CONCEPTS OF GRAPH THEORY AND ITS APPLICATIONS

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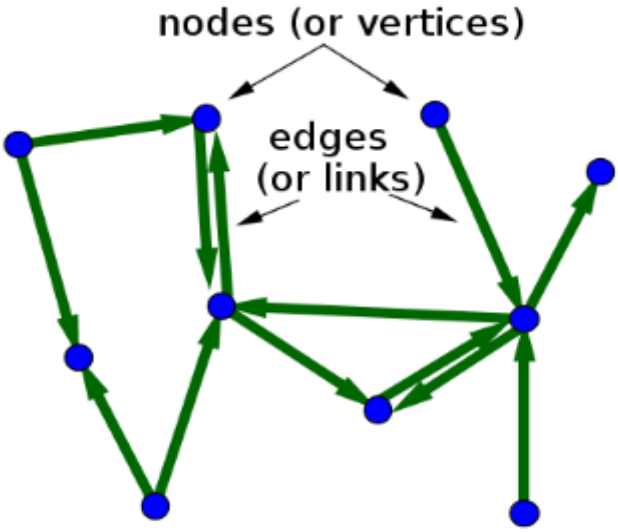
***Abstract**: The main objective of this paper is to introduce the main concepts of Graph theory and study about connected graph, Eulerian graph, Hamiltonian graph etc.* Graph theory is a branch of mathematics which has wide application in the area of mathematics as well as in other branches of science. This paper aims to emphasize the applications of graph theory in our daily life, in Computer science, Operation Research, Chemistry etc.

***IndexTerms* – Graph, Eulerian Graph, Hamiltonian Graph, Operation Research.**

I. INTRODUCTION

In 1736, Euler introduced graph theory to solve Konigsberg bridge problem. Graph theory in mathematics means the study of graphs. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In general, a graph is represented as a set of vertices (nodes or points) connected by edges (arcs or line).The graph with vertices V and edges E is written as G(V,E).An undirected graph G consists of set of vertices V and a set of edges E such that each edges is associated with an unordered pair of vertices . A directed graph is a graph with set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another. A directed graph is sometimes called a digraph or directed network. In contrast, a graph where the edges are bidirectional is called an undirected graph. When drawing a directed graph, the edges are typically drawn as arrows indicating the direction, as illustrated in the following figure.

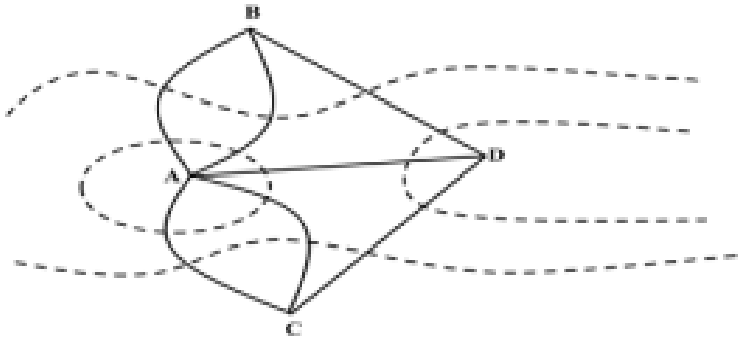


A directed graph with 10 vertices (or nodes) and 13 edges.

Moreover, a graph with no self loops and parallel edges is called a simple graph and a graph with self loops and parallel edges is called a pseudograph. Furthermore, Graph theory is used in many areas such as communications, Engineering, Physical Sciences etc. In areas of computer science such as switching theory and logical design, artificial intelligence, computer graphics, graph theory is also very useful.

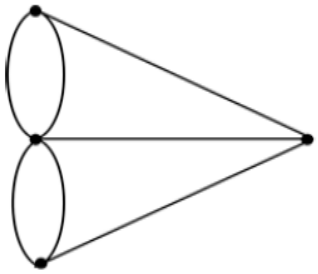
II. ORIGIN OF EULERIAN GRAPH

Graph theory began in 1736 when Leonhard Euler solved a problem that had been puzzling the good citizens of the town of Konigsberg in Prussia (now Kaliningrad in Russia). The river called Pregel flows through the city Konigsberg (located in Russia) dividing the city into four land regions, two are river banks and two are islands or delta formation. The four land regions were connected by 7 bridges. The people wanted to know if it were possible to start at any location in town, cross every bridge exactly once, and return to the starting location.



The bridges of Konigsberg problem

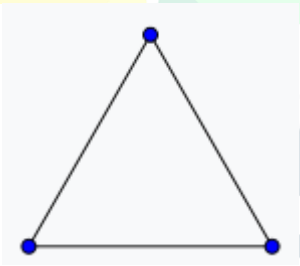
Euler suggested and further explained that it is impossible to do so by using the terminology of points (representing the land regions) and lines (representing the bridges). He abstracted the case of Konigsberg by eliminating all unnecessary features. He drew a picture consisting of "dots" that represented the landmasses and the line-segments(edges) representing the bridges that connected those land masses. The resulting picture looked somewhat similar to the figure shown below.



Euler proposed that any given graph can be drawn with each edge traversed exactly once if and only if it has zero or exactly two nodes with odd degrees. The graph obeying this condition is called Eulerian circuit or path. In other words, a simple path in a graph G is called Euler path if it traverses every edge of graph exactly once.

III. FUNDAMENTAL DEFINITIONS

a) Complete Graph- A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. It is also known as universal graph. For example-



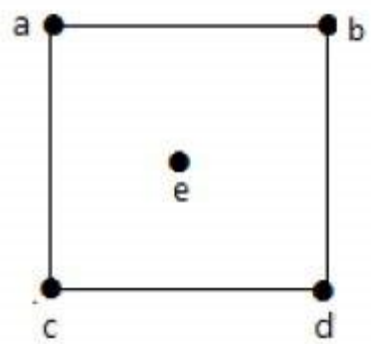
A complete diagraph is a directed graph in which every pair of distinct vertices is connected by a pair of unique edges (one in each direction).

b) Degree in a Graph - It is the number of vertices adjacent to a vertex V. It is denoted by deg(V). In a simple graph with n number of vertices, the degree of any vertices is – $\deg(V) \leq n - 1 \; \forall \; V \in G$

A vertex can form an edge with all other vertices except by itself. So the degree of a vertex will be up to the number of vertices in the graph minus 1. This 1 is for the self-vertex as it cannot form a loop by itself. If there is a loop at any of the vertices, then it is not a Simple Graph.

c) Degree of a Vertex in an Undirected Graph - An undirected graph has no directed edges.

Example 1 Take a look at the following graph -



In the above graph,

deg(a) = 2, deg(b) = 2, deg(c) = 2, deg(d) = 2, and deg(e) = 0.

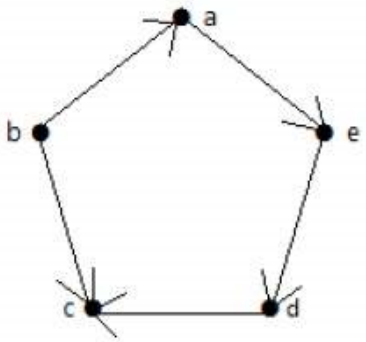
The vertex 'e' is an isolated vertex. The graph does not have any pendent vertex. A vertex with degree one is called a pendent vertex. A vertex with degree zero is called an isolated vertex.

Degree of a Vertex in a Directed Graph- In a directed graph, each vertex has an indegree and an outdegree.

Indegree of a Graph- Indegree of a vertex V is the number of edges which are coming into the vertex V.

Outdegree of a Graph-Outdegree of a vertex V is the number of edges which are going out from the vertex V.

Example 2 Take a look at the following directed graph. Vertex 'a' has an edge 'ae' going outwards from vertex 'a'. Hence its outdegree is 1. Similarly, the graph has an edge 'ba' coming towards vertex 'a'. Hence the indegree of 'a' is 1.

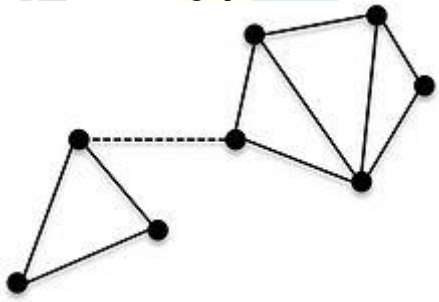


The indegree and outdegree of other vertices are shown in the following table −

Vertex	Indegree	Outdegree
a	1	1
b	0	2
c	2	0
d	1	1
e	1	1

Connected Graph- A graph G is said to be connected graph if there is atleast one path between every pair of vertices in G.

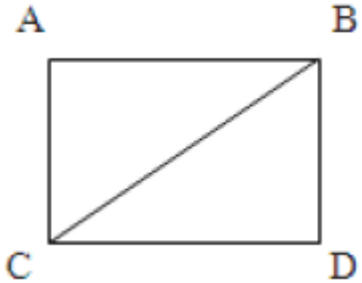
Disconnected Graph- A graph G which is not a connected graph is called disconnected.



This graph becomes disconnected when the dashed edge is removed.

Euler Path: A simple path in a graph G is called Euler Path if it traverses every edge of graph exactly once.

Example: Below figure has Euler Path BDCABC as each edge appears exactly once.



Euler circuit: Euler Circuit is a circuit in graph G which traverses every edge of graph exactly once. Euler Circuit is simply a closed Euler path. It is also called Euler line.

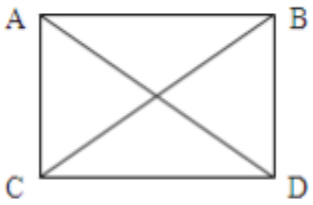
Eulerian Graph: A graph which contain either Euler Path or Euler Circuit is called Eulerian Graph.

Hamiltonian Path: A Hamiltonian Path in a connected graph is a path which contains each vertex of graph exactly once.

Hamiltonian Circuit: A Hamiltonian circuit is a circuit that contains each vertex of graph exactly once except for the first vertex, which is also the last.

Hamiltonian Graph: A graph which possesses either Hamiltonian circuit or Hamiltonian path is called a Hamiltonian graph.

Example: Below figure has Hamiltonian circuit ABCDA as each vertex appears exactly once except A. So it is Hamiltonian graph.



IV. APPLICATIONS OF GRAPH THEORY

Applications in Google map: Now a days, Google map is a very useful tool for travelling anywhere in the world. Using Google map we can find all routes from one place to any other place and also can find the shortest route. In case of Google map, we can consider the places as vertices of graph and the routes as the edges. Then the software of Google map, when we find the routes between two places, it will find all edges between these two places or vertices and also gives the shortest edge as the shortest path.

Applications in Internet: Internet is a very useful invention of modern science. In the working technique of internet the concepts of graph theory are used. In case of connectivity of internet, all the users are considered as vertices and the connection between them are edges. Similarly, in case of social networking sites one friend is connected to other friends and his friends are also connected to others. If we consider the friends as vertices of graph and define an edge in between them, then it will form a graph.

Graphs in Chemistry: Graph theory is used in chemistry for mathematical modeling of chemical phenomena. We can make natural model of a molecule where vertices represent atoms and edges represent bond. There is a branch of mathematical chemistry called Chemical graph theory (CGT) which deals with the non trivial applications of graph theory to solve molecular problems.

Applications in Operation Research: Graph theory is a very useful tool in operation research. There are some Operation research problems that can be solved using graphs. In transportation problem, when we need to minimize the transportation cost or maximize the profit, then the graph theoretical approach is very useful. It is also used in different assignment problems such as assigning different peoples to different jobs, manage time table for school, college etc.

Applications in Computer Science: There is a major role of graph theory in computer science. Graph theory concepts are used to develop the algorithm of different programs. There are some algorithms such as. (1). Shortest path algorithm in a network. (2). Finding minimum spanning tree. (3). Algorithms to find the cycles in a graph etc.

V. CONCLUSION

The main objective of this paper is to present the importance of graph theory in different branches of science and our everyday life. This paper is valuable for students and researchers to get the overview of graph theory and its application in diverse fields like everyday life, computer science, Operation Research, Chemistry.

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