

# SLIDING MODE CRUISE CONTROL BASED ON HIGH GAIN OBSERVER

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**Abstract-** Vehicular and highway automation has demonstrated superior safety performance, increased capacity, reduced fuel consumption and enhanced overall comfort and performance for drivers. This paper proposes an exponential high gain, sliding-mode observer (SMO) as a cruise control strategy for a longitudinal vehicle model. First, longitudinal model of vehicle is analysed and transfer function of model is obtained. Thereafter, sliding mode high gain observer is designed considering uncertainties and external disturbances. The model with integrated high gain observer is simulated on MATLAB environment. Numerical simulations on longitudinal vehicular model with designed Sliding mode controller demonstrate faster convergence, improved position and speed tracking.

**Index Terms-** SMO (sliding mode observer), HGO (high gain observer), Estimation, Lyapunov function.

## I. INTRODUCTION

Automatic vehicle speed control or cruise control is one of the widely researched topics in the automotive industry. [1]. Most of research in the literature embodies the objectives of range policy, string stability and human comfort & safety [2-8]. Few studies are mainly focused on reducing the model uncertainty and external disturbance, since the vehicle dynamics are highly nonlinear and uncertain [9]. Some researchers have tried to develop model based lower level controller which determines control inputs using the vehicle parameters and inverse dynamics [10]. Furthermore, linearized vehicle model has also been considered in literature to design gain scheduling linear quadratic controller for throttle and brake actuation [11]. The problem of cruise control system is to maintain the output speed of the system as set by input signal. This can be achieved by various control strategies such as proportional-integral derivatives (PID) controller, state-space controller and fuzzy logic based controllers. [12]. In modelling of a cruise control, it is vital that model takes into account all of the important parameters, which directly or indirectly affect the overall performance of the system. [13] Post modelling, the designing of appropriate controller along with stability analysis based on linear state-space model or transfer function is performed. [13]

Sliding-mode control is a well-established method for handling disturbances and modelling uncertainties through the concepts of sliding surface design and equivalent control. Based on the same concept, sliding-mode observers (SMOs) have been developed in literature for robust estimation of system states [14]– [22]. State estimation of nonlinear systems has been an active field of

research in the last few decades. High-gain observers have received wide attention for uniformly observable single output systems [23]- [25]. A constant gain observer is a special class of nonlinear systems that does not require the nonlinear transformation [26]. Furthermore, there is significant improvement in the existing design of the HGO with incorporation of the nonlinear system into the gain design strategy [27]. The Lyapunov-based approach is followed to tackle the problems of state observation in the presence of bounded uncertainties/unknown inputs in HGO.

In this article, a high gain observer based SMC is designed and integrated to the vehicle dynamics on MATLAB Simulink Platform. The output of the system is controlled by the controller in order to provide the desired speed at which the car is to be maintained in the presence of uncertainties and external disturbance.

The remainder of this paper is organized as follows. Section II presents the cruise description and modelling. Section III presents the design of high gain observer that incorporates an SMO. In Section IV, design of sliding-mode controller is discussed. Section V presents the results of cruise control. Section VI embodies conclusion.

## II. SYSTEM DISCRIPTION

The purpose of the cruise control system is to regulate the vehicle speed so that it follows the driver's command and maintains the speed. The Vehicle block represents a two-axle vehicle body in longitudinal motion as shown in fig (1). The vehicle can have the same or a different number of wheels on each axle. The vehicle wheels are assumed identical in size. The vehicle can have a center of gravity (CG) positioned at or below the plane of travel.

The vehicle block accounts for body mass, aerodynamic drag, road incline, and weight distribution between axles due to acceleration and road profile. It may optionally include pitch and suspension dynamics. The vehicle does not move vertically relative to the ground.

The vehicle axles are parallel and form a plane. The longitudinal,  $x$ , direction lies in this plane and perpendicular to the axles. If the vehicle is traveling on an incline slope,  $\beta$ , the normal,  $z$ , direction is not parallel to gravity but is always perpendicular to the axle-longitudinal plane.

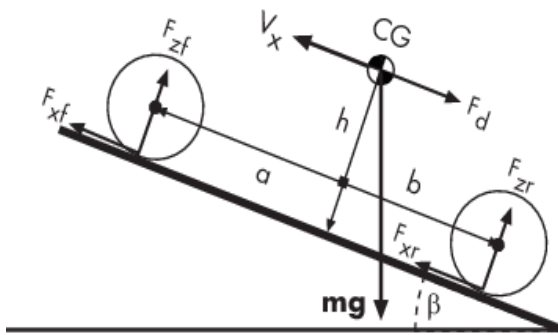


Fig. 1. Longitudinal vehicle model

TABLE I. PARAMETERS OF LONGITUDINAL VEHICLE

Parameters	Meaning	Unit
$g$	Gravitational acceleration	$m/s^2$
$\beta$	Incline Angle	rad
$m$	Mass of Vehicle	kg
$h$	Height of vehicle from CG	m
$V_x$	Velocity of the vehicle	$m/s$
$V_w$	Wind speed	$m/s$
$F_{xf}, F_{xr}$	Longitudinal forces on each wheel at the front and rear ground contact points, respectively	N
$F_{zf}, F_{zr}$	Normal load forces on each wheel at the front and rear ground contact points, respectively	N
$A$	Vehicle cross section area	$m^2$
$C_d$	Aerodynamic drag coefficient	
$\rho$	Mass density of air	$Kg/m^3$
$F_d$	Aerodynamic drag force	N
$n$	Number of wheels on each axle	

Case: When  $V_x > 0$ , the vehicle moves forward,

When  $V_x < 0$ , the vehicle moves backward,

When  $V_w > 0$ , the wind is headwind,

When  $V_w < 0$ , the wind is tailwind,

By applying newton's second law for z direction (i.e no vertical acceleration) ,[]

$$0 = w * \text{Cos}\beta - F_{zf} - F_{zr} + R_{hz}$$

$$m_{ax} = \frac{w}{g} a_x$$

$$= m \frac{du}{dt}$$

$$= F_{xr} + F_{xf} - w * \text{Sin}\beta - R_{xr} - R_{xf} - D_A - R_{hx} \quad (1)$$

The aerodynamic drag force depends on the relative velocity between vehicle and the surrounding air, given by-

$$D_A = 0.5 * \rho * C_d * A(V_x + V_w)^2$$

The rolling resistance arises due to work of deformation on the tire and road surface and is roughly proportional to the normal force on tire-

$$R_x = R_{xf} + R_{xr} = f(F_{zf} + F_{zr})$$

A simple model of the longitudinal motion of a vehicle can be used to determine changes in the vehicle forward motion due to grades, breaking, accelerating with,  $R_{hx} = 0$ ,  $F_{xr} = 0$  &  $a_x = du/dt$ ,

At equilibrium  $du/dt=0$

$$F_{x0} = mg\text{Sin}\beta_0 + \rho mg\text{Cos}\beta_0 + 0.5 * \rho C_d A(V_x + V_w)^2 \quad (2)$$

The actuator and the vehicle propulsion system are modelled as cascaded of first order

$$\frac{C_1}{(1 + sT)}$$

However, a problem of non-linearity arises, one way to overcome this problem is to linearize all of the state-equations by differentiating both left and right hand sides of the equations.

The linearized model provides a transfer function can be obtained by solving the state-equations for the ratio of  $\Delta V(s) / \Delta U(s)$ .

$$\frac{\Delta V(s)}{\Delta U(s)} = \frac{KC_1}{(\tau s + 1)(1 + sT)}$$

Where  $\tau = \frac{m}{\rho C_d A(V_x + V_w)}$

$$K = 1/ \rho C_d A(V_x + V_w)$$

$$\frac{\Delta V(s)}{\Delta U(s)} = \frac{0.0144}{(s^2 + 0.018s + 0.006)}$$

Which can be represented as

$$\left. \begin{aligned} \dot{x}_1 &= \theta \\ \dot{x}_2 &= -0.018x_2 - 0.006x_1 + 0.0144 u(t) \\ y &= x_1 \end{aligned} \right\} \quad (3)$$

## II. DESIGN OF HIGH GAIN OBSERVER

Design a high gain observer as

$$\left. \begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \frac{\alpha_1}{\epsilon} (y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= -0.018\hat{x}_2 - 0.006\hat{x}_1 + 0.0144 u + \frac{\alpha_2}{\epsilon^2} (y - \hat{x}_1) \end{aligned} \right\} \quad (4)$$

Where  $\alpha_1$  &  $\alpha_2$  are the positive values, and  $\epsilon \ll 1$ .

Let  $g_1 = \frac{\alpha_1}{\epsilon}$ ,  $g_2 = \frac{\alpha_2}{\epsilon^2}$  then we have,

$$\dot{\hat{x}}_1 = \hat{x}_2 + g_1 (y - \hat{x}_1)$$

$$\dot{\hat{x}}_2 = -0.018\hat{x}_2 - 0.006\hat{x}_1 + 0.0144 u + g_2 (y - \hat{x}_1)$$

Where  $\tilde{x} = x - \hat{x}$

Hence, from above equations,

$$\left. \begin{aligned} \tilde{x}_1 &= -g_1 \tilde{x}_1 + \tilde{x}_2 \\ \tilde{x}_2 &= -g_2 - a \tilde{x}_2 \\ y &= x_1 \end{aligned} \right\} \quad (5)$$

i.e  $\dot{\tilde{x}} = A\tilde{x}$

where  $A = \begin{bmatrix} -g_1 & 1 \\ -g_2 & -a \end{bmatrix}$ ,  $\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$

It is assume that if A is Hurwitz, then  $\tilde{x}$  will converge to zero exponentially, and can be expressed as

$$\|\tilde{x}(t)\| \leq \phi_0 \|\tilde{x}(t_0)\| e^{-\sigma_0(t-t_0)} \quad (6)$$

Where  $\phi_0$  and  $\sigma_0$  are positive constants.

Consider  $\sigma_0$  is related to the minimum eigenvalue of A; the smaller the  $\epsilon$  value is, the bigger the  $h_i$  value is, and then the smaller the minimum eigenvalue of A is, the bigger the  $\sigma_0$  value is.

Therefore, the convergence of  $\|\tilde{x}(t)\|$  depends on  $\epsilon$ , and high gain observer will improve convergence precision of  $\|\tilde{x}(t)\|$  greatly.

### III. SLIDING MODE CONTROLLER DESIGN

The system is designed to drive and then constrain the system state to lie within a neighbourhood of the switching function. Trajectory of a system is designed by a sliding mode controller.

Let the sliding surface is represented by “s”. The surface “s” is chosen to reduced order dynamics when constrained to plane.

The sliding mode function is-

$$s = ce + \dot{e}$$

where  $c > 0$ .

The tracking error and its derivative value is-

$$\hat{s} = c\hat{e} + \dot{\hat{e}}$$

where  $\hat{e} = \theta_d - \hat{\theta}$  and  $\dot{\hat{e}} = \dot{\theta}_d - \dot{\hat{\theta}}$

The sliding mode controller is

$$u(t) = \frac{1}{k} (\ddot{\theta}_d + a\hat{\theta} + b\theta + \eta\hat{s} + c\dot{\hat{e}}) \quad (7)$$

where  $\eta > 0$ ,  $\hat{e} = \theta_d - \hat{\theta}$  and  $\hat{s} = c\hat{e} + \dot{\hat{e}}$ .

Let the Lyapunov function for the controller

$$V_s = \frac{1}{2} s^2$$

Since

$$\ddot{e} = \theta_d'' - \ddot{\theta} = \theta_d'' + a\dot{\theta} + b\theta - ku$$

$$\dot{s} = c\dot{e} + \ddot{e} = c\dot{e} + \theta_d'' + a\dot{\theta} + b\theta - ku$$

then

$$\dot{s} = c\dot{e} + \theta_d'' + a\dot{\theta} + b - (\theta_d'' + a\hat{\theta} + b\theta + \eta\hat{s} + c\dot{\hat{e}})$$

$$= c\dot{\hat{e}} + a\dot{\tilde{\theta}} - \eta\hat{s} = -\eta s + \eta\tilde{s} + c\dot{\hat{e}} + a\dot{\tilde{\theta}}$$

$$= -\eta s + \eta (-c\tilde{\theta} - \dot{\tilde{\theta}}) + c(-\dot{\tilde{\theta}}) + a\dot{\tilde{\theta}}$$

$$= -\eta s + \eta (-c\tilde{\theta} - \dot{\tilde{\theta}}) + c(-\dot{\tilde{\theta}}) + a\dot{\tilde{\theta}}$$

$$= -\eta s - \eta c\tilde{\theta} + (a - \eta - c)\dot{\tilde{\theta}}$$

where  $\tilde{\theta} = \theta - \hat{\theta}$ ,  $\dot{\tilde{\theta}} = \dot{\theta} - \dot{\hat{\theta}}$ ,

$$\tilde{e} = e - \hat{e} = -\theta + \hat{\theta} = -\tilde{\theta},$$

$$\tilde{\dot{e}} = -\dot{\tilde{\theta}} \text{ and } \tilde{s} = s - \hat{s}$$

$$= c\tilde{e} + \tilde{\dot{e}} = -c\tilde{\theta} - \dot{\tilde{\theta}}$$

Then,

$$\begin{aligned} \dot{V}_s &= -\eta s^2 + s(-\eta c\tilde{\theta} + (a - \eta - c)\dot{\tilde{\theta}}) \\ &= -\eta s^2 + k_1 s\tilde{\theta} + k_2 s\dot{\tilde{\theta}}, \end{aligned}$$

where  $k_1 = -\eta c$  and  $k_2 = a - \eta - c$ .

$$\text{Since } k_1 s\tilde{\theta} \leq \frac{1}{2} s^2 + \frac{1}{2} k_1^2 \tilde{\theta}^2$$

$$\text{and } k_2 s\dot{\tilde{\theta}} \leq \frac{1}{2} s^2 + \frac{1}{2} k_2^2 \dot{\tilde{\theta}}^2$$

$$\begin{aligned} \dot{V}_s &\leq -\eta s^2 + \frac{1}{2} s^2 + \frac{1}{2} k_1^2 \tilde{\theta}^2 + \frac{1}{2} s^2 + \frac{1}{2} k_2^2 \dot{\tilde{\theta}}^2 \\ &= -(\eta - 1)s^2 + \frac{1}{2} k_1^2 \tilde{\theta}^2 + \frac{1}{2} k_2^2 \dot{\tilde{\theta}}^2 \end{aligned} \quad (8)$$

where  $\eta > 1$ .

As per the, Lyapunov function for the closed system is improved and designed as

$$V = V_s + V_0,$$

where  $V_0 = \frac{1}{2} \tilde{x}^T \tilde{x} = \frac{1}{2} \tilde{\theta}^2 + \frac{1}{2} \dot{\tilde{\theta}}^2$ .

Since  $\dot{V}_0 = \tilde{x}^T \dot{\tilde{x}} = \tilde{x}^T A \tilde{x}$ , and  $\dot{V}_0$  converges to zero exponentially, i.e.,

$$\tilde{x}^T A \tilde{x} \leq \chi(\bullet) e^{-\sigma_0(t-t_0)},$$

where  $\chi(\bullet)$  is a K-class function  $\|\tilde{x}(t_0)\|$

Then

$$\begin{aligned} \dot{V} &\leq -(\eta - 1)s^2 + \frac{1}{2} k_1^2 \tilde{\theta}^2 + \frac{1}{2} k_2^2 \dot{\tilde{\theta}}^2 + \tilde{x}^T A \tilde{x}, \\ &= -\eta_1 V_s - \frac{1}{2} \eta_1 \tilde{\theta}^2 - \frac{1}{2} \eta_1 \dot{\tilde{\theta}}^2 + \frac{1}{2} (k_1^2 + \eta_1) \tilde{\theta}^2 + \frac{1}{2} (k_2^2 + \eta_1) \dot{\tilde{\theta}}^2 + \tilde{x}^T A \tilde{x} \end{aligned}$$

$$\leq -\eta_1 V + \chi(\bullet) e^{-\sigma_0(t-t_0)}$$

where  $\eta_1 = 2(n - 1) > 0, \sigma_0 > 0, x$

$$= [\theta \quad \dot{\theta}]^T$$

The solution of

$$\dot{V} \leq -\eta_1 V \chi(\bullet) e^{-\sigma_0(t-t_0)}$$

$$V(t) \leq e^{-\eta_1(t-t_0)} V(t_0) + \chi(\bullet) \int_{t_0}^t e^{-\eta_1(t-\tau)} e^{-\sigma_0(\tau-t_0)} d\tau$$

$$= e^{-\eta_1(t-t_0)} V(t_0) \chi(\bullet) e^{-\eta_1 t + \sigma_0 t_0} \int_{t_0}^t e^{\eta_1 \tau} e^{-\sigma_0 \tau} d\tau$$

$$\begin{aligned}
 &= e^{-\eta_1(t-t_0)}V(t_0) \frac{\chi(\bullet)}{\eta_1-\sigma_0} e^{-\eta_1 t + \sigma_0 t_0} e^{(\eta_1-\sigma_0)\tau} |_{t_0}^t \\
 &= e^{-\eta_1(t-t_0)}V(t_0) \frac{\chi(\bullet)}{\eta_1-\sigma_0} e^{-\eta_1 t + \sigma_0 t_0} (e^{(\eta_1-\sigma_0)t} - e^{(\eta_1-\sigma_0)t_0}) \\
 &= e^{-\eta_1(t-t_0)}V(t_0) + \frac{\chi(\bullet)}{\eta_1-\sigma_0} (e^{\sigma_0(t-t_0)} - e^{-\eta_1(t-t_0)})
 \end{aligned}$$

i.e.,

$$\lim_{t \rightarrow \infty} V(t) \leq \theta$$

Since,  $V(t) \geq 0$ , then we have  $t \rightarrow \infty, V(t) = 0$  and  $V(t)$  to zero exponentially, the convergence precision depends on  $\eta_1, i. e. \eta$ .

#### IV. RESULT & DISCUSSION

The system is modelled using Matlab/Simulink as shown in figure 2. The speed of cruise system, based on an exponential high gain observe is tracked. A closed system exponential convergence is also analysed.

The following parameters were chosen for simulation:  $k=.014; a=.018; b=.006; c=5; xite=10, k=.014; h1=400-a; h2=40000;$

The initial states are  $x_1(0) = 0.20, x_2(0) = 0$ , and ideal position signal is set as  $\theta_d = \text{sint}$ .

Fig. 2 shows the simulation model of controller and vehicle.

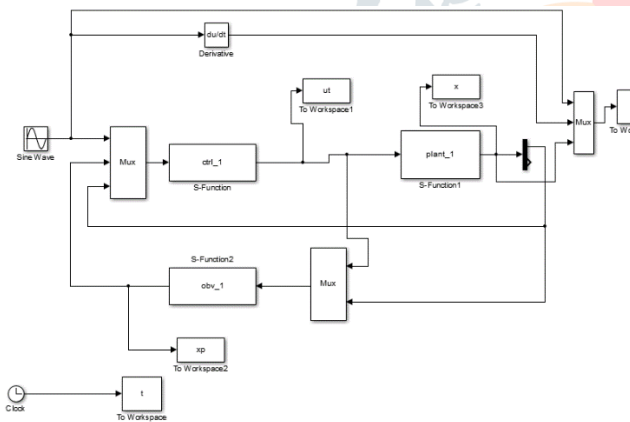


Fig 2. Block diagram of simulate cruise control with SMC

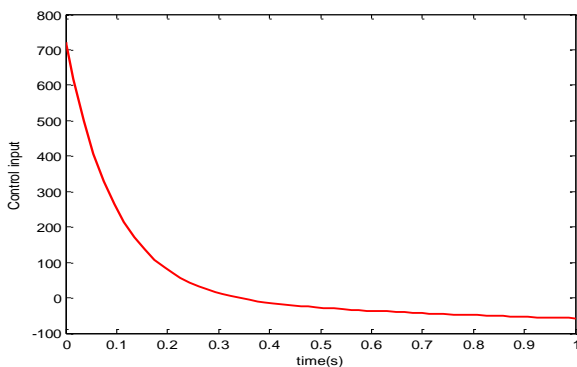


Fig 3. Control input for plant

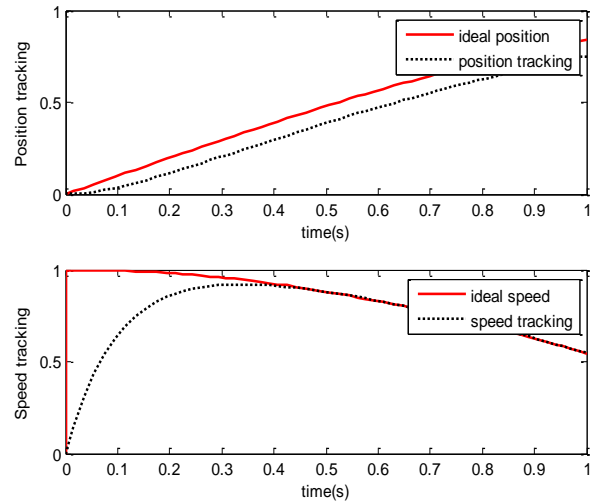


Fig 4. Speed & Position tracking

The simulation results of control input of vehicle is shown in Figures (3). Improved estimation performance can be clearly observed from control input. It is seen that the speed tracking and position of vehicle in fig. (4) shows a better results and gives small error which enhance the cruise trajectory.

#### V. CONCLUSIONS

The mathematical model for cruise control system has been derived. A robust HGO design is developed for a cruise system by incorporating a sliding-mode term into the nonlinear observer to improve estimation accuracy. Vehicle model along with Sliding mode observer is simulated in MATLAB Simulink environment. Simulation results obtained show superior cruise trajectory, speed tracking and minimised position error.

#### REFERENCES

- [1] Kim, Hakgu, and Kyongsu Yi. "Combined throttle and brake control for vehicle cruise control: A model free approach." *Intelligent Vehicles Symposium (IV), 2013 IEEE*. IEEE, 2013.
- [2] Choi, Sungwoo, et al. "Model-free control of automotive engine and brake for Stop-and-Go scenarios." *Control Conference (ECC), 2009 European*. IEEE, 2009.
- [3] Naranjo, José Eugenio, et al. "ACC+ Stop&go maneuvers with throttle and brake fuzzy control." *IEEE Transactions on intelligent transportation systems* 7.2 (2006): 213-225.
- [4] Martinez, J-J., and Carlos Canudas-de-Wit. "Model reference control approach for safe longitudinal control." *American Control Conference, 2004. Proceedings of the 2004*. Vol. 3. IEEE, 2004.
- [5] Levine, W., and Michael Athans. "On the optimal error regulation of a string of moving vehicles." *IEEE Transactions on Automatic Control* 11.3 (1966): 355-361.
- [6] Zhou, Jing, and Huei Peng. "Range policy of adaptive cruise control vehicles for improved flow stability and string stability." *IEEE Transactions on intelligent transportation systems* 6.2 (2005): 229-237.
- [7] Wang, Junmin, and Rajesh Rajamani. "Should adaptive cruise-control systems be designed to maintain a constant time gap between vehicles?." *IEEE Transactions on Vehicular Technology* 53.5 (2004): 1480-1490.
- [8] Fancher, P., Z. Bareket, and R. Ervin. "Human-centered design of an ACC-with-braking and forward-crash-warning system." *Vehicle System Dynamics* 36.2-3 (2001): 203-223.
- [9] Moon, Seungwuk, Ilki Moon, and Kyongsu Yi. "Design, tuning, and evaluation of a full-range adaptive cruise control system

- with collision avoidance." *Control Engineering Practice* 17.4 (2009): 442-455.
- [10] Liang, Chi-Ying, and Huei Peng. "Optimal adaptive cruise control with guaranteed string stability." *Vehicle system dynamics* 32.4-5 (1999): 313-330.
- [11] Goodrich, Michael A., and Erwin R. Boer. "Model-based human-centered task automation: a case study in ACC system design." *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans* 33.3 (2003): 325-336.
- [12] Shakouri, Payman, et al. "Adaptive cruise control system: comparing gain-scheduling PI and LQ controllers." *IFAC Proceedings Volumes* 44.1 (2011): 12964-12969.
- [13] V. I. Utkin, *Sliding Modes in Control and Optimizations*. Berlin, Germany: Springer-Verlag, 1992.
- [14] B. L. Walcott and S. H. Zak, "State observation of nonlinear uncertain dynamical systems," *IEEE Trans. Autom. Control*, vol. AC-32, no. 2, pp. 166-170, Feb. 1987.
- [15] S. Drakunov and V. Utkin, "Sliding mode observers. Tutorial," in *Proc. 34th IEEE Conf. Decision Control*, New Orleans, LA, 1995, pp. 3376-3378.
- [16] J. P. Barbot, T. Boukhobza, and M. Djemai, "Sliding mode observer for triangular input form," in *Proc. 35th IEEE Conf. Decision Control*, Kobe, Japan, 1996, pp. 1489-1490.
- [17] C. Edwards, S. K. Spurgeon, and R. J. Patton, "Sliding mode observers for fault detection and isolation," *Automatica*, vol. 36, no. 4, pp. 541-553, Apr. 2000.
- [18] Y. Xiong and M. Saif, "Sliding mode observer for nonlinear uncertain systems," *IEEE Trans. Autom. Control*, vol. 46, no. 12, pp. 2012-2017, Dec. 2001.
- [19] A. J. Koshkouei and A. S. I. Zinober, "Sliding mode state observation for non-linear systems," *Int. J. Control*, vol. 77, no. 2, pp. 118-127, Jan. 2004.
- [20] K. C. Veluvolu, Y. C. Soh, and W. Cao, "Robust discrete-time nonlinear sliding mode state estimation of uncertain nonlinear systems," *Int. J. Robust Nonlinear Control*, vol. 17, no. 9, pp. 803-828, Jun. 2007.
- [21] K. C. Veluvolu, Y. C. Soh, and W. Cao, "Robust observer with sliding mode estimation for nonlinear uncertain systems," *IET Control Theory Appl.*, vol. 1, no. 5, pp. 1533-1540, Sep. 2007.
- [22] J. P. Gauthier, H. Hammouri, and S. Othman, "A simple observer for nonlinear systems applications to bioreactors," *IEEE Trans. Autom. Control*, vol. 37, no. 6, pp. 875-880, Jun. 1992.
- [23] J. P. Gauthier and I. A. K. Kupka, "Observability and observers for nonlinear systems," *SIAM J. Control Optim.*, vol. 32, no. 4, pp. 975-994, Jul. 1994.
- [24] F. Deza, E. Busvelle, J. P. Gauthier, and D. Rakotopara, "High gain estimation for nonlinear systems," *Syst. Control Lett.*, vol. 18, no. 4, pp. 295-299, Apr. 1992.
- [25] K. Busawon, M. Farza, and H. Hammouri, "Observer design for a special class of nonlinear systems," *Int. J. Control*, vol. 71, no. 3, pp. 405-418, Oct. 1998.
- [26] K. Busawon and J. De León-Morales, "An observer design for uniformly observable non-linear systems," *Int. J. Control*, vol. 73, no. 15, pp. 1375-1381, 2000.
- [27] A. N. Atassi and H. K. Khalil, "Separation results for the stabilization of nonlinear systems using different high-gain observer design," *Syst. Control Lett.*, vol. 39, no. 3, pp. 183-191, Mar. 2000.