M L ESTIMATION OF THE AVAILABILITY MEASURE OF IDENTICALE TWO-OMPONENT SYSTEM IN PRESENCE OF CCFS FOLLOWS WEIBULL LAW

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Abstract :

This study aimed to assess the maximum likelihood estimation method for the Availability transient and steady state Failure function of Identical two component repairable system . It is assumed that the system under the influence of common-cause failures (CCFs). The CCFs and individual failures assumed to follows weibull law and Occurs with some chance of failures .Numerical evidences are provided to justify the use of M L estimation procedure in the cause of system Availability transient and study state Failure functions .

Keyword: Availability, series, parallel system, CCFS failure, MLE.

1. INTRODUCTION

Reliability analysis vary powerful tool in industrial, electrical, electronics and nuclear power plants. In 1971 have identified common cause failures (CCFs) was identified which is the event of harmonized failure of components of a system due to external causes instead of outage of components themselves . CCFs greatly reduce the reliability indices under its influence. Billiton &Allan[1983] discussed the role of CCFS. Atwood[1986] and Atwood & Stevenson [1982] and Meachum and Atwood [1983] used BFR model for CCFS in the area of nuclear power plants. The Quantification and estimation of CCFs rates were discussed by them. Chari [1988] and Chari and U S Manyam [2003] have studied the concept of CCFs to arrive at the expression of Reliability indices like Reliability R(t), meantime between failure E(t) and Frequency Failure F(t) functions using Monrovian approach. This paper attempts the estimation of Reliability R(t), meantime between failure E(t) and Frequency Failure F(t) functions for parallel and series system in the context of Common Cause Failures.

2. ASSUMPTIONS

1. The system has two components, which are stochastically independent.
2. The system is affected by individual as well as common cause failures.
3. The components in the system will fail singly at the constant rate β_a and failure probability is P_1
4. The components may fail due to common causes at the constant rate β_c and with failure probability is P_2 such that $P_1 + P_2 = 1$.
5. Time occurrences of CCS failures and individual failures follow Wei-bull law.
6. The individual failures and CCS failures occurring independent of each other.
7. The failed components are serviced singly and service time follows exponential distribution with rate of service .

3. NOTATIONS

β_I : Individual failure rate.

β_c : Common cause failure rate.

μ : Service rate of individual components

A(t) : System Availability function.

$A_s(t)$: Availability function of series system.

$\hat{A}_s(t)$: ML estimate of Reliability function of series system.

$A_p(t)$: Availability function of two component parallel system.

$\hat{A}_p(t)$: Maximum Likelihood Estimate of time dependent Reliability function for parallel system.

$A_s(\infty)$: Availability function of series system.

$\hat{A}_s(\infty)$: ML estimate of Reliability function of series system.

$A_p(\infty)$: Availability function of two component parallel system.

$\hat{A}_p(\infty)$: Maximum Likelihood Estimate of time dependent Reliability function for parallel system.

$\hat{\beta}_I$: sample estimation of individual failure rate.

$\hat{\beta}_c$: sample estimation of common cause failure rate.

$\hat{\mu}$: Sample estimate of service time of the components.

n= Sample size.

N = Number of simulated samples.

4. MODEL

Under the stated assumptions Markovian model can be formulated to drive the

Reliability function R(t) under the influence of individual as well as CCS and the Markovian graph is given in fig.3.1. The quantities are $\beta_0, \beta_{1,}, \beta_2$

$\beta_0 = \beta_I P_1$, β_1=2$\beta_I P_1$, β_2= $\beta_c P_2$, μ_1= μ, μ_2= 2 μ.

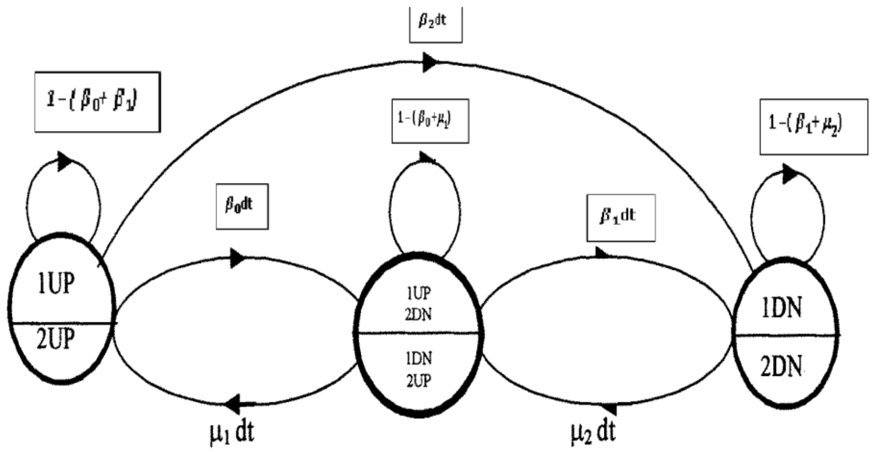

Fig. 3.1: Markov Graph For Two Component System With Individual And Common Cause Failures.

From the Markov graph the equations were formed and the probabilities of the various state of the systems i.e. P_o(t), P_1 (t), P_2(t) are derived (see Chari [1991 a]).

5. AVAILABILITY FUNCTION OF TWO-COMPONENT IDENTICAL SYSTEM

The time dependent function of availability for series and parallel systems are as follows.

(a) Series system

The transient state expression of Availability function for series system is given by

$$A_s(t) = \frac{(2\mu^2)}{2\mu^2 + 4\beta_I P_1 \mu + 3\beta_c P_2 \mu + (H_1 e^{(\gamma_2 t)} - H_2 e^{(\gamma_3 t)})/(\gamma_2 - \gamma_3)} \quad \text{------- (1)}$$

Where

$$H_1 = \frac{(\gamma_3^2 + 3\gamma_3\mu + 2\mu^2)}{\gamma_3} \quad \text{------- (2)}$$

$$H_2 = \frac{(\gamma_4^2 + 3\gamma_4\mu + 2\mu^2)}{\gamma_4} \quad \text{------- (3)}$$

$$\gamma_3 = \left(\frac{-(\beta_I P_1 + \beta_c P_2 + 3\mu) + \sqrt{(\mu - \beta_I P_1 - \beta_c P_2)^2 - 4\beta_c P_2 \mu}}{2} \right) \quad \text{------- (4)}$$

$$\gamma_4 = \left(\frac{-(\beta_I P_1 + \beta_c P_2 + 3\mu) + \sqrt{(\mu - \beta_I P_1 - \beta_c P_2)^2 - 4\beta_c P_2 \mu}}{2} \right) \quad \text{------- (5)}$$

where

β_I, β_c& μ are the individual, common cause failure and service rates respectively.

(b) Parallel system

Time dependent expression of Availability function of parallel system is given by

$$A_p(t) = \frac{(2\mu(\mu+2\beta_I P_1+\beta_c P_2))}{2\mu^2+4\beta_I P_1 \mu+3\beta_c P_2 \mu+2\beta_I^2 P_1^2+\beta_I \beta_c P_1 P_2+[\frac{H_3 e^{(\gamma_5 t)}-H_4 e^{(\gamma_6 t)}}{\gamma_6-\gamma_5}]} \quad \text{------(6)}$$

Where

$$H_3 = \frac{(\gamma_3^2+3\gamma_5(\mu+\beta_I P_1)+2\mu(\mu+2\beta_I P_1+\beta_c P_2))}{\gamma_5} \quad \text{------ (7)}$$

$$H4 = \frac{(\gamma_6^2+3\gamma_6(\mu+\beta_I P_1)+2\mu(\mu+2\beta_I P_1+\beta_c P_2))}{\gamma_6} \quad \text{------ (8)}$$

$$\gamma_5 = \left(\frac{-(3\beta_I P_1+\beta_c P_2+3\mu)+\sqrt{(\mu+\beta_I P_1+\beta_c P_2)^2-8\beta_c P_2\mu}}{2}\right) \quad \text{------- (9)}$$

$$\gamma_6 = \left(\frac{-(3\beta_I P_1+\beta_c P_2+3\mu)-\sqrt{(\mu+\beta_I P_1+\beta_c P_2)^2-8\beta_c P_2\mu}}{2}\right) \quad \text{----- (10)}$$

where

β_I, β_c& μ are the individual, common cause failure and service rates respectively.

6. STEADY STATE ABILITY FUNCTION

The steady state Availability functions for series and parallel system as follows.

(a) Series system

The expression of steady state Availability function for series system is given by

$$A_s(\infty) = \frac{(\mu_1 \mu_2)}{(\mu_1\mu_2+\mu_2\beta_1+\mu_2\beta_2+\mu_1\beta_2+\beta_0\beta_1+\beta_1\beta_2)} \quad \text{-------(11)}$$

where

$$\beta_0 = \beta_I P_1,\ \beta_1 = 2\beta_I P_1,\ \beta_2 = \beta_c P_2,\ \mu_1 = \mu,\ \mu_2 = 2\mu \quad \text{--------(12)}$$

(b) Parallel system

The expression of steady state Availability function of parallel system is given by

$$A_p(\infty) = \frac{(2\beta_I(2\beta_c P_1\mu + P_2\beta_I\beta_c+\beta_I\beta_c))}{(2\beta_I(2\beta_c P_1\mu + P_2\beta_I\beta_c+\beta_I\beta_c)+P_2\beta_I^2\mu+P_2\mu^2(2P_1\beta_c+P_2\beta_I))} \quad \text{------- (13)}$$

where

β_I, β_c& μ are the individual ,Common cause failure and service rates respectively

7.ESTIMATION OF STEADY STATE AVAILABILITY FUNCTION- ML ESTIMATION APPROCH

This section discusses the Maximum likelihood estimation approach for estimating Reliability function of two component parallel and series systems, which is under the influence of Individual as well as common cause failures.

Let X_1, X_2, $X_3 ... X_n$,, be a sample of 'n' number of times between individual failures which will obey weibul law.

Let Y_1 Y_2, Y_3 ...Y_n,, be a sample of 'n number of times between common cause system failures assume to follow weibul law.

Let Z_1 Z_2, Z_3 ...Z_n,, be a sample of 'n` number of times between service of the component assume to follows exponential law.

(a) Series system

The time dependent expression of Availability function for series system is given by

$$\hat{A}_s(t) = \frac{(2\hat{\mu}^2)}{2\hat{\mu}^2 + 4\hat{\beta}_I P_1 \hat{\mu} + 3\hat{\beta}_c P_2 \hat{\mu} + \left[\left(H_1 e^{(\gamma_2 t)} - H_2 e^{(\gamma_3 t)}\right)\right]/(\gamma_2 - \gamma_3)} \quad \text{------- (14)}$$

Where

$$H_1 = \frac{(\gamma_3^2 + 3\gamma_3\hat{\mu} + 2\hat{\mu}^2)}{\gamma_3} \quad \text{------- (15)}$$

$$H_2 = \frac{(\gamma_4^2 + 3\gamma_4\hat{\mu} + 2\hat{\mu}^2)}{\gamma_4} \quad \text{--------(16)}$$

$$\gamma_3 = \left(\frac{-(\hat{\beta}_I P_1 + \hat{\beta}_c P_2 + 3\hat{\mu}) + \sqrt{(\hat{\mu} - \hat{\beta}_I P_1 - \hat{\beta}_c P_2)^2 - 4\hat{\beta}_c P_2 \hat{\mu}}}{2}\right) \quad \text{------- (17)}$$

$$\gamma_3 = \left(\frac{-(\hat{\beta}_I P_1 + \hat{\beta}_c P_2 + 3\hat{\mu}) - \sqrt{(\hat{\mu} - \hat{\beta}_I P_1 - \hat{\beta}_c P_2)^2 - 4\hat{\beta}_c P_2 \hat{\mu}}}{2}\right) \quad \text{------- (18)}$$

where

β_I, β_c& μ are the individual ,Common cause failure and service rates respectively.

(b) Parallel system

Time dependent expression of Availability function of parallel system is given by

$$\hat{A}_p(t) = \frac{\left(2\hat{\mu}(\hat{\mu} + 2\hat{\beta}_I P_1 + \hat{\beta}_c P_2\ )\right)}{2\hat{\mu}^2 + 4\hat{\beta}_I P_1 \mu + 3\hat{\beta}_c P_2 \hat{\mu} + 2\hat{\beta}_I^2 P_1^2 + \hat{\beta}_I \hat{\beta}_c P_1 P_2 + \left[\frac{H_3 e^{(\gamma_5 t)} - H_4 e^{(\gamma_6 t)}}{\gamma_6 - \gamma_5}\right]} \quad \text{------- (19)}$$

Where

$$H_3 = \frac{(\gamma_3^2+3\gamma_5(\hat{\mu}+\hat{\beta}_I P_1)+2\hat{\mu}(\hat{\mu}+2\hat{\beta}_I P_1+\hat{\beta}_c P_2))}{\gamma_5} \quad \text{------- (20)}$$

$$H_4 = \frac{(\gamma_6^2+3\gamma_6(\hat{\mu}+\hat{\beta}_I P_1)+2\hat{\mu}(\hat{\mu}+2\hat{\beta}_I P_1+\hat{\beta}_c P_2))}{\gamma_6} \quad \text{------- (21)}$$

$$\gamma_5 = \left(\frac{-(3\hat{\beta}_I P_1+\hat{\beta}_c P_2+3\hat{\mu})+\sqrt{(\hat{\mu}+\hat{\beta}_I P_1+\hat{\beta}_c P_2)^2-8\hat{\beta}_c P_2\hat{\mu}}}{2}\right) \quad \text{------- (22)}$$

$$\gamma_6 = \left(\frac{-(3\hat{\beta}_I P_1+\hat{\beta}_c P_2+3\hat{\mu})-\sqrt{(\hat{\mu}+\hat{\beta}_I P_1+\hat{\beta}_c P_2)^2-8\hat{\beta}_c P_2\hat{\mu}}}{2}\right) \quad \text{-------(23)}$$

Where

$\hat{\beta}_I$, $\hat{\beta}_c$& $\hat{\mu}$ are the individual ,Common cause failure and service rates respectively.

8. ESTIMATION OF STEADY STATE AVAILABILITY FUNCTION ML ESTIMATION APPROCH

The estimation of study state Availability function for Series and parallel system as follows.

(a) Series system

The expression of steady state Availability function for series system is given by

$$\hat{A}_s(\infty) = \frac{(\hat{\mu}_1\hat{\mu}_2)}{(\hat{\mu}_1\hat{\mu}_2+\hat{\mu}_2\hat{\beta}_1+\hat{\mu}_2\beta_2+\hat{\mu}_1\hat{\beta}_2+\hat{\beta}_0\hat{\beta}_1+\hat{\beta}_1\hat{\beta}_2)} \quad \text{--------(24)}$$

where

$$\hat{\beta}_0 = \hat{\beta}_I P_1, \quad \hat{\beta}_1 = 2\hat{\beta}_I P_1, \quad \hat{\beta}_2 = \hat{\beta}_c P_2, \; \hat{\mu}_1 = \hat{\mu}, \; \hat{\mu}_2 = 2\hat{\mu} \quad \text{--------(25)}$$

(b) Parallel system

The expression of steady state Availability function of parallel system is given by

$$A_p(\infty) = \frac{(2\hat{\beta}_I(2\hat{\beta}_c P_1\hat{\mu} + P_2\hat{\beta}_I\hat{\beta}_c+\hat{\beta}_I\hat{\beta}_c))}{(2\hat{\beta}_I(2\hat{\beta}_c P_1\hat{\mu} + P_2\hat{\beta}_I\hat{\beta}_c+\hat{\beta}_I\hat{\beta}_c)+P_2\hat{\beta}_I^2\hat{\mu}+P_2\hat{\mu}^2(2P_1\hat{\beta}_c+P_2\hat{\beta}_I))} \quad \text{--------(26)}$$

Where

$\hat{\beta}_I$, $\hat{\beta}_c$& $\hat{\mu}$ are the individual ,Common cause failure and service rates respectively.

Now Maximum likelihood estimates Availability functions $A_s(t)$, $A_p(t)$, $A_s(\infty)$,$A_p(\infty)$, which shown in equations (1),(6), (11) and (13) are given by the functions $\hat{A}_s(t)$ $\hat{A}_p(t)$, $\hat{A}_s(\infty)$ and $\hat{A}_p(\infty)$which are shown in equations (14),(19),(24)and (26) are respectively

9. INTERVAL ESTIMATION OF AVAILABILITY OF FAILURES

let us considered

$\sqrt{n}\,[(\hat{\beta}_I, \hat{\beta}_c, \hat{\mu})-(\beta_I, \beta_c, \mu)] \sim N_3(0, \Sigma)$ for $n \to \infty$

Where $\Sigma = (\sigma_{ij})_{3\times3}$ co-variance matrix

$\Sigma = \text{dig}\,(\beta_I^2, \beta_c^2, \mu^2)$

Also from Rao (1974) we have

$\sqrt{n}\,[A(t) - \hat{A}(t)] \sim N(0,\sigma_\theta^2)$ as $n\to \infty$ and θ is the vector

By the properties of M L method of estimation $\hat{A}$ (t), $\hat{A}$ (∞) is CAN estimate of A(t) and A(∞)respectively also $\sigma^2_{(\theta)}$ be
The estimator of $\sigma^2_{(\theta)}$
Where

$(\hat{\theta}) = (\hat{\beta}_I, \hat{\beta}_c, \hat{\mu})$

nature of estimates are not established so far.

8. SIMULATION AND VALIDITY :

For a range of specified values of the rates of individual (β_I), common cause failures(β_c) and service rates(μ) and for the samples of sizes n = 5 (5) 30 are simulated using computer package developed in this paper and the sample estimates are computed for N = 10000 (20000) 100000 and mean square error (MSE) of the estimates for $A_s(t)$, $A_P(t)$, $A_s(\infty)$,$A_p(\infty)$ were obtained and given in tables [Tab.1] The tables and graphs are seen in the. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interest to note that for a sample size as low as five (n=5) also M L estimate is still seen to be reasonably good giving near accurate estimate in this case. This shows that ML method of estimator is quite useful in this context.

Table-1: Results of the simulations for all function , but this is a sample of steady availability function for series system

with $\beta_I = 0.02$, β_c=0.03, μ=1,p_1=0.5 and k=1

Sample size n = 5				
N	$A_S(\infty)$	$\hat{A}_s(\infty)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.959003	0.970669	0.000117	(0.657462 1.000000)
30000	0.959003	0.967795	0.000051	(0.657462 1.000000)
50000	0.959003	0.951586	0.000033	(0.657462 1.000000)
70000	0.959003	0.962577	0.000014	(0.657462 1.000000)
90000	0.959003	0.983442	0.000081	(0.657462 1.000000)

Sample size n = 10				
N	$A_S(\infty)$	$\hat{A}_s(\infty)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.959003	0.968251	0.000092	(0.745781 1.000000)
30000	0.959003	0.956576	0.000014	(0.745781 1.000000)
50000	0.959003	0.954883	0.000018	(0.745781 1.000000)
70000	0.959003	0.942828	0.000061	(0.745781 1.000000)
90000	0.959003	0.940959	0.000060	(0.745781 1.000000)

Sample size n = 25				
N	$A_S(\infty)$	$\hat{A}_s(\infty)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.959003	0.949061	0.000099	(0.824150 1.000000)
30000	0.959003	0.960506	0.000009	(0.824150 1.000000)
50000	0.959003	0.960378	0.000006	(0.824150 1.000000)
70000	0.959003	0.945203	0.000052	(0.824150 1.000000)
90000	0.959003	0.964530	0.000018	(0.824150 1.000000)

Sample size n = 15				
N	$A_S(\infty)$	$\hat{A}_s(\infty)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.959003	0.970776	0.000118	(0.784908 1.000000)
30000	0.959003	0.960349	0.000008	(0.784908 1.000000)
50000	0.959003	0.923705	0.000158	(0.784908 1.000000)
70000	0.959003	0.963655	0.000018	(0.784908 1.000000)
90000	0.959003	0.958845	0.000001	(0.784908 1.000000)

Sample size n = 30				
N	$A_S(\infty)$	$\hat{A}_s(\infty)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.959003	0.949073	0.000099	(0.835899 1.000000)
30000	0.959003	0.955586	0.000020	(0.835899 1.000000)
50000	0.959003	0.963603	0.000021	(0.835899 1.000000)
70000	0.959003	0.964210	0.000020	(0.835899 1.000000)
90000	0.959003	0.957334	0.000006	(0.835899 1.000000)

The graphic for the simulations of steady availability function for series system

with $\beta_I = 0.02$, β_c=0.03, μ=1,p_1 =0.5 and k=1

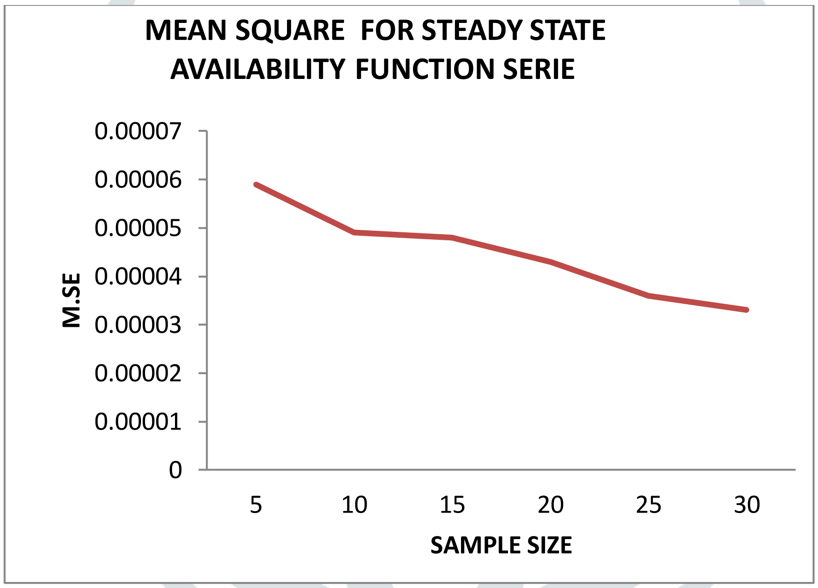

Fig-1:mean Square For Steady State Availability Function Series

CONCLUSIONS:

This paper attempts to evaluate the estimate of the Availability for transient and steady state function s in the presence of common cause and individual failure. The ML method proposed here is giving almost accuracy estimation in case of sample size 10 and above which is verified by the simulation in the absence analytical approach. Also these results suggested the ML estimate is reasonable good and gives accurate estimates even for sample size n=5 therefore this paper identifies the use of thee an ML method of estimator justified through empirical means estimation of the Availability of two component system in presence of CCFs as well as individual failures.

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