L}{h}$

The displacement in width direction, $U_y = \frac{d_{32}*V*W}{h}$

The displacement in the thickness direction, $U_z = \frac{d_{33}*V*h}{h}$

V=voltage applied.

Table 3. Comparison between the finite element results and analytical solution

Displacement (m)	FEM results (ANSYS) x10^{-6}	Analytical X10^{-6}	% Error
U_x (length direction)	2.11	2.08	1.44
U_y (width direction)	0.699	0.712	1.96
U_z(thickness direction)	0.052	0.0591	0.33

3.2 STATIC ANALYSIS

The simply supported beam and piezoelectric patch are constructed and the static analysis is carried out using ANSYS 18.1. From the static analysis, the strain values for different beam materials with patches at different locations are obtained.

3.2.1 FINITE ELEMENT ANALYSIS OF A SIMPLY SUPPORTED BEAM WITHOUT PATCH

The static analysis is carried out on the beam without the patch. From this, the strain in X-direction across the thickness (neutral axis to –Z side) is noted at the outer surface. The following Table-4 shows the strain at the outer surface of the beam.

Table 4.Strain comparison for different beam materials without patch

Beam	Aluminium	Brass	Magnesium	Titanium
Strain at outer (µs)-FEM-(ANSYS)	1730.3	1170.5	2653.1	1137

3.2.2FINITE ELEMENT ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FULL PATCH

The static analysis is carried out on the beam with a full patch on the top and bottom of the beam. From this, the strain in X-direction across the thickness (neutral axis to –Z side) is noted at the outer surface. The following Table-5 shows the strain at the outer surface of the patch.

Table 5. Strain comparison for different beam materials with full patch

Beam	Aluminium	Brass	Magnesium	Titanium
Strain at outer (µs)-FEM-(ANSYS)	34.8	31.2	33.09	31.25

3.2.3 FINITE ELEMENT ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FIVE PAIRS OF PATCHES

The static analysis is carried out on the beam with five pairs of patches on the top and bottom of the beam. From this, the strain in X-direction across the thickness (neutral axis to –Z side) is noted at the outer surface. The following Table-6 shows the strain at the outer surface of the patch.

Table 6. Strain comparison for different beam materials with five pairs of patches

Beam	Aluminium	Brass	Magnesium	Titanium
Strain at outer (µs)-FEM-(ANSYS)	975.7	865.9	1413	841.1

3.2.4 FINITE ELEMENT ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH THREE PAIRS OF PATCHES

The static analysis is carried out on the beam with three pairs of patches on the top and bottom of the beam. From this, the strain in X-direction across the thickness (neutral axis to –Z side) is noted at the outer surface.

Table 7. Strain comparison for different beam materials with three pairs of patches

Beam	Aluminium	Brass	Magnesium	Titanium
Strain at outer (µs)-FEM-(ANSYS)	1286	869	1972	844

3.2.5 FINITE ELEMENT ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH ONE PAIR OF PATCHES

The static analysis is carried out on the beam with one pair of patches on the top and bottom of the beam. From this, the strain in X-direction across the thickness (neutral axis to –Z side) is noted at the outer surface. The following Table-8 shows the strain at the outer surface of the patch.

Table 8. Strain comparison for different beam materials with one pair of patches

Beam	Aluminium	Brass	Magnesium	Titanium
Strain at outer (µs)-FEM-(ANSYS)	1271	862	1935	838

3.3 MODAL ANALYSIS

The beam and piezoelectric patch are constructed and the modal analysis is carried out using ANSYS 18.1. Modal analysis helps you to understand how a structure vibrates (frequency, damping, and mode shapes). Modal analysis can be used for troubleshooting, simulation and prediction, and design optimization. From the modal analysis, the natural frequencies are calculated for different beam materials with patches at different locations.

3.3.1 MODAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITHOUT PATCH

Modal analysis is carried out on the beam without the patch. The natural frequencies for six modes are calculated for different beam materials. The following Table-9 & Table-10 shows the comparison of the results between the finite element solution and analytical solution and both the results were in good agreement.

The analytical formulae for calculating the natural frequencies of a simply supported beam is

$$\omega_r = r^2\pi^2\sqrt{{EI}/{mL^4}} \quad \text{rad/sec}$$

Where E= young's modulus of the beam; I= moment of inertia
m= mass per unit length; L= length of the beam; r= mode number
Natural frequency, $f_r = \frac{\omega_r}{2\pi}$ Hz

Table 9. Natural frequency comparison between the FEM result and analytical solution for different beam materials

Beam	1st Mode	Analytical	2nd Mode	Analytical	3rd Mode	Analytical
Aluminium	805.96	790	3243.2	3160	7217.2	7110
Brass	544.96	535.4	2192.9	2141.6	4880	4818.6
Magnesium	811.2	792	3264.5	3168	7264.5	7128
Titanium	769.8	751	3096.9	3004	6891.7	6759

Table 10. Natural frequency comparison between the FEM result and analytical solution for different beam materials

Beam	4th Mode	Analytical	5th Mode	Analytical	6th Mode	Analytical
Aluminium	12703	12640	19820	19750	28440	28389.6
Brass	8644	8566.4	13415	13385	19332	19281.6
Magnesium	13050	12979.2	20050	19800	28614	28512
Titanium	12120	12016	18900	18775	27092	27036

3.3.2 MODAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FULL PATCH

Modal analysis is carried out on the beam with a full patch on the top and bottom of the beam. The natural frequencies for six modes are calculated for different beam materials are shown in Table-11.

Table 11. The natural frequency of different beam materials with full patch

Beam	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode
Aluminium	1089.9	4232.3	6982.2	8193.4	10917	11102
Brass	893.84	3482	5781	6837.8	9081.5	9349.9
Magnesium	1117.5	4315.0	7081.3	8202.3	10999	11035
Titanium	1042.3	4059.8	6728.2	7980.2	10572	10917

3.3.3 MODAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FIVE PAIRS OF PATCHES

Modal analysis is carried out on the beam with five pairs of patches on the top and bottom of the beam. The natural frequencies for six modes are calculated for different beam materials are shown in Table-12.

Table 12. The natural frequency of different beam materials with five pairs of patches

Beam	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode
Aluminium	889.94	3422.9	6761.8	7121.8	10023	10375
Brass	757.13	2932.8	5586.7	6097.8	8606.1	8649.9
Magnesium	863.17	3287.3	6808.3	6865.7	9624.7	10369
Titanium	890.44	3449.8	6540.7	7169.4	10103	10149

3.3.4 MODAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH THREE PAIRS OF PATCHES

Modal analysis is carried out on the beam with three pairs of patches on the top and bottom of the beam. The natural frequencies for six modes are calculated for different beam materials are shown in Table-13.

\ **Table 13. The natural frequency of different beam materials with three pairs of patches**

Beam	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode
Aluminium	683	2781	6224	6719	10451	11459
Brass	572	2217	4884	5620	8319	8480
Magnesium	616	2556	6022	6414	9879	11655
Titanium	719	2872	6067	6874	10475	10802

3.3.5 MODAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH ONE PAIR OF PATCHES

Modal analysis is carried out on the beam with one pair of patches on the top and bottom of the beam. The natural frequencies for six modes are calculated for different beam materials are shown in Table-14.

Table 14. The natural frequency of different beam materials with one pair of patches

Beam	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode
Aluminium	693	3118	5508	7400	10135	10862
Brass	564	2190	4179	5317	6738	8118
Magnesium	630	3051	5526	7309	10240	10570
Titanium	722	3040	5340	7183	9407	10917

3.4 HARMONIC ANALYSIS

The beam and piezoelectric patch are constructed and the Harmonic analysis is carried on different beam materials with patches at different locations using ANSYS 18.1.The harmonic force of 100N is applied at the centre of the beam. A harmonic analysis is used to determine the response of the structure under a steady-state sinusoidal (harmonic) loading at a given frequency.The following Table-15 shows the amplitude of vibration control for different materials with patches at different locations.

Table 15. The amplitude of vibration control for different beam materials with patches at different locations.

	Aluminium (mm)	Brass (mm)	Magnesium (mm)	Titanium (mm)
Beam without patch	38	5.4	31	5.2
Beam with Full patch	0.56	0.8	0.49	0.97
Beam with 5 pairs of Patches	5	1.5	1.8	3
Beam with 3 pairs of Patches	5.6	1.7	4.2	3.5
Beam with 1 pair of Patches	14.7	1.9	7.1	3.7

3.4.1 HARMONIC ANALYSIS OF A SIMPLY SUPPORTED BEAM WITHOUT PATCH

Harmonic analysis is carried out on the beam without the patch. The harmonic force of 100N is applied at the centre of the substrate. The following Fig 6 shows the amplitude of the vibration control of the beam.

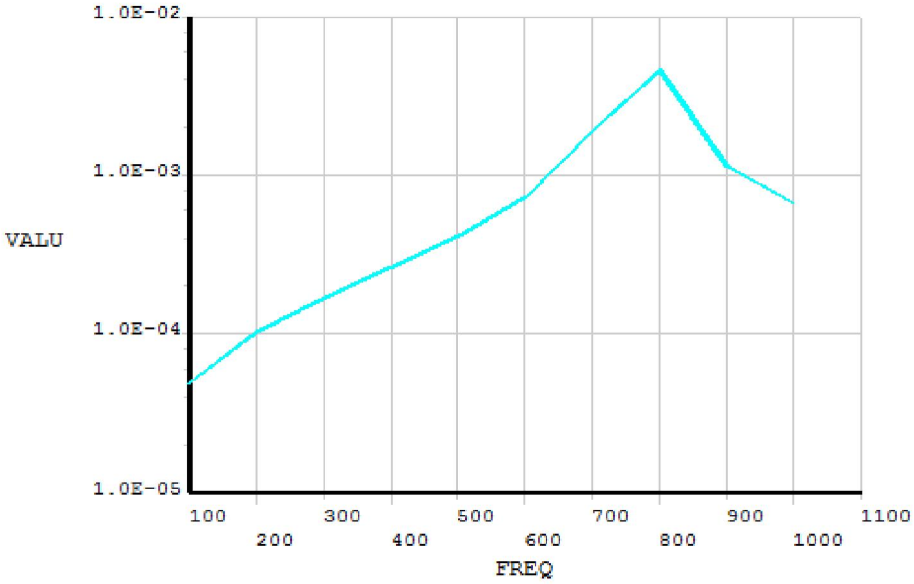

Fig6:Frequency vs. Amplitude results for Aluminium

3.4.2 HARMONIC ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FULL PATCH

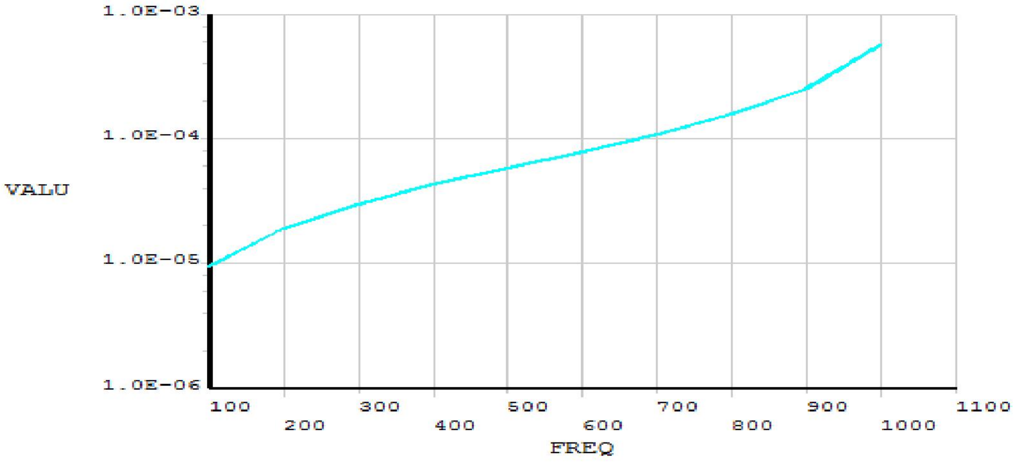

Fig7:Frequency vs. Amplitude results for Aluminium

3.4.3 HARMONIC ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH FIVE PAIRS OF PATCHES

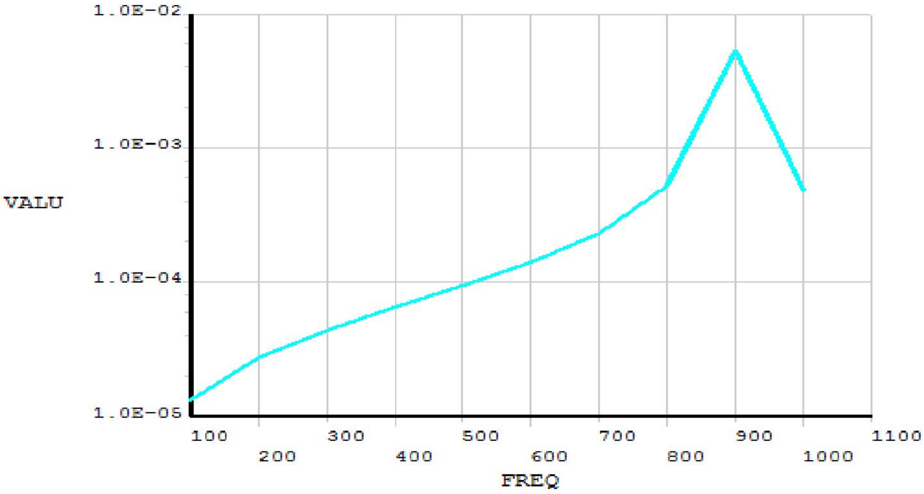

Fig8:Frequency vs. Amplitude results for Aluminium

3.4.4 HARMONIC ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH THREE PAIRS OF PATCHES

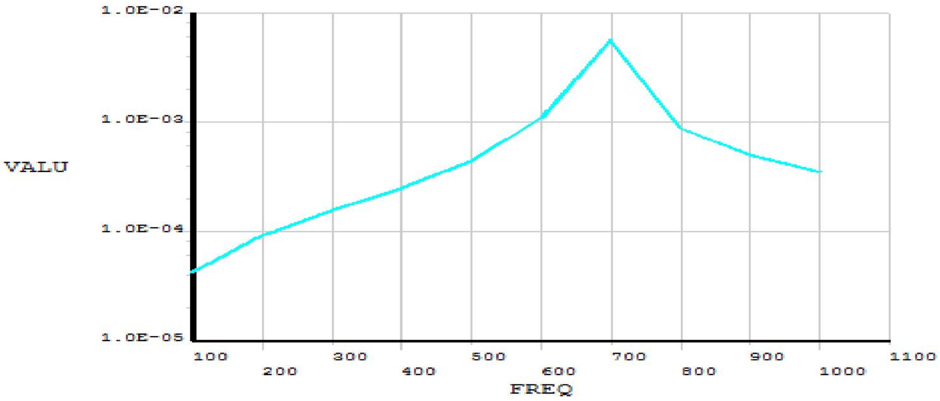

Fig9:Frequency vs. Amplitude results for Aluminium

3.4.5 HARMONIC ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH ONE PAIR OF PATCHES

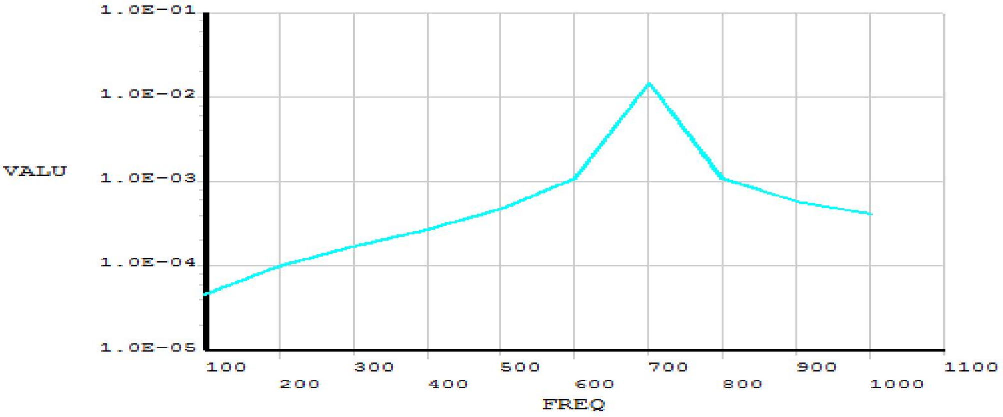

Fig10:Frequency vs. Amplitude results for Aluminium

4. CONCLUSION

The present work involves the vibration control of simply supported beam using piezoelectric patches based on finite element method. The beam and piezoelectric patch are constructed and the static, modal and harmonic analysis is carried out. The aluminium, brass, magnesium and titanium used as beam material. To investigate the effect of the number of actuators pairs on the deformation control, three sets of actuators pairs are considered: all the five pairs of actuators, three pairs of patches at the middle and at two ends of the beam and one pair of patches at the middle of the beam.

Based on the present analysis, the following conclusions were made:

- Static analysis is performed to determine the displacements along the length, width and depth direction's of Piezo single layer subjected to Voltage to validate FE Modelling of Piezoelectric materials
- Static analysis is performed to determine the strain values for different beam materials with patches at different locations. For maximum effectiveness, the patches must be placed in high strain regions and away from areas of low strains.
- Modal analysis is performed to determine the natural frequencies of six modes for different beam materials with patches at different locations.
- Harmonic analysis is performed to determine the amplitude of vibration control for different materials with patches at different locations from Frequency vs. Amplitude curves and from this the best vibration control is obtained when the patch is attached over the full length of simply supported beam.

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