Evaluation of Optimum Control Input and State trajectory in Hamiltonian form and System Matrix via Linear Differential Inclusions Optimization for non linear system

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***Abstract* --**This paper proposes the design of performance index with Hamiltonian form in discrete time system. The control system is explained by a non linear dynamical state space form. Euler Lagrange Multiplier is applied to this non linear system to get the value of optimum input and state. For implication purpose, the non linear system is expressed as time varying state space form via Linear Differential Inclusions process. The System Controller of state variable with Integral Square Error is expressed here by mean value theorem.

Keywords: Hamiltonian, Euler Lagrange Multiplier, Linear Differential Inclusions, Time Varying State Space Form

I. INTRODUCTION :

Optimum Control is acquainted as real life based application of Calculus of Variation. Research in the field of Optimization was processed towards the direction of Dynamic Programming Principle with the basis of minimum energy, time and regulator problem [1]. The sub optimal control law was applied to demonstrate good robustness properties in continuous differential equation by adjusting the gain controller parameter. The evolution of System is realized with the help of system controllers which are characterized by control plant input, u € $\mathbb{R}^n$ where, u = $u_1, u_2 \ldots, u_n$ and obviously n≥1. The continuous control input u(.) € U_t $\forall$ t $[t_0, t_f]$. Here U_t is Space of real-valued vector function. In the problem of Control System, the value function as being by performance index is to be optimized in control system. The provision of discrete time system with optimum control was adopted by R.E Kalman in the early 1960's when the dynamic system was not liable to uncertainties. In the last four decades, several techniques have been introduced to control the disturbances, uncertainties of the external and internal parameters of the system. In this section we first explain the Hamiltonian expression of Euler Lagrange equation in discrete time system [4].

In Section II, a numerical problem is taken to solve the optimum plant input and the value of state trajectory in non linear system.

In Section III, The Integral Square Error is evaluated in non linear time varying system.

II. HAMILTONIAN DISCRETE TIME OPTIMIZATION

Consider, the discrete time non linear system as,

$x_{k+1} = \Gamma(x_k, u_k)$ with initial condition x_0 ….. (1)

In the above Eq^n(1), x_k is a state vector of nth dimension matrix and u_k is a vector of control input of mth dimension matrix.

Let the performance index J for ($k \in N$) time duration be developed for this system as,

$$J(n) = V(N, x_N) + \sum_{k=i}^{N-1}[\Gamma_1^k(x_k,u_k) + \lambda_{k+1}^T(\Gamma_2^k(x_k,u_k) - x_{k+1}] \quad ..... (2)$$

Here in Eq^n(2) 'n' is the number of sequence of samples, V is the fixed final state variable. $\Gamma_i^k\big|_{i=1,2}$ are the multiplier controller variable and λ_{k+1} is the Lagrange Multiplier.

Defining Hamiltonian form for Eq^n(2),-

$$H^k(x_k,u_k) = \Gamma_1^k(x_k,u_k) + \lambda_{k+1}^T\Gamma_2^k(x_k,u_k) \quad ...... (3)$$

Comparing Eq^n(2) and Eq^n(3) we can write,-

$$J(n) = V(N,x_N) - \lambda_n^T x_n + H(x_i,u_i) + \sum_{k=i+1}^{N-1}[H^K(x_k,u_k) - \lambda_k^T x_k] \quad .... (4)$$

To Examine the increment in J due to increment in all the variables x_k, u_k, λ_k. We assume the final time 'N' to be fixed.

From Eq^n(4) we have,-

$$dJ(n) = (V_{x_n} - \lambda_N)^T dx_n + (H_{x_i}^i)^T dx_i + (H_{x_i}^i)^T du_i$$
$$+ \sum_{k=i+1}^{N}[(H_{x_k}^k - \lambda_k)^T dx_k + (H_{u_k}^k)^T du_k] + \sum_{k=i+1}^{N}(H_{\lambda_k}^{k-1} - x_k)^T d\lambda_k \quad ...... (5)$$

Where, $H_{x_k}^k = \dfrac{\partial H^k}{\partial x_k}$ and so on. ...... (6)

Necessary conditions for a constrained minimum are given by,-

$$x_{k+1} = \frac{\partial H^k}{\partial \lambda_{k+1}} \quad ...... (7)$$

$$\lambda_k = \frac{\partial H^k}{\partial x_k} \quad .... (8)$$

$$0 = \frac{\partial H^k}{\partial u_k} \quad .... (9)$$

k= 1, 2, …., i, ….., N-1

III. NUMERICAL PROBLEM FORMULATION

Consider, for (n, x) plane with plant input, 'u' is illustrated by the non linear equation as –

$$\frac{\partial x}{\partial n} = (-\sin x^n) + u \quad .... (10)$$

with the boundary condition x(0) = 1 and u(0) = 1, the performance measuring parameter,

$$J = \frac{1}{2}\sum_{n=0}^{N-1}(3x^n + u^2) \quad .... (11)$$

The (n, x) plant is expressed with its Hamiltonian form as,

$$H(x(n),u) = \frac{3}{2}x^n + \frac{1}{2}u^2 + \lambda((-\sin x^n) + u)$$

$$=> H(x(n),u) = \frac{3}{2}x^n + \frac{1}{2}u^2 + \lambda(\frac{\partial x}{\partial n})$$

(from Eq^n(10) we have, $(\frac{\partial x}{\partial n} = (-\sin x^n) + u)$) ……..(12)

In Eq^n(12), 'λ' is called Lagrange Multiplier

Applying Euler Lagrange equation on Eq^n(12) we have,

$$\left(\frac{\partial H(x(n),u)}{\partial x}\right)-\frac{d}{dn}\left(\frac{\partial H(x(n),u)}{\partial\left(\frac{\partial x}{\partial n}\right)}\right)=0$$

$$=> {}^{3}\!/_{2}\, nx^{n-1}-\frac{d}{dn}(\lambda)=0$$

$$=> \frac{d}{dn}(\lambda)={}^{3}\!/_{2}\, nx^{n-1}$$

$$=> \lambda={}^{3}\!/_{2}\, x^{n}$$

………. (13)

From Eq^n(12), the stationary condition is expressed by,-

$$\frac{\partial H(x(n),u)}{\partial u}=0$$

$$=> u+\lambda=0 \qquad …….. (14)$$

$$=> u=-\lambda=-\frac{3}{2}x^{n}$$

So, the plant Optimum input is $u^*=-\frac{3}{2}(x^{n})^*$ ……. (15)

Now, From Eq^n(10) the optimum state trajectory will be,-

$$x^*(n)=\int_{n_0}^{N}\left(\frac{\partial x}{\partial n}\right)^* dn \qquad ….. (16)$$

Using Table of Integral , we have solved the value of optimum state. Here it is considered that the initial time starts from zero and goes up to the infinity then the optimum state will be,-

$$x^*(n)=\sin\left(\frac{\pi}{2n}\right)\Gamma^*\left(\frac{1}{n}\right)+\frac{3}{2}\frac{(x^{n+1})^*}{n+1} \qquad ….. (17)$$

In Eq^n(17), the 'Γ*(.)' is called the Optimum gamma function [6].

IV. DESCRIBING NON LINEAR TIME VARYING SYSTEM

Now, the description of Eq^n(1) is represented by the state space form in discrete time varying system as [3],-

$$\frac{\partial x(n)}{\partial(n)}=(\alpha+\Gamma E(n)G(n))x(n))\Big|_{n}^{N} \qquad ……….. (18)$$

where, α € $^{n\times n}$, Γ € $^{n\times nm}$, G € $^{np\times n}$ are irreversible matrix and E(n, N) is an unknown time varying matrix depends on the value of Γ and G.

The Performance Index is developed for this system (Eq^n(18)) as,-

$$J\left(\frac{\partial x(n)}{\partial(n)},n\right)=\sum_{k=0}^{N-1}\Gamma[x(k,N),u(k,N),S(k,N)] \qquad ……… (19)$$

In the above Performance Index (Eq^n(19)), the Optimum value of the input be u*(x, n).

Now to apply quadratic regulator for the non linear time varying system via Linear Differential Inclusions, the Eq^n(18) is described by a non linear dynamic model in state space form:

$$\frac{\partial x(n)}{\partial(n)}=\sum_{k=0}^{N}\theta(k,N)M_k x(k,N) \qquad ………… (20)$$

where, $\theta(k,N)\geq 0$ and $\sum\theta(k,N)=1$ is needed for minimum value $\forall N>0$ and k>0. Here the regulator M_k € ^{nxn} being dimensional matrix elements.

Now, in practice the performance index is optimized as the objective value function expressed in the form of time square error. The deviation of the signal in between actual and feedback in closed loop is minimize at the input of control elements is minimized by optimization of the performance index.

Using mean value theorem, the running cost of state variable for this non linear system with integral square error (e(n)) is expressed as,-

$$\Gamma_{n\in n_1,n_2}(x(n,N)) = \begin{bmatrix} \alpha + \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} e(n_1,N)e(n_2,N) \\ \frac{\partial \Gamma_{n_1}}{\partial x_{n_2}}\left(\frac{\partial \Gamma_{n_1}}{\partial x_{n_2}}(x(n,N) - \frac{\partial \Gamma_{n_1}}{\partial x_{n_2}}(0)\right) \end{bmatrix} x(n,N) \qquad \ldots\ldots\ldots\ldots\ldots (21)$$

If it is defined, E_n is the vector transition of e(n, N), then $E_n \{e(n_1, N) \mid e(n_2, N)\} = (0, \ldots, 0,1,0, \ldots,0)^T$

V. CONCLUSION

In this paper, it was evaluated to get the value of an optimum plant input and optimum state trajectory for a non linear system using Euler Lagrange Multiplier. The Hamiltonian form of Nonlinear Discrete Time system was proposed for some necessary condition. Finally, using Linear Differential Inclusions we evaluate Integral Square Error. Study and analysis for multi varying parameter system may be considered as future scope of work.

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