Under-sampling Patterns for Compressive Sensing MRI

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Abstract— Magnetic Resonance Imaging (MRI) is an efficient and popular technique used in biomedical field to capture images of different body organs. One of the limitations associated with this technique is its slow process of scanning. This is a major problem when patients comport and compliance is considered. There are several approaches developed for rapid MRI scan. But these techniques will not provide significant reduction in scan time since all these are developed by satisfying Nyquist sampling criteria. Recently the theory of undersampling referred as Compressive sensing (CS) has been developed. By applying CS into MRI, imaging time can be reduced significantly. One of the key requirements in compressive sensing MRI is the sampling pattern used. The K- space sampling pattern in CS MRI should be optimum to reconstruct the image exactly. In this manuscript non uniform under-sampling patterns and their usefulness in compressive sensed MRI is discussed and compared in terms of Signal to Noise Ratio (SNR) and execution speed using different reconstruction algorithm.

Index Terms— Nyquist sampling criteria, CS, MRI, reconstruction algorithm, SNR, execution speed.

I. Introduction

MRI is a popular technique is in practice for the diagnosis of patients and research in biomedical area [1]. Reduction in scan time is one of the major research areas in the MRI process. Recently compressive sensing (CS) is used in MRI called as compressive sensed MRI (CS MRI). This leads to a significant reduction in scanning time.

CS is the philosophy of undersampling of a signal by violating sampling criteria and reconstructing it without any aliasing. Compressive sensing combines compression and sampling into a single step by capturing only important samples. CS theory is based on three major requirements namely Sparsity, under-sampling process and reconstruction algorithm [2]. Sparsity is the measure of number of non zero components in the signal. If the signal contains very few significant components then it is called sparse. Some of the MR images are naturally sparse and others become sparse [3] in the transformed domain. In MRI, information about spatial frequencies within the image is extracted from K- space [1]. Sampling the full K- space data provides sufficient data according to the Nyquist criteria. However this becomes time consuming. Applying CS theory allows random under-sampling of the K- space. Signal energy within the K- space is not uniform. The samples contain significant information are concentrated at the center and lower energy samples are present near the edges. Due to this property of MR images variable density pseudorandom under-sampling [4] of K- space is possible. In this paper, simple approach to design an efficient K-space under-sampling pattern, namely, the variable density radial and spiral patterns for sampling MR images in K- space and its implementation for CS MRI reconstruction is demonstrated.

The detailed description of the manuscript is given in next sections which comprises of theoretical description of CS, sampling patterns, optimization algorithms, results and discussions with conclusion.

II. Theoretical description of CS

CS is the principle of reconstructing a sparse data from a under sampled set of points exactly [5]. Fundamental concept of CS theory is understood by the matrix theory behind this [5]. Consider a signal ‘s’ which can be sparse by using the basis function φ. Then the equation (1) is

$$s = \sum_{i=1}^{n} x_i \varphi_i \quad = \varphi x \qquad (1)$$

where x_i is the weighting coefficient. The signal ‘s’ can be under sampled using the sampling function Φ with mxn size, then the measured data y is given in equation (2) [6].

$$y = \Phi s = \Phi\varphi x = Ax \qquad (2)$$

Since it is under-sampling the matrix A will have number of columns (n) greater than the number of rows (m). To reconstruction of the signal s is based on the solution of the equation (2). Since m<n, there will be infinite number of solutions available. To reconstruct the signal exactly the challenge is to choose the right solution. There are several algorithms extensively used to solve this optimization problem [7]. All of these work on the principle of norm function or Total Variation (TV). The l_p norm of a data 's' is written as

$$\|s\|_p = \left(\sum_{i=0}^{n} |s_i|^p\right)^{\frac{1}{p}} \qquad (3)$$

With p=1, $\|s\|_1$ is known as l_1 norm and p=2, $\|s\|_2$ is called as l_2 norm, l_1 minimization optimization problem is given as [8]

$$\min_x \|x\|_1 \text{ subject to } \quad y = Ax \qquad (4)$$

TV is the gradient estimation; TV of s is described as [9]

$$TV(s) = \sum_{i,j} \left\| s_{i+1,j} - s_{i,j}, s_{i,j+1} - s_{i,j} \right\|_2 \qquad (5)$$

where ij is the pixel position.

III. Sampling patterns for CS MRI

In MRI, data is acquired from the K- space rather than image space. The sampling rate of K-space determined by desired field of view (FOV) [10]. Acquiring data lower than the sampling rate leads to aliasing. CS allows the acquisition of K-space data randomly below the Nyquist rate using sampling pattern. But the question is which type of sampling pattern [9] is optimal? In MRI variable density sampling pattern can be used since MR images contain significant information at the center than the outer periphery [10]. CS can increase the acquisition speed in MRI by decreasing the number samples [11] selected from the K- space. Sampling of K- space is done by using K- space trajectory. Basically there are three types of trajectories namely cartesian, radial and spiral. Cartesian sampling trajectory is simple and most commonly used sampling pattern. Since K- space is non uniform with most of the information is centrally concentrated, trajectories like radial and spiral provide higher SNR.

Radial sampling trajectory [1] has a dense sampling at the center of K- space with reduced density towards the edges. Hence this pattern will be well suited for CS MRI. Spiral trajectory is similar to the radial collects more data near the center of K- space. These collected samples are actually the scanned body part Fourier samples. The image reconstruction involves the application of inverse Fourier transform on the sampled data.

IV. CS optimization algorithms

There are several algorithms extensively used to solve reconstruction problem in compressive sensing. All of these work on the principle of minimization of norm function or TV. This reconstruction is based on the solution of the following constrained optimization problem [10]

$$\min \|\varphi m\|_1 \quad \text{subject to } \|F_s m - y\|_2 < \epsilon \qquad (6)$$

where y=k-space data measured from MRI scanner, F_S=under-sampled Fourier coefficients and m=reconstructed image, ϵ = noise level controller. Among all solutions that are found, only one solution which is compressible by the selected sparsity transform should be chosen.

There are different types of algorithms existing for the CS reconstruction problem. This manuscript focuses on four algorithms. These are tested on 128x128 head phantom image reconstruction and the results are summarized.

A. Nesterov's algorithm (NESTA)

NESTA algorithm is proposed research work of Nesterov that is helpful in solving both the l_1 and TV problems. This iterative algorithm is helpful in CS reconstruction process. This algorithm defines the optimization problem as $\min_x \|x\|_1$ subject to $\|Ax - y\|_2{}^2 \le \epsilon$.

B. Reconstruction from Partial Fourier algorithm (RecPF)

The RecPF algorithm is basically used in the reconstruction of the MRI images. This algorithm is initiated by Yang, Zhang, and Yin. RecPF algorithm is helpful in solving both the l_1 and TV problems. The algorithm defines the optimization problem as [10]:

$$\min_x \|x\|_{TV} + \frac{\lambda}{2} \|kx - y\|^2{}_1 \qquad (6)$$

where k is the linear operator, λ is the regularization parameter. This algorithm is well suited for fast MRI reconstruction due to its higher speed and quality.

C. Two-step Iterative Shrinkage/Thresholding algorithm (TwIST)

TwIST algorithm is the faster version IST. This algorithm is also helpful in solving both the l_1 and TV problems. In TwIST algorithm optimization process is defined using the objective function given by:

$$f(x) = \frac{1}{2}\|y - kx\|_2{}^2 + \lambda\Phi(x) \qquad (7)$$

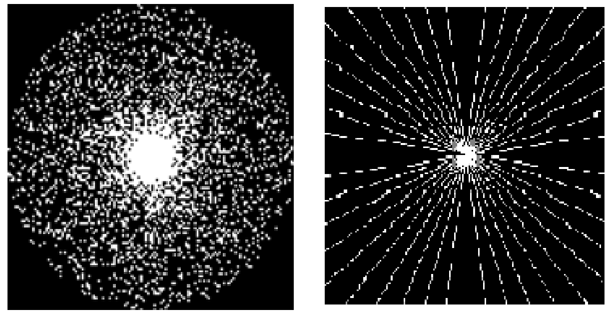
where y=observed data, k=linear operator, λ=regularization parameter and ϕ=convex regularizer.

D. Split Augmented Lagrangian shrinkage algorithm (SALSA)

SALSA is based on variable splitting to obtain an equivalent constrained optimization formulation technique, which is also called as an augmented Lagrangian [10] method. In this algorithm definition of optimization problem is given as $f(x) = \frac{1}{2}\|y - kx\|^2 + \lambda\Phi(x)$.

V. Simulation results

The reconstruction results obtained for the four algorithms given above for the CS MRI using an image of head phantom are presented. These algorithms sample the data in frequency domain. The spiral and radial sampling patterns used for pseudo random under- sampling are as shown in Fig: 1.



(a) (b)

Fig: 1 Sampling pattern (a) Spiral (b) Radial

The execution process uses 18% of the Fourier coefficients. The reconstruction results using radial pattern is given in [11] and using spiral pattern is as shown in Fig: 2 and Fig: 3. Comparison of the algorithms is performed with respect to SNR and execution time. Table: 1 shows the tabulation of results.

Table: 1 Comparison of algorithms.

Algorithm	Radial		Spiral	
	Comparison parameters		Comparison parameters	
	execution time (sec)	SNR (dB)	execution time (sec)	SNR (dB)
NESTA	3.8	11.79	3.9	9.27
RecPF	4.17	39.00	4.80	33.88
TwIST	7.29	44.59	6.02	35.71
SALSA	9.20	35.63	10.23	43.68

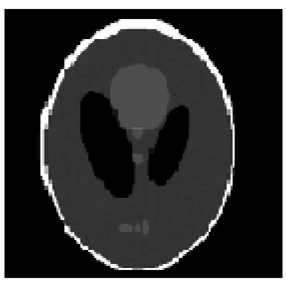


Fig: 2 Input phantom

(a) (b) (c) (d)

Fig: 3 Image reconstruction using (a) NESTA (b) RecPF (c) TwIST and (d) SALSA

It has been found that RecPF is a simple algorithm and as good as TwIST in performance. Further it is observed that NESTA is having high speed with poor SNR. Comparisons also show that TwIST has highest SNR quality with high execution time. It is also observed that Radial and Spiral sample patterns are quite similar having high density at the center. SNR comparisons show that Radial sampling pattern works better than the Spiral sampling.

VI. Conclusion

MRI is efficient biomedical tool with very slow acquisition process. To decrease the scan time in MRI compressive sensing can be used. This also reduces the burden on patients. CS exploits the sparsity in MR images. The pseudo random sampling patterns Radial and Spiral are designed and implemented in CS MRI. Simulation results show the performance analysis of these patterns. In CS MRI non cartesian sampling patterns are preferred.

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