A STUDY ON SINGLE SERVER FUZZY QUEUE CHARACTERISTICS BY FLEXIBLE α-CUTS METHOD

[1]D.Devi , [2] p.kaviarasi

[1]Head of the department, [2]M.phil.,scholar

[1]Department of Mathematics,

[1]Kamban College of arts and science for womens, Tiruvannamalai, India.

Abstract :

In real life system, number of system is described using queueing system .In this paper , we have discussed about the two kinds of fuzzy arithmetics to analysis fuzzy queues characteristics ,namely fuzzy arithmetic based on zadeh's extension principles and fuzzy arithmetic based on alpha cuts and intervals arithmetic .Here we have a numerical problems to find the flexible alpha cuts of a factory, this illustrate approach. The expected customer waiting time and the expected customer number in a single server fuzzy retrial queue are computed at steady state

Keywords:

Fuzzy queue characteristics, fuzzy arithmetic, extension principle, interval arithmetic, flexible alpha-Cuts and intervals arithmetic, fuzzy retrial queue.

I. INTRODUCTION:

Queueing systems is mainly analysis of banking ,factories ,telecommunication systems, computer systems, telephone systems ... This topic approach to correct alpha-cuts needed in the membership functions . An arriving customer finding the server busy, waits in a queue until the moment when the server is available to receive him, and leaves the system after service .In fuzzy queueing system literature, almost all studies have applied fuzzy set theory to analyze fuzzy queues characteristics.

shows that membership functions can following a new process which we denominate "*flexible α-cuts method*". Explain about the two fuzzy arithmetics based respectively on extension principle and a *alpha* cuts and intervals arithmetic, which are needed in system characteristics factories computation .The optimization mathematical programs called Parametric Non Linear Programming (PNLP) .

II. FUZZY ARITHMETIC BASED ON α-CUTS AND INTERVALS ARITHMETIC:

- **α-CUTS ARITHMETIC:**

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers. The fuzzy addition $\oplus$, substraction $\ominus$, multiplication $\odot$ or division $\oslash$ of $\tilde{A}$ and $\tilde{B}$ are defined through their a α-cuts ($0 \le \alpha \le 1$) which are closed and bounded real intervals. If

$\tilde{A}_\alpha$ =$[A^{L}(\alpha),A^{U}(\alpha)]$ and $\tilde{B}_\alpha$ = $[B^{L}(\alpha),B^{U}(\alpha)]$ represent respectively the a α-cuts of $\tilde{A}$ and the a α-cuts of $\tilde{B}$, then and *Buckley*, give the basic operations relative to a α-cuts of $\tilde{A}$ and $\tilde{B}$ as follows

$$[\tilde{A}\oplus\tilde{B}]_\propto = \tilde{A}_\alpha + \tilde{B}_\alpha = [A^{L}(\alpha),A^{U}(\alpha)]+[B^{L}(\alpha),B^{U}(\alpha)]$$

$$[\tilde{A} \ominus \tilde{B}]_\propto = \tilde{A}_\alpha - \tilde{B}_\alpha = [A^{L}(\alpha),A^{U}(\alpha)]-[B^{L}(\alpha),B^{U}(\alpha)]$$

$$[\tilde{A}\otimes\tilde{B}]_\propto = \tilde{A}_\alpha . \tilde{B}_\alpha = [A^{L}(\alpha),A^{U}(\alpha)].[B^{L}(\alpha),B^{U}(\alpha)]$$

$$[\tilde{A} \oslash \tilde{B}]_\propto = \frac{\tilde{A}_\alpha}{\tilde{B}_\alpha} = \frac{[A^{L}(\alpha),A^{U}(\alpha)]}{[B^{L}(a),B^{U}(a)]}$$

- ***Interval arithmetic:***

1) ***Buckley :***

Let $[a_1, b_1]$ and $[a_2, b_2]$ be two closed and bounded real intervals. If $\star$ denotes addition, substraction, multiplication or division, then

$$[a_1 , b_1] \star [a_2 , b_2] = [\alpha , \beta]$$

For division, it is assumed that $0 \notin [a_{2,}, b_2]$. With basic operations, is developed as follows :

$$[a_1,b_1]+[a_2,b_2] = [a_1 + a_{2,} b_1 + b_2]$$
$$[a_1,b_1]-[a_2, b_2] = [a_1 - b_{2,} b_1 - a_2]$$

$$[a_1 , b_1] . [a_2 , b_2] = [\min\{a_1 a_2, a_1 b_2, b_1 a_2, b_1 ab_2\}, \max\{a_1 a_2, a_1 b_2, b_1 a_{2,} b_1 ab_2\}$$

$$\frac{[a_1, b_1]}{[a_2, b_2]} = \left[\min\left\{\frac{a_1}{a_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{b_1}{b_2}\right\}, max\left\{\frac{a_1}{a_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{b_1}{b_2}\right\}\right]$$

III. TWO ARITHMETIC:

- **Fuzzy arithmetic based on extension principle:**

One of powerful methods of extension principle is the fuzzy arithmetic based on this principle. This arithmetic permits to extend a classical binary operation $*$ in $\mathbb{R}$ to a fuzzy binary operation $* \, in \, \mathbb{F}(\mathbb{R})$ such as $\forall z \epsilon \mathbb{R}, \forall \tilde{M}, \tilde{N} \epsilon \mathbb{F}(\mathbb{R})$.

$$\eta_{M*N}(z) = \sup\{min\{\eta_{\tilde{M}}(x), \eta_{\tilde{N}}(y)\} \mid x \epsilon \mathbb{R}, y \epsilon \mathbb{R} \text{ and } x * y = z\}$$

- **Definition :**

1. **Buckely:**

A fuzzy set Ã is said a fuzzy number if Ã is said a fuzzy subset of $\mathbb{R}$ *such as:*
(i) core(Ã) $\neq \emptyset$;
(ii) $\tilde{A}_\alpha$ are all closed and bounded subintervals of $\mathbb{R}$ *;*
(iii) supp(Ã) *is bounded.*
If Ã is a fuzzy number, each real number x such as $\eta_{\tilde{A}_\alpha}(x) = 1$ *is said* modal value, mode *or* mean value *of* Ã. *We will denote the set of all fuzzy numbers by* $\mathbb{F}(\mathbb{R})$

2. ***Mukeba*etal:**

Let Ã and $\tilde{B}$ be two fuzzy numbers. Ã is said less than $\tilde{B}$ (Ã < $\tilde{B}$) if only if
$\forall x \in supp(\tilde{A})$; $\forall y \in supp(\tilde{B})$, $x < y$. *In other words,*
$\tilde{A} < \tilde{B} \Leftrightarrow \sup\{supp(\tilde{A})\} < \inf\{supp(\tilde{B})\}$.

3. ***Hanss :***

The membership function of a fuzzy set Ã can be expressed in terms of characteristic functions of its α-cuts .
Formula

$$\eta_{\tilde{A}}(x) = \cup_{\alpha \epsilon [0,1]} \min\{\alpha, \eta_{\tilde{A}_\alpha}(x)\}$$

$$\eta_{\tilde{A}_\alpha}(x) = \begin{cases} 1 \; if \; x \epsilon \tilde{A}_\alpha \\ 0 \; otherwise \end{cases}$$

IV. The process flexible α-cuts method:

This present process can be applied in fuzzy model only when the analytical crisp queue formula and the fuzzy queue input parameters are known. Suppose that we are in want to determine a characteristic $\tilde{\psi}$ of a fuzzy queue whose input parameters are fuzzy numbers $\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3 \ldots \ldots \tilde{\xi}_n$, and suppose also that ψ and $\xi_1, \xi_2, \xi_3 \ldots . \xi_n$ are respectively the same characteristic and the same parameters in crisp model. If the analytical formula ψ of is known, it is often expressed by $\psi = f(\xi_1, \xi_2, \xi_3 \ldots . \xi_n)$; Where $\tilde{f}$ is a real multivalued function using basics operations ”$+, -, \times \; et, \div$ " in $\mathbb{R}$. By means of *Zadeh* ’s extension principle, the crisp characteristic ψ is extended to the following fuzzy characteristic $\tilde{\psi} = \tilde{f}(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3 \ldots \ldots \tilde{\xi}_n)$; where is $\tilde{f}$ a fuzzy multi valued function using basics fuzzy operations ”$\oplus, \ominus, \odot \; et \; \oslash$” in $\mathbb{F}(\mathbb{R})$.To determine the fuzzy queue characteristic $\tilde{\psi}$ by flexible a-cuts approach, it suffices
determine $\tilde{\psi}$ by α-cuts and intervals arithmetic.

In idea to illustrate the practical use of the approach, we propose and solve a numerical example in the following.

V. Problem

Consider a factor worked member 5500 in electronical department. This ward has provided to each of them a computing network in which there is 3 computers and one high speed printer. The operation consists in printing an worker identity card . The printer can product around 6 cards per second. The 3 operators affected to computers send around 3 requests per second to the printer. If an arriving request encounters the printer idle, it enters in exponential service, triggers a printing and one member is served.

Solution

In the terms of queueing theory, the computing network provided for each electoral ward above is identifiable as a markovian single server retrial queue with patient customers and ill-known parameters denoted FM/FM/1-R .

Using fuzzy sets theory, these parameters can be written as the following triangular fuzzy numbers:

The requests sent to the printer are considered as customers and thus, arrival rate is $\tilde{\lambda}$=(4,5,6),

The printer is considered as the server and its speed as service rate $\tilde{\mu}$ = (7,8,9),

The buffer storage is an orbit and the repeated attempts are retrials rate $\tilde{\theta}$ = (3,4,5)

Suppose that l ;m and q denote these queues parameters in crisp model and assume that $E(W)$, $E(N)$, $E(W_b)$ and $E(N_b)$ are respectively the expected request waiting time in the system, the expected requests number in the system, the expected request waiting time in the buffer and the expected .The traditional queueing theory, if the system is stable, that is $\frac{\lambda}{\mu} < 1$, these measures are computed from the following

$$\mathrm{E}(W_b)=\frac{\lambda}{\mu-\lambda}\left(\frac{1}{\mu}+\frac{1}{\theta}\right) \longrightarrow 1 \qquad \mathrm{E}(N_b)=\frac{\lambda^2}{\mu-\lambda}\left(\frac{1}{\mu}+\frac{1}{\theta}\right) \longrightarrow 2$$

By Zadeh's extension principle likewise. Further to these considerations, the a α-cuts of , $\tilde{\lambda}, \tilde{\mu}$ *and* $\tilde{\theta}$ are successively

$$\tilde{\lambda}_\alpha = [\alpha+4, -\alpha+6] \quad , \quad \tilde{\mu}_\alpha = [\alpha+7, -\alpha+9\,] \quad , \quad \tilde{\theta}_\alpha = [\alpha+3, -\alpha+5]$$

Expected request waiting time in the system E($\widetilde{W}$)

$$[E(\widetilde{w})]_\alpha = \left(\frac{\tilde{\lambda}_\alpha}{\tilde{\mu}_\alpha - \tilde{\lambda}_\alpha}\right).\left(\frac{\tilde{\lambda}_\alpha + \tilde{\theta}_\alpha}{\tilde{\lambda}_\alpha.\tilde{\theta}_\alpha}\right)$$

Replacing $\tilde{\lambda}, \tilde{\mu}$ *and* $\tilde{\theta}$

$$[E(\widetilde{w})]_\alpha = \left(\frac{[\alpha+4,-\alpha+6]}{[\alpha+7,-\alpha+9]-[\alpha+4,-\alpha+6]}\right).\left(\frac{[\alpha+4,-\alpha+6]+[\alpha+3,-\alpha+5]}{[\alpha+4,-\alpha+6].[\alpha+3,-\alpha+5]}\right)$$

$$[E(\widetilde{w})]_\alpha = \left(\frac{[\alpha+4,-\alpha+6]}{[2\alpha+1,-2\alpha+5]}\right).\left(\frac{[2\alpha+7,-2\alpha+11]}{[\alpha+4,-\alpha+6].[\alpha+3,-\alpha+5]}\right) \qquad \text{(By formula)}$$

$$[E(\widetilde{w})]_\alpha = \left(\frac{[\alpha+4,-\alpha+6]}{[2\alpha+1,-2\alpha+5]}\right).\left(\frac{[2\alpha+7,-2\alpha+11]}{[minF_1(\alpha), maxF_1(\alpha)]}\right)$$

Where $minF_1(\alpha)$ *and* $maxF_1(\alpha)$ are solutions of the two following parametric non linear programs (PNLP),

$$\{minF_1(\alpha) = \min\{f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\}$$

$$f_{11}(\alpha) = (\alpha+4)(\alpha+3)$$
$$f_{12}(\alpha) = (\alpha+4)(-\alpha+5)$$
$$f_{13}(\alpha) = (-\alpha+6)(\alpha+3)$$
$$f_{14}(\alpha) = (-\alpha+6)(-\alpha+5)$$

$$\{maxF_1(\alpha) = \max\{f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\}$$

$$f_{11}(\alpha) = (\alpha+4)(\alpha+3)$$
$$f_{12}(\alpha) = (\alpha+4)(-\alpha+5)$$
$$f_{13}(\alpha) = (-\alpha+6)(\alpha+3)$$

$$f_{14}(\alpha) = (-\alpha+6)(-\alpha+5)$$

$$minF_1(\alpha) = f_{11}(\alpha) = (\alpha+4)(\alpha+3)$$
$$maxF_1(\alpha) = f_{14}(\alpha) = (-\alpha+6)(-\alpha+5)$$

$$[E(\widetilde{w})]_\alpha = \left(\frac{[\alpha+4,-\alpha+6]}{[2\alpha+1,-2\alpha+5]}\right).\left(\frac{[2\alpha+7,-2\alpha+11]}{[\alpha+4,-\alpha+6].[\alpha+3,-\alpha+5]}\right)$$

$$=\frac{[minF_2(\alpha),maxF_2(\alpha)]}{[minF_3(\alpha),maxF_3(\alpha)]}$$

Similarly for $minF_3(\alpha), maxF_3(\alpha), minF_4(\alpha), maxF_4(\alpha)$

$$[E(\widetilde{w})]_\alpha = \left(\frac{(\alpha+4)(2\alpha+7)}{(-2\alpha+5)(-\alpha+6)(-\alpha+5)}\right).\left(\frac{(-\alpha+6)(-2\alpha+11)}{(2\alpha+1)(\alpha+4)(\alpha+3)}\right) \quad \longrightarrow \quad 3$$

Expected requests number in the system $E(\widetilde{N})$

Using Little's law, the expected requests number in the system $\boldsymbol{E(\widetilde{N})}$ is obtained progressively as follows:

$$[E(\widetilde{N})]_\alpha = \tilde{\lambda}_\alpha.\ [E(\widetilde{w})]_\alpha$$

$$=(\alpha+4)(-\alpha+6).\left(\frac{(\alpha+4)(2\alpha+7)}{(-2\alpha+5)(-\alpha+6)(-\alpha+5)}\right).\left(\frac{(-\alpha+6)(-2\alpha+11)}{(2\alpha+1)(\alpha+4)(\alpha+3)}\right)$$

$$=[\,minF_5(\alpha), maxF_5(\alpha)]$$

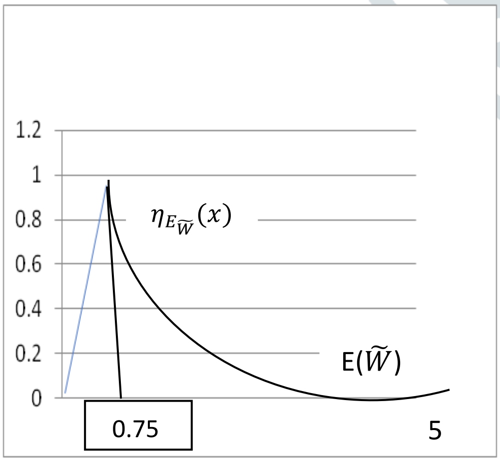
Again the process PNLP,then finally

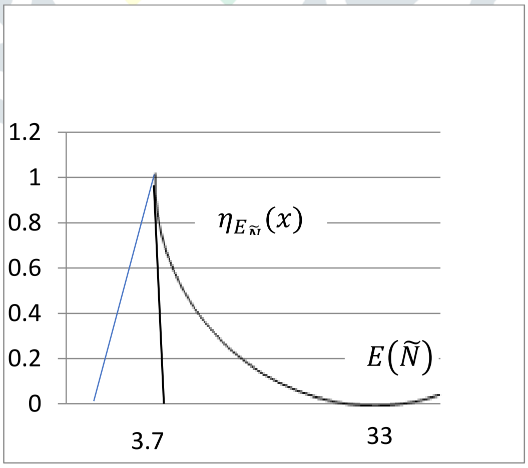
$$[E(\widetilde{N})]_\alpha = \left(\frac{(\alpha+4)(\alpha+4)(2\alpha+7)}{(-2\alpha+5)(-\alpha+6)(-\alpha+5)}\right).\left(\frac{(-\alpha+6)(-\alpha+6)(-2\alpha+11)}{(2\alpha+1)(\alpha+4)(\alpha+3)}\right) \quad \longrightarrow \quad 4$$

Because the stability condition $\tilde{\lambda} < \tilde{\mu}$ of the network process is verified. If we successively put $\alpha = 0$ and $\alpha = 1$ in Eq. 3 and 4 we obtain respectively$[E(\widetilde{w})]_0$=[0.18,5.5], $[E(\widetilde{w})]_1$ =*0.75}*, $[E(\widetilde{N})]_0$=[0.*74*,*33*], $[E(\widetilde{N})]_1$ ={ 3.75*}*. The first equality $[E(\widetilde{w})]_1$=[0.18,5.5**]** indicates that the expected request waiting time in the system is between *0.18* and 5 seconds. The second gives the modal value of $[E(\widetilde{w})]_\alpha$ which means that the most possible value of the expected request waiting time in the system is 0.75 second. Similarly, the third and the fourth equalities mean that the expected number of requestsin the system is between 0.*74* and 33, and its most possible value is 3.3.

If $\alpha -$ runs from 0 to 1, the bounds of real intervals in Eq. (3) and (4) describe the membership functions graphsof fuzzy characteristics E($\widetilde{W}$) and E($\widetilde{N}$) represented on figures 1 and 2 as below.

Figure 1: Membership function graph of $E(\widetilde{W})$ Figure 2: Membership function graph of $E(\widetilde{N})$.





VI. Conclusion

Good results if the number of fuzzy queue parameters exceeds 3. By its help, request waiting time and requests number in a fuzzy computer system have been successfully .Arithmetics, the paper has introduced a new approach called *flexible alpha-cuts method* which accomplishes the same task using only one type of fuzzy arithmetic. Analysis of fuzzy queues having atmost 3 fuzzy parameters for reliable calculations, clear and immediate.

VII. References:

1) M. Jeeva, E. Rathnakumari, Fuzzy Cost Computations of M/M/1 and M/G/1 Queueing Models, British Journal of Mathematics and Computer Science, 4 (1) (2014) 120-132.*https:/doi.org/10.9734/BJMCS/2014/4353*

2) V.G. Kulkarni and B. D. Choi, Retrial Queues with Server Subject to Breakdowns and Repairs, Queueing Systems.7, 191–208 (1990).

3) J. P. Mukeba, R. Mabela, B. Ulungu, Performance Measures of a Product Form Queueing Network with Fuzzy Parameters, Journal of Fuzzy Sets Valued Analysis, 2015 (1) (2015) 68-77.

4) L. A. Zadeh, "The Concept of a Linguistic Variable and its Application to Approximate Reasoning- I", Information Sciences (8), 199–249 (1975).

5) E. S. Lee and R. J. Li, Analysis of Fuzzy Queues, Computers and Mathematics Applications.17, 1143–1147 (1989).

6) C. Kao and C.C. Li and S.P. Chen, Parametric Programming to the Analysis of Fuzzy Queues, Fuzzy Sets and Systems 107, 93–100 (1999)

7) R. Kalayanaraman, N. Thillaigovindan, G. Kannadasan, A Single Server Fuzzy Queue with Unreliable Server, International Journal of Computational Cognition, 8 (1) (2010) 1-4.

8) J.-C., Ke, H.-I., Huang and C.-H. Lin., On Retrial Queueing Model with Fuzzy Parameters." Physical A: Statistical Mechanics and its Applications 374 (1), 272–280 (2007).