Octagon-Cell Interconnection Networks: Performance Parameters and Its Analysis

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Abstract: This paper presents the performance parameters of Octagon-Cell Interconnection Networks through various Lemmas. These parameters have been analyzed in contrast to other networks such as Hypercube, 3D Hexagonal, 2D Torus, Ring, Linear Array, Cube-Connected Cycle and Binary Tree. The parameters of this network have been proved through Lemmas in terms of depth of the network and number of nodes.

***Index terms*: Interconnection Networks, Octagon-Cell Network, Scalable Network, Parallel System, Undirected Graph.**

I. INTERCONNECTION NETWORK AND PARALLEL SYSTEMS

Interconnection network can be defined as a "programmable system that enables fast data communication between components of a digital system". It plays a major role in the performance of parallel systems. Parallel machines enable us to break a single problem into parallel tasks and process them concurrently, thereby reducing the application processing time significantly. The overall performance of parallel systems depends very much upon the type of interconnection network being used in the system. In many parallel processing systems, an interconnection network is used for transportation of data between processors or between processor and memory modules.

In the field of supercomputing, parallel processing system plays an important role. The success of any multicomputer system depends on the type of interconnection network being used in the system that provides a communication medium among multiple processors [6]-[8]. There are many factors, which may affect the choice of an appropriate interconnection network for the underlying parallel computer. These factors include performance requirement, scalability, incremental expandability, partitionability, simplicity, distance span, physical constraints, reliability and repairability, expected workloads and cost constraints. Many researchers have proposed different interconnection networks for parallel processing architecture.

Hypercube is among the wide variety of interconnection network structures proposed for parallel computing systems. It has received much attention due to its beneficial properties incorporated in the topology [1]. The embedability, symmetry, stronger resilience and simple routing have made hypercube superior to many other multicomputer networks. However, in a hypercube structure, the number of communication ports and channels per processor increases with the increase of the network size, which becomes the disadvantage of the structure [1]. Binary tree has extremely low bisection width and other interconnection networks have also some drawbacks.

Therefore, we had carried out the research study [5] to develop a new interconnection network to meet the demands of massively parallel systems. We had studied the pros and cons of these existing networks and developed a new interconnection network called Octagon-Cell Network (OCN), suitable for massively parallel systems [5].

In this paper, we have developed and proved Lemmas for performance parameters of OCN. These parameters have also been analyzed in contrast to that of other networks.

II. REPRESENTATION OF OCTAGON-CELL NETWORK

We have proposed Octagon-Cell Network in [5]. This network is represented as an undirected graph, where the nodes represent processors or memories in the system and edges represent communication links between nodes. The structure of Octagon-Cell Network is recursive in nature. It is scalable and can be expanded to any depth. An Octagon-Cell has eight nodes with eight edges. It has d levels numbered from 1 to d with depth d. Level 1 represents one Octagon-Cell. Level 2 represents eight Octagon-Cells surrounding the Octagon-Cell at level 1. Level 3 represents 16 Octagon-Cells surrounding the 8 Octagon-Cells at level 2 and so on.

Each node in Octagon-Cell Network is represented by a pair (X,Y), where X denotes the line number in which the node exists and Y denotes serial number of the node in that line. Number inside the Octagon-Cell represents the level number of octagon-Cell Network [5].

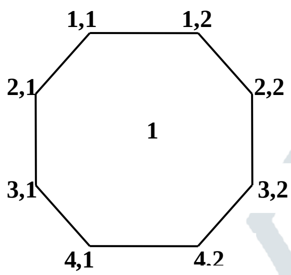

Figure 1: Octagon-Cell Network of Depth 1

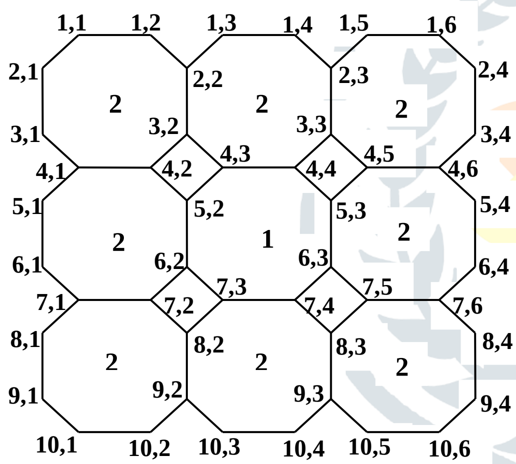

Figure 2: Octagon-Cell Network of Depth 2

- Number of nodes in the network is described through the following Lemmas:

Lemma 1

In an Octagon-Cell Network, the number of nodes at level i is: $N_i = 8(4i - 3)$.

Lemma 2

The total number of nodes in Octagon-Cell Network of depth d is: $N = 8d(2d - 1)$

We have proved the above Lemmas in [5]. The formula to find the depth of the Octagon-Cell Network is described through the following Lemma.

Lemma 3

The number of levels, which is the depth of Octagon-Cell Network needed to accommodate 'N' number of nodes, is:
$d = [1 + \sqrt{(1 + N)}]/4$

Proof

Lemma 2 provides the formula to find the total number of nodes in Octagon-Cell Network of depth d.
It is given by: $N = 8d(2d - 1) = 16d^2 - 8d$.
This implies $16d^2 - 8d - N = 0$. Solving this quadratic equation, we have two solutions. Those are:
$d = [1 + \sqrt{(1 + N)}]/4$ or $[1 - \sqrt{(1 + N)}]/4$.
Considering the positive one,
$d = [1 + \sqrt{(1 + N)}]/4$.
Putting $N = 8$, we get $d = 1$.
Putting again $N = 48$, we get $d = 2$ and so on.

- The number of Octagon-Cells in the Octagon-Cell Network is described through the following Lemmas.

Lemma 4

In an Octagon-Cell Network, the number of Octagon-Cells at level i is:
$C_i = 8(i - 1)$, for $i > 1$.
For $i = 1$, $C_i = 8(i - 1) + 1$

Proof

Octagon-Cell Network with level 1 represents one Octagon-Cell. Therefore $C_1 = 1$. This can be represented as $C_i = 8(i - 1) + 1$, for $i = 1$. This is clear from Fig 1. Level 2 represents eight Octagon-Cells surrounding one Octagon-Cell at level 1 (Fig 2). Similarly, Level 3 represents sixteen Octagon-Cells surrounding eight Octagon-Cells at level 2 and so on. It can be seen that level (i + 1) has eight Octagon-Cells in addition to those at level i (for $i > 1$).

Therefore $C_2 = 8$, for $i = 2$. C_2 can be represented as: $8(2 - 1) = 8$, at level 2.
$C_3 = 8 + 8 = 16$, for $i = 3$. C_3 can be represented as: $8(3 - 1) = 16$, at level 3.
From the above, the number of Octagon-Cells at level i is given by $C_i = 8(i - 1)$, for $i > 1$.

Lemma 5

The number of Octagon-Cells in Octagon-Cell Network of depth d is:
$C = 4d^2 - 4d + 1$.

Proof

Lemma 4 proves the formula to find the number of Octagon-Cells in the Octagon-Cell Network at each level i. That is: $C_i = 8(i - 1)$, for $i > 1$. For $i = 1$, $C_i = 8(i - 1) + 1$. It can be seen that level i + 1 has eight more Octagon-Cells in addition to those at level i, for $i > 1$.

Therefore, the number of Octagon-Cells in this network of depth d is equal to:

$C = [\sum_{i=1}^{d} 8(i-1)] + 1$

$= [\sum_{i=1}^{d} (8i - 8)] + 1$

$= \sum_{i=1}^{d} 8i - \sum_{i=1}^{d} 8 + 1$

$= [8\{d(d + 1)\}/2] - 8d + 1 = 4d^2 - 4d + 1$

Thus, $C = 4d^2 - 4d + 1$, the number of Octagon-Cells in Octagon-Cell Network of depth d.

Lemma 6

The number of Octagon-Cells in Octagon-Cell Network is:

$C = [1 - \sqrt{(1 + N)}]/2 + N/4$, where N is the number of nodes in Octagon-Cell Network of depth d.

Proof

Putting $d = [1 + \sqrt{(1 + N)}]/4$ (Lemma 3), in the formula for $C = 4d^2 - 4d + 1$, it is obtained: $C = [1 - \sqrt{(1 + N)}]/2 + N/4$

III. PERFORMANCE PARAMETERS

Following performance parameters are being used for evaluating Octagon-Cell Network.

- Number of Nodes
- Number of Links
- Node Degree
- Diameter
- Bisection Width
- Network Cost
- Average Distance
- Message Traffic Density

Number of Links

The number of links in OCN is formulated in the following Lemmas.

Lemma 7

The number of links in OCN at level i is:

$L_i = 4(12i - 11)$, for $i > 1$. For $i = 1$,

$L_i = 4(12i - 11) + 4$ gives the number of links at level 1.

Proof

For i = 1, $L_1 = 4(12.1 - 11) + 4 = 8$. It is well known (Fig 1) that the number of links in OCN at level 1 is 8. Similarly, the number of links at level 2 and 3 are 52 and 100 respectively. It can be seen that level (i + 1) has 48 more links in addition to those at level i, for $i > 1$. Therefore the number of links at level i is: $L_i = 52 + 48(i - 2)$

$= 52 + 48i - 96$

$= 4(12i - 11)$, for $i > 1$.

Lemma 8

The number of links in OCN at each level is: $L_i = 4[3\sqrt{(1 + N)} - 8]$, where $i > 1$ and N is the number of nodes in OCN of depth $d > 1 (d = i)$.

For i = 1, $L_i = 4[3\sqrt{(1 + N)} - 8] + 4$, where N is the number of nodes in OCN of depth $d = 1$.

Proof

Lemma 7 proves the formulae to find number of links in OCN at each level that are:

$L_i = 4(12i - 11)$, for $i > 1$ and

$L_i = 4(12i - 11) + 4$, for $i = 1$.

Considering i = d and putting the formula:

$d = [1 + \sqrt{(1 + N)}]/4$ (Lemma 3) in the right side of the equations for L_i, we get:

$L_i = 4[12\{[1 + \sqrt{(1 + N)}]/4\} - 11]$ for $i > 1$ and

$L_i = 4[12\{[1 + \sqrt{(1 + N)}]/4\} - 11] + 4$,

for i = 1.

Evaluating these equations, we get:

$L_i = 4[3\sqrt{(1 + N)} - 8]$ for $i > 1$, where N is the number of nodes in OCN of depth $d > 1$ $(d = i)$ and $L_i = 4[3\sqrt{(1 + N)} - 8] + 4$ for i = 1, where N is the number of nodes in OCN of depth $d = 1$.

Lemma 9

The number of links in OCN of depth d is:

$L = 4(6d^2 - 5d + 1)$.

Proof

Lemma 7 proves the formula to find the number of links in OCN at each level i. That is: $L_i = 4(12i - 11)$, for $i > 1$.

For i = 1, $L_i = 4(12i - 11) + 4$. It can be seen that level i + 1 has 48 more links in addition to those at level i, for $i > 1$. Therefore, the number of links in OCN of depth d is equal to:

$L = \left[\sum_{i=1}^{d} 4(12i - 11)\right] + 4$

$= \left[\sum_{i=1}^{d} (48i - 44)\right] + 4$

$= \left[\sum_{i=1}^{d} 48i - \sum_{i=1}^{d} 44\right] + 4$

$= [48\{d(d + 1)\}/2] - 44d + 4$

$= 24d^2 - 20d + 4$

$= 4(6d^2 - 5d + 1)$

Therefore $L = 4(6d^2 - 5d + 1)$, the total number of links in OCN of depth d.

Lemma 10

The number of links in OCN is:
$L = 2[(3/4)N - \sqrt{(1 + N)} + 1]$, where N is the number of nodes in OCN of depth d.

Proof

Putting the depth of the network: $d = [1 + \sqrt{(1 + N)}]/4$ (Lemma 3), in the formula for $L = 4(6d^2 - 5d + 1)$, it is obtained: $L = 2[(3/4)N - \sqrt{(1 + N)} + 1]$

Node Degree

The node degree of an interconnection network is defined as the maximum number of edges that a node can have in the network [1]. The network topology that secures constant node degree is highly desirable, because it facilitates modularity in building blocks for scalable systems [1]-[4].

Lemma 11

The node degree of OCN remains constant for ($d > 1$), that is 3.

Proof

From Fig 1, it is very clear that each node of OCN is connected to two links, so the node degree of OCN of depth 1 is two. For $d > 1$, each node is connected to three links except the nodes at the border of OCN. Therefore, the node degree of OCN of depth 2 is three. If we expand this network, it is clear that the node degree remains constant at three. Therefore, the node degree of OCN is constant for $d > 1$.

Diameter

The diameter 'D' of a network is the maximum shortest path between any two nodes [1]-[4]. The number of links traversed measures the path length. The network diameter indicates the maximum number of distinct hops between any two nodes and it should be as small as possible. This will not only reduce the traversing time for messages, but also minimize message density in the links of the network [1].

Lemma 12

The diameter of OCN is: $4(2d - 1)$, where d is the depth of OCN.

Proof

For depth 1(Fig 1), considering two farthest nodes, it is clear that the maximum shortest path from a node 'u' to a node 'v' is 4. Similarly for depth 2 (Fig 2), the maximum shortest path is 12. It can be seen that level (i + 1) has 8 more diameter than that at level i.

So, for depth 1, the diameter D = 4.
For depth 2, D = 4 + 8.
We can write $D = 4 + 8(2 - 1)$ for depth 2.
Similarly $D = 4 + 8 + 8$, which can be written as: $D = 4 + 8(3 - 1)$ for depth 3.
Therefore $D = 4 + 8(d - 1)$ for depth d. This implies $D = 4(2d - 1)$, which is the diameter of Octagon-Cell Network of depth d.

Lemma 13

The diameter of OCN is: $2[\sqrt{(1 + N)} - 1]$, where N is the number of nodes in OCN of depth d.

Proof

Putting the depth of the network,
$d = [1 + \sqrt{(1 + N)}]/4$ (Lemma 3), in the formula for diameter $D = 4(2d - 1)$, it is obtained: $D = 4 [2\{[1 + \sqrt{(1 + N)}]/4\} - 1]$. This implies $D = 2[\sqrt{(1 + N)} - 1]$
For N = 8, we get D = 4. Similarly for N = 48, we get D = 12 and so on.

Bisection Width

When a given network is cut into two equal halves the minimum number of edges along the cut is called the channel bisection width b [1]-[4].

Lemma 14

The bisection width of OCN is: b = 2d, where d is the depth.

Proof

From Fig 1, it is clear that there are at least two links along the cut when the network is cut into two equal halves. Therefore, the bisection width of OCN with depth 1 is two. The bisection width of the network of depth 2 is four (Fig 2). Similarly, the network of depth 3 has the bisection width six. This sequence of numbers formulates the formula for bisection width of OCN of depth d, which is 2d.

Lemma 15

The bisection width of OCN is:
$[1 + \sqrt{(1 + N)}]/2$, where N is the number of nodes in OCN of depth d.

Proof

Putting depth of the network,

d = [1 + √(1 + N)]/4 (Lemma 3) in the formula for b = 2d, it is obtained:
b = 2[1 + √(1 + N)]/4. Evaluating this equation, we get b = [1 + √(1 + N)]/2.

Network Cost

Cost is an important performance parameter to evaluate a network topology. It is defined as the product of node degree and diameter D.

Lemma 16

The network cost of OCN is: 12(2d − 1), for d > 1 (d is the depth of the network)

Proof

OCN has constant node degree 3 for d > 1. It has the diameter D = 4(2d − 1). So network cost = 3[4(2d − 1)] = 12(2d − 1), for d > 1.

Lemma 17

The network cost of OCN is:
6[√(1 + N) – 1], where N is the number of nodes in OCN of depth d > 1.

Proof

Putting depth of the network,
d = [1 + √(1 + N)]/4 (Lemma 3) in the formula for network cost = 12(2d − 1), for d > 1, we get: 12[2{[1 + √(1 + N)]/4} – 1].
Evaluating this equation, we obtain network cost = 6[√(1 + N) – 1], where N is the number of nodes in OCN of depth d > 1.

Average Distance ($\bar{d}$)

Average Distance is the summation of distance of all nodes from a given source node over the total number of nodes.

$$\bar{d} = \frac{\sum_{i=1}^{N-1} d(S,K_i)}{N}, \quad (K_i \neq S)$$

N is the total number of nodes in OCN of depth d. S is any given source node. K_i are the nodes other than the source node.

Message Traffic Density (ρ)

Message Traffic Density is defined as: $\rho \equiv \bar{d}N/L$, where L is the total number of links, N is the total number of nodes in OCN of depth d and $\bar{d}$ is the average distance.

$$\rho = \frac{\sum_{i=1}^{N-1} d(S,K_i)}{4(6d^2 - 5d + 1)}.$$ Here d is the depth of OCN.

IV. RESULT ANALYSIS FOR OCTAGON-CELL NETWORK

Octagon-Cell Network is scalable. This network can be applicable for massively parallel systems, as hundreds of thousands of nodes can be connected with just three links per node and for other superiority of its performance parameters. Octagon-Cell Network provides multiple advantages such as constant node degree, small diameter, and high bisection width, large number of nodes with fewer connections, low network cost in comparison to that of some other networks. The results of performance parameters of Octagon-Cell Network are presented in the form of graphs.

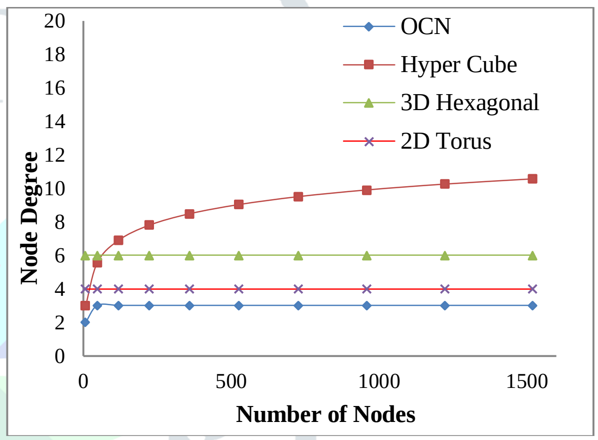

Figure 3: Comparison Graph for Node Degree versus Number of Nodes

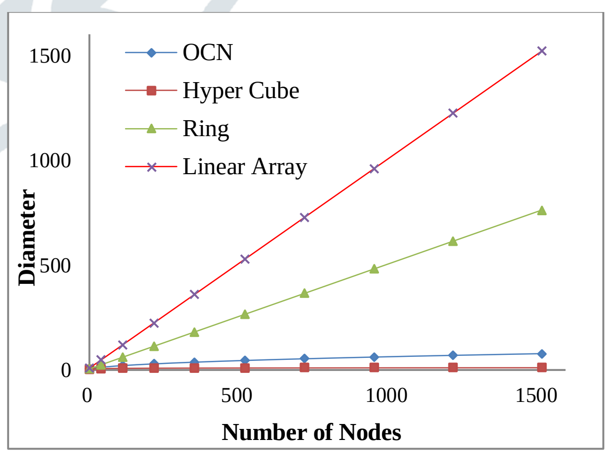

Figure 4: Comparison Graph for Diameter versus Number of Nodes

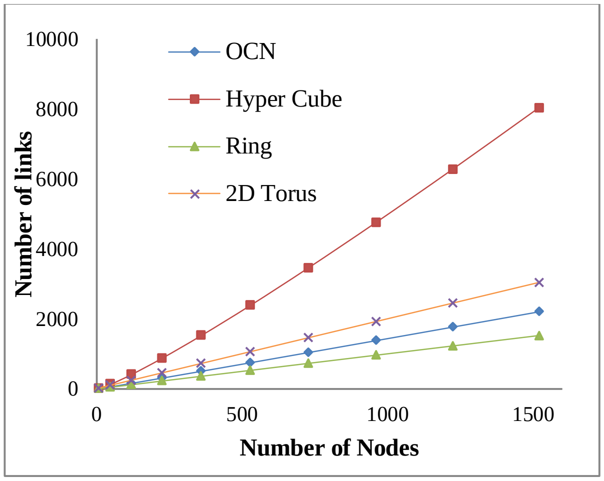

Figure 5: Comparison Graph for Number of Links versus Number of Nodes

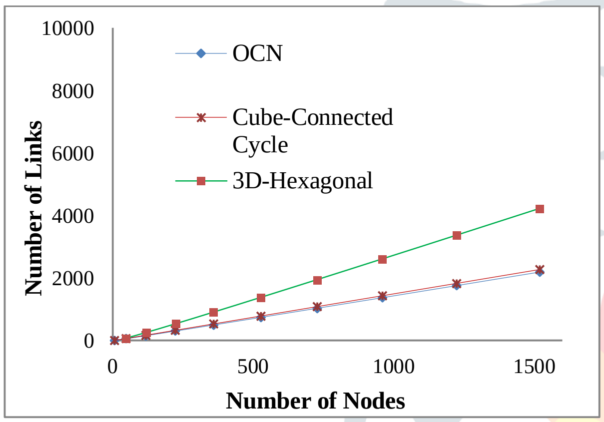

Figure 6: Comparison Graph for Number of Links versus Number of Nodes

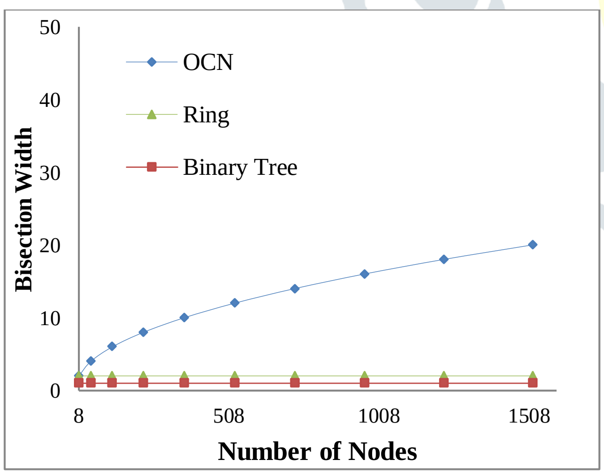

Figure 7: Comparison Graph for Bisection Width versus Number of Nodes

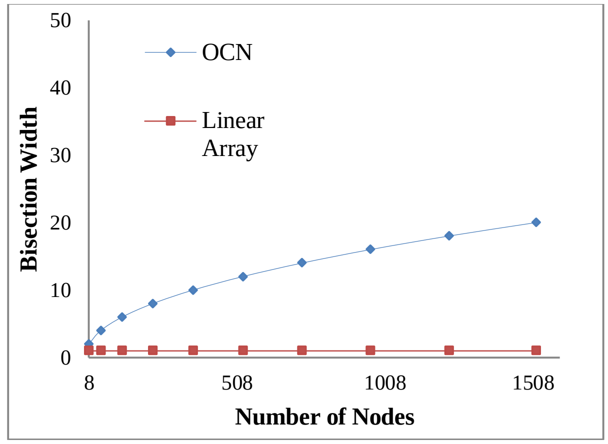

Figure 8: Comparison Graph for Bisection Width versus Number of Nodes

V. CONCLUSION

Octagon-Cell Network carries better features in comparison to some other interconnection networks. We developed and proved various formulae for different performance parameters of this network of any depth. The comparison graphs of different performance parameters of Octagon-Cell Network against that of other interconnection networks show the goodness of this network for being applicable to massively parallel systems.

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