Stabilization of Cart Inverted pendulum using type 2 fuzzy PID Controller

Shraddha D. Tale
Department of Electrical Engineering
Walchand College of Engineering
sangli

Prof. N.V. Patel
Department of Electrical Engineering
Walchand College of Engineering
Sangli

***Abstract*—Nowadays simple and good excludable controllers are used for control many complex process and a nonlinear system. For designing, such type of controller for inverted pendulum for stabilization and prove the performance of controller system. The cart inverted pendulum system is nonlinear and highly unstable system. for stabilization of cart and pendulum many control strategy used, The cart moved horizontal direction one meter track and pendulum is stabilized upright position. The cart inverted pendulum system equation are nonlinear so that the relation between cart and angle pendulum difficult to calculate. In this paper controller design for stabilization of pendulum. The interval type 2 fuzzy PID controller are designed for stabilization of pendulum angle and cart position with uncertainty model. The degree of freedom is also provided by interval type 2 fuzzy set can be used for improve the performance of system.**

***Keywords*—cart Inverted pendulum stabilization; Intervay Type 2 fuzzy controller; PID controller;**

I. INTRODUCTION

The inverted pendulum on cart is highly unstable system. The problem in also dynamics of system because cart inverted pendulum system is complex system. The cart inverted pendulum system is multivariable, nonlinear, non-minimum phase system. The pendulum is connect to cart, pendulum is incorporate with two degree of freedom. It operate with degree of freedom provided the horizontal motion of cart. The inverted pendulum main benchmark problem is highly unstable system and that's way problem in dynamics of pendulum.

The inverted pendulum is unstable system that's way pendulum fall any direction we have to stabilized the pendulum in upright position. It required exact control signal for stabilization. So many control strategy has been designed (PID, fuzzy controller, LQR, state feedback controller) for stabilization of pendulum. The dc motor is used, it gives the control voltage to cart. The control force is applied on cart cart move in horizontal direction in one meter track and pendulum attached to hinged of cart. Control force is applied on cart and pendulum is stabilized upright position.

There is two states of inverted pendulum one is stable state and another is unstable state. The pendulum is at vertical position at o=o this is the unstable state. When o=π there is stable state. At unstable state it has a some disturbance is there,it has infelicitous state that's way it highly nonlinear system. For that stabilization linear controller is designed. The inverted pendulum system is single input multiple output (SIMO), the input is in dc voltage form and output is cart position and pendulum angle. The output is measured by the optical encoder is attached to the cart.

In principle on cart inverted pendulum system, it has many application and on this control approach proceed it has application like robotic system, mobile wheeled inverted, humoid robots, and on this benchmark pendulum priciple is used for like Segway, rocket lunching and huge lifting crane.

The fuzzy controller designed for the stabilization of cart inverted pendulum using this linear controller the exact linear control force applied and stabilized the pendulum at upright position. The fuzzy logic controller is important controller that it is totally rule based system. The human decision troupe with rule collection. The type 2 fuzzy controller is extension version of type 1 fuzzy.

II. MATHEMATICAL MODELLING OF INVERTED PENDULUM

A. Mathematical Modelling Analysis

The mathematical modeling is the important for the cart inverted pendulum system. The mathematical modeling of system is describe the dynamics of system in form of set of equation. For cart inverted pendulum system can be represented by different mathematical model, different method are used for describe the model of system.

It can be get in form of input initial condition either it can be represent in transfer function form or state space form. The transfer function is used for the single input single output (SISO)- LTI system and transfer function state space representation with multi input multi output (MIMO) system.

In this chapter we introduced the cart inverted pendulum system and mathematical modeling of system. There is two methods to analyzed the modeling of system.one is lagrangies method and another is newton low. but we used newton low method and obtain the transfer function of system, we get linearized model of system and present into in state space form. The motion of inverted pendulum consist of translational movement and rotational movement, from this movement characteristic we describe the model of inverted pendulum.

. The following is the parameter table that gives the value of the various parameters that has been adopted from the Feedback Digital Pendulum Manual.

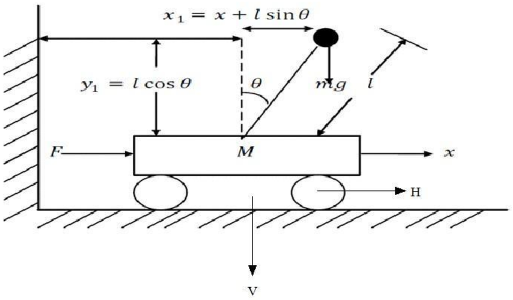

Fig-1. Configuration inverted pendulum on cart

Let H the horizontal component of reaction force and V be vertical component of reaction force. Let x1 be the horizontal component of co-ordinates of Centre of Gravity (COG) and y will be vertical component co-ordinate of COG

$$x1 = x + l\sin\theta \qquad (1.1)$$

$$y1 = l\cos\theta \qquad (1.2)$$

$$\dot{x1} = \dot{x} + l\dot{\theta}\cos\theta \qquad (1.3)$$

$$\dot{y1} = -l\dot{\theta}\sin\theta \qquad (1.4)$$

For analyze translational motion use newton first low motion apply the net force on body is equal to the product of mass and acceleration.

$$H = m\ddot{X}1$$

The friction component of force that opposes the linear motion of the cart

$$F = M\ddot{X} + b\dot{X} + H \qquad (1.5)$$

we get

$$F = (m + M)\ddot{X} + b\dot{X} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}\sin\theta$$

The vertical reaction as

$$V = m\ddot{Y}1 = -m\ddot{\theta}L\sin\theta - m\dot{\theta}\,L\cos\theta$$

The torque equation given by

$$-H\cos\theta\,L + (v + mg)\sin\theta = I\ddot{\theta} + b\dot{\theta}$$

Thus

$$(I + mL^2)\theta = -mLX\cos\theta - b\theta + mgL\sin\theta$$

Two equilibrium condition for CIPS at θ=0(vertically up)

And θ=Π

For linearized model up position

$\sin\theta = \theta$, $\cos\theta = 1$

We get simplified transfer

$$\frac{\theta(s)}{F(s)} \quad \frac{sml}{-s^3[(M+m)I+Mml^2]+s^2[d(M+m)+b(I+ml^2)]-s[bd+mgl(M+m)]+mglb}$$

Symbol	Parameter	Value	Unit
M	Cart Mass	2.4	Kg
M	Pole Mass	0.23	Kg
L	Pole Length	0.4	m
B	Cart Friction coefficient	0.05	Ns/m
d	Pendulum damping coefficient	0.005	Nms/m
I	Moment of Inertia of the pole	0.099	Kg.m^2
g	Gravity	9.81	m/s^2

After substituting value of parameter, we get transfer function θ=0

$$\frac{\theta(s)}{F(S)} = \frac{0.092s}{0.34869s^3+0.14895s^2+2.3733s-0.0451}$$

$$\frac{X(s)}{F(S)} = \frac{0.1358s^2+0.005S-0.9025}{0.34869s^4+0.14895s^3-2.3733s^2-0.0451S}$$

So we, get linearized state space model

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-b(I+ml^2)}{I(M+m)+Mml^2} & \frac{-m^2l^2g}{I(M+m)+Mml^2} & \frac{mdl}{I(M+m)+Mml^2}\\ 0 & 0 & 0 & 1\\ 0 & \frac{mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & \frac{-d(M+m)}{I(M+m)+Mml^2}\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \frac{(I+ml^2)}{I(M+m)+Mml^2}\\ 0\\ \frac{-ml}{I(M+m)+Mml^2}\end{bmatrix}u$$

$$\begin{bmatrix}\dot{X}\\ \ddot{X}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.0195 & 0.2381 & 0.0013\\ 0 & 0 & 0 & 1\\ 0 & 0.0132 & 6.8073 & -0.0377\end{bmatrix}\begin{bmatrix}X\\ \dot{X}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ 0.3895\\ 0\\ -0.2638\end{bmatrix}U$$

$$Y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}X\\ \dot{X}\\ \theta\\ \dot{\theta}\end{bmatrix}$$

.

III. DESIGN INTERVAL TYPE 2 FUZZY CONTROLLER

The interval type 2 fuzzy logic controller is design for stabilization of pendulum. The designing IT2FC is better than than the PID controller for simplicity. The interval type2 fuzzy controller intensify the performance of controller for nonlinear system. The type 2 fuzzy controller produced superior control performance and handle nonlinear dynamics, parameter uncertainty, noise and disturbance. For internal constitution of IT2FC is same to type 1 fuzzy controller. The vital dissimilarity of output of fuzzy set or rule based of interval type 2 fuzzy logic controllers.

The type reducer require to turn into type 1 fuzzy set before performed the difuzzyfication process. The footprint of uncertainty (FOU) the degree of freedom to improve the control execution of system. The type 2 fuzzy controller give the easy plain scaling surface than type 1 fuzzy controller. The internal constitution of type 2 fuzzy controller are more complex than type 1 fuzzy controller. The design of IT2FC depend on the PID control law.

For design IT2FC some methods exit, the type 2 fuzzy controller is consist of one single input signal i.e. error and output in form of designing parameter of membership function of interval type 2 fuzzy set. To design and supervise the type 2 fuzzy set is in form of error domain.one inference mechanism is presented in output of type 2 fuzzy PID controller. The conventional PID controller is cascaded with type 2 fuzzy controller, mapping error signal in membership function. After performing the normalization method, the input variable of error signal is within the range of [-1 1].

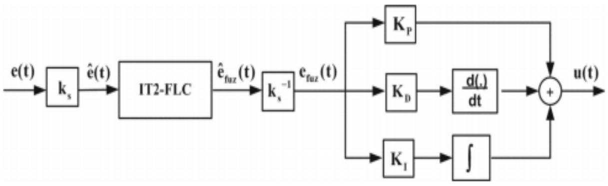

Fig.2.- control scheme interval type 2 fuzzy PID

The error signal represented in e= r-y, r is reference signal and y is output of process. The error signal scaled and limited in following evolution.

The input to the fuzzy controller is decided by ,

$$k_s = \begin{cases} +1 & k_s e(t) > +1 \\ k_s & |k_s e(t)| < +1 \\ -1 & k_s e(t) < -1 \end{cases}$$

k_s= scaling factor

$\hat{e}$(t)= input variable of the IT2FC structure

$$k_s = \frac{1}{r_0 - y_0}$$

r_0 and y_0 are the initial reference and initial process output

After performing simple normalization the error $\hat{e}$ within range [-1 1]

A rescaling is execute to the output of error fuzzyfie follows

$e_{fuz}(t) = \hat{e}_{fuz}(t) k_s$

The another controller structure is PID controller is cascade with fuzzy controller

$$u(t) = k_p e_{fuz}(t) + k_I \int e_{fuz}(t) + k_D \frac{d_{efuz}(t)}{d_t}$$

k_p - Proportional gain

k_I - Integral gain

k_D - Derivative gain

For tuning of PID controller gain various methods are used and give exact gain value of PID controller. Here PID controller is designed with less overshoot and less settling time. The PID gain value are kp ki and kd for smoother control action three control action designed. The response is track the reference of input signal. The proportional gain give the smoother control action and process output converges to reference

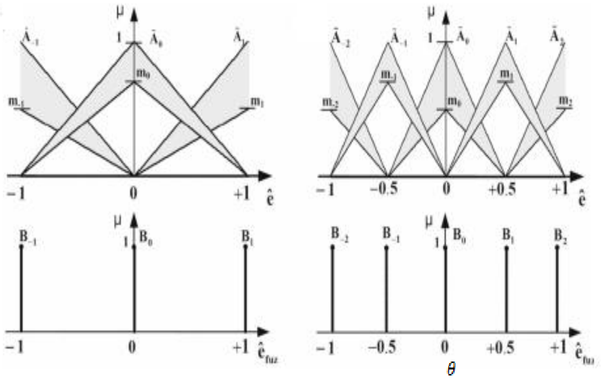

Fig.3.- smooth and aggressive membership function

(i) If the value of µA˜ 1(e)(i.e. m1) decreases/increases then the value of el fuz decreases/increases, respectively.

(ii) If the value of µA˜ 0(e)(i.e. m0) decreases/increases then the value of er fuz increases/decreases, respectively. In the light of (i) & (ii) reminding that the defuzzified output of an IT2-FLC (efuz) is the average value of er fuz and el fuz values.

(iii) If the value of m0 is decreased while m1 is increased then the value of efuz is increased since the values of both er fuz and el fuz are increased. Thus, an aggressive control action is obtained.

(iv) If the value of m0 is increased while m1 is decreased then the value of efuz is decreased since the values of both er fuz and el fuz are decreased. Thus, a smooth control action is obtained.

The proposed controller design method can be easily extended and generalized to IT2 FLCs with alternative interval type-2 fuzzy sets representing the input domain. However, it should be noted that the key principle of the proposed design methodology is "simplicity". For that reason, a simple rule base with triangular IT2-FSs is preferred. If a satisfactory closed loop control performance can be achieved via this simple structure, the use of alternative IT2-FSs; such as Gaussian, Sigmoidal, etc.; is not necessary and will obviously increase the computational cost

Simulation and reesults

The fuzzy controller designed for stabilized the inverted pendulum. From the nonlinear equation we design the Simulink model of inverted pendulum system. The designed fuzzy controller try to move cart in proper with appropriate speed. The controller is activated only when the pendulum reaches the upright position.

The experiment is carried on the system with simulation represent the figure. The equilibrium point θ=0 at unstable position and pendulum is holding this position due to minute friction, due to this control signal is non zero. The interval type 2 fuzzy PID controller stabilized the pendulum.

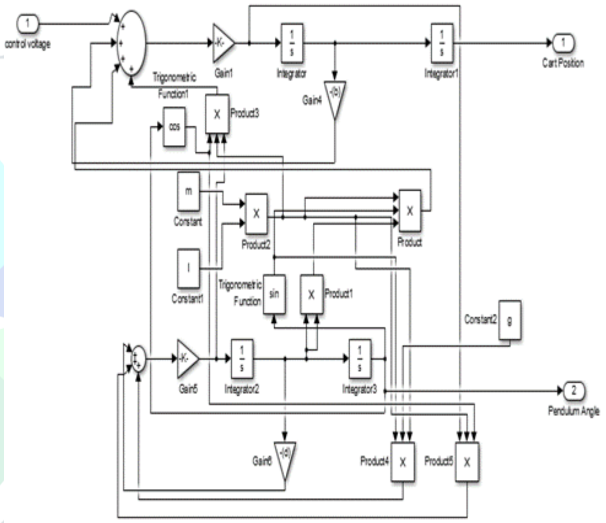

Fig.4.- Nonlinear model of CIPS

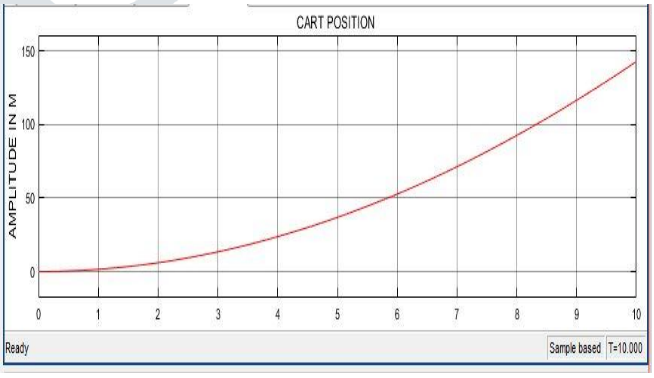

Fig.5.- performance of cart position

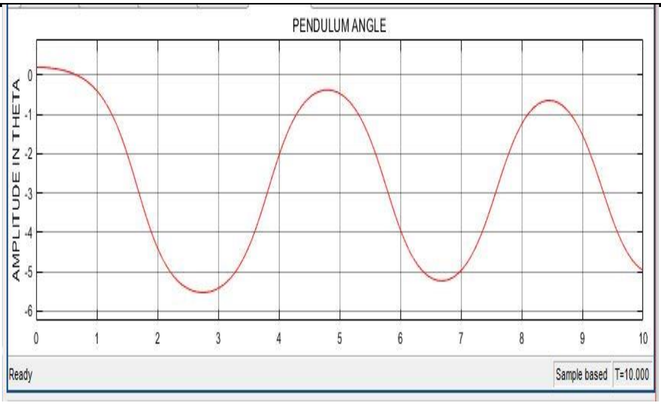

Fig.6. performance of pendulum angle

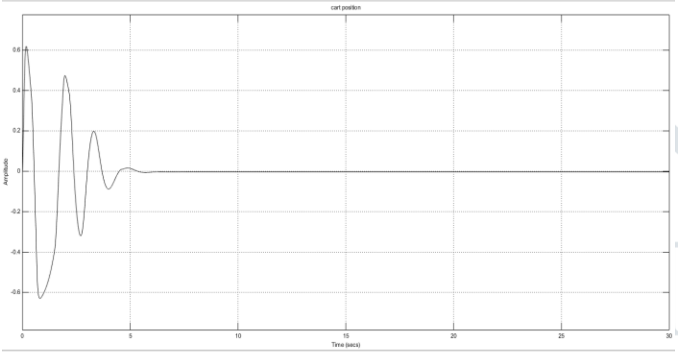

Fig.6. performance of cart position

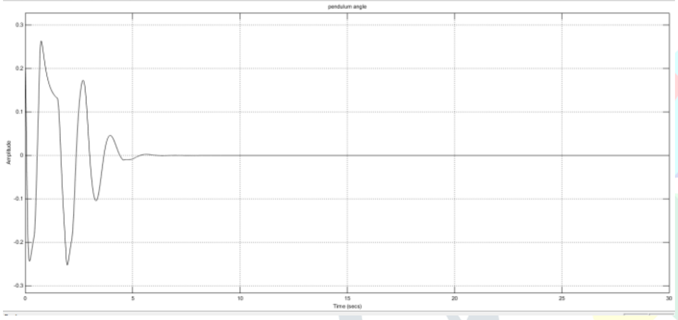

Fig.7. performance of pendulum angle

IV. CONCLIUSION

In this study, a simple design method for single input interval type-2 fuzzy PID controllers has been proposed. The proposed interval type-2 fuzzy controller structure still keeps the most preferred features of the PID controller such as simplicity and easy design.

The most important feature of the proposed IT2-FLC is its single input type-2 fuzzy internal structure which gives the opportunity to derive the output in a closed form in terms of the defined tuning parameters , i.e. the extra degrees of freedom provided by the antecedent interval type-2 fuzzy sets. Thus, the type-2 fuzzy controller output can be explicitly defined in the error domain based on the derived closed form

The closed loop control performances of the proposed type-2 fuzzy control structure have been compared with conventional controllers on benchmark linear and nonlinear processes. The transient state and disturbance rejection performances of the implemented control structures are compared.

Hence, the main contribution of the presented a type-2 fuzzy controller design method is to enhance the closed loop system performance via the extra degrees of freedom provided by the antecedent interval type-2 fuzzy sets and stabilized the pendulum at upright position.

Identify applicable sponsor/s here. If no sponsors, delete this text box (*sponsors).*

REFERENCES

[1] Kumbasar, T., 2014. A simple design method for interval type-2 fuzzy PID controllers. Soft Computing, 18(7), pp.1293-1304.rs

[2] El-Bardini, Mohammad, and Ahmad M. El-Nagar. "Interval type-2 fuzzy PID controller for uncertain nonlinear inverted pendulum system." ISA transactions 53, no. 3 (2014): 732-743.Deletion:.

[3] Khosla, Anita, G. Leena, and M. K. Soni. "Interval Type-2 Fuzzy Logic Controller to Control the Velocity and Angle of Inverted Pendulum." International Journal of Intelligent Systems and Applications 6, no. 7 (2014): 44.

[4] Yamakawa, Takeshi. "Stabilization of an inverted pendulum by a high-speed fuzzy logic controller hardware system." Fuzzy sets and Systems 32, no. 2 (1989): 161-180.

[5] Roose, Ahmad Ilyas, Samer Yahya, and Hussain Al-Rizzo. "Fuzzy-logic control of an inverted pendulum on a cart." Computers & Electrical Engineering 61 (2017): 31-47.

[6] Castillo, Oscar. "Design of Stable Type-2 Fuzzy Logic Controllers." Type-2 Fuzzy Logic in Intelligent Control Applications. Springer, Berlin, Heidelberg, 2012. 49-61.

[7] Radhamohan, Singh Vivekkumar☺, M. O. N. A. Subramaniam, and M. J. Nigam. "Fuzzy swing-up and stabilization of real inverted pendulum using single rulebase." Journal of Theoretical and Applied Information Technology 14.1/2 (2010): 43-49

[8] Ichtev, Alexandar K. "Fuzzy controller for swing-up and stabilization of inverted pendulum." Intelligent Systems, 2008. IS'08. 4th International IEEE Conference. Vol. 1. IEEE, 2008

[9]. Castillo, Oscar. "Design of stable Type-2 Fuzzy logic controllers." Type-2 Fuzzy Logic in Intelligent Control Applications. Springer, Berlin, Heidelberg, 2012. 49-61.

[10]. Tripathi, Sandeep Kr, Himanshu Panday, and Prerna Gaur. "Robust control of inverted pendulum using fuzzy logic controller." Engineering and Systems (SCES), 2013 Students Conference on. IEEE, 2013.

[11] Abuelenin, Sherif M. "Decomposed interval Type-2 fuzzy systems with application to inverted pendulum." Engineering and Technology (ICET), 2014International Conference on. IEEE, 2014.

[12]Yi, Jianqiang, Naoyoshi Yubazaki, and Kaoru Hirota. "Upswing and stabilization control of inverted pendulum system based on the SIRMs dynamically connected fuzzy inference model." Fuzzy Sets and Systems 122.1 (2001): 139-152.