Self-Phase Modulation of Tightly Focused q-Gaussian Laser Beam in Parabolic Plasma Channels

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Abstract

Self-phase modulation (SPM) of q-Gaussian laser beams interacting nonlinearly with plasma channels has been investigated theoretically. Due to intensity gradient over the cross section of the laser beam the transverse component of Ponderomotive force becomes finite. This results in redistribution of carriers in the illuminated portion of the channel. The resulting index of refraction of the illuminated region of the channel resembles to that of graded index fibres that produces tight focusing of the laser beam. The reduction in transverse dimensions of the laser beam in turn leads to spread in transverse momentum of its photons. This transverse momentum spread then modifies the axial phase of the laser beam. Following Hamiltonian formulation, a set of coupled differential equations for the evolution of spot size and axial phase of the laser beam has been obtained. The equations so obtained have been solved numerically so see the effect of laser and channel parameters on the evolution of beam envelope.

Keywords

q-Gaussian, Virial theory, Self-Focusing, Gouy Phase, Plasma Channel.

Introduction

Self-phase modulation refers to change in the spatial frequency (axial phase) of a focused beam compared to a simple plane wave [1]. Due to SPM the spacing between the phase fonts of an optical beam increases due to reduction in local phase velocity. In nonlinear optical media (the media whose optical and dielectric properties are a function of intensity), SPM of an optical beam originates as a consequence of modification of its transverse dimensions. The intensity dependence of the index of refraction in turn makes it a function of transverse coordinates due to some amplitude structure over the cross section of the laser beam. For lasers operating in TEM_{00} mode this amplitude structure is generally bell shaped having maximum amplitude at the axis of the beam. The resulting profile of the index of refraction of the medium resembles to that of a graded index fiber resulting in tight focusing of the laser beam. As converging beams going through the focus have finite spatial extent in the transverse plane, the uncertainty relation then induces some distribution over the transverse and consequently longitudinal wave vectors. The net effect of this distribution over wave vectors is modulation of the axial phase of the laser beam.

Over past few decades self-modulation of axial phase of the axial phase of laser beams in nonlinear media has been drawing attention of the researchers due to its relevance in a number of applications and physical problems. In wave optics it explains the phase shift obtained by the secondary wavelets emerging from primary wave front. In the working of lasers, it decides the resonant frequencies of various transverse modes in laser cavity. Applied physics problems also rely on SPM. A potential example is optical trapping of particles where it produces lateral trapping force [2] and also provides a mechanism for the tracking of trapped particles [3, 4]. Moreover, a number of schemes for higher harmonic generation [5,6] of optical beams use the concept of longitudinal phase shift to meet the phase matching condition.

Laser beams differing in intensity profile behave differently in nonlinear media. However, from literature review it has been found that most of the earlier investigations on SPM of laser beams basec on the fact that irradiance over the cross section of a laser beam emitted from a laser operating in its fundamental mode i.e., $TEM_{00}$is having ideal Gaussian profile. However, in actual the irradiance over the cross section of a laser beam differs from ideal Gaussian profile[7] due to cavity imperfections. The actual irradiance over the cross section of the beam has been modelled by Nakattsutsumi et al[8] through Tsalli's [9] q-Gaussian distribution. The aim of this paper is to give first theoretical investigation on SPM of q-Gaussian laser beams in preformed plasma channels.

Mathematical formulation

The model equation governing the evolution of an optical beam through a plasma channel whose equilibrium electron density increases quadretically with radial distance from the axis of laser beam is

$$\mathbf{2ik_0\frac{\partial A}{\partial z}=\nabla_\perp^2 A+\frac{1}{c^2}[\omega_{p0}^2 e^{-\beta AA^*|_{r=0}}-\left(\omega_{p0}^2+\frac{r^2}{r_{ch}^2}\omega_{pch}^2\right)e^{-\beta AA^*}]A} \quad (1)$$

with, $\omega_{p0}^2=\frac{4\pi e^2}{m}n_0$ and $\omega_{pch}^2=\frac{4\pi e^2}{m}\Delta n$. Here, n_0 is the axial electron density of the channel, Δn is the difference in the electron density at the edge of the channel and that at the axis and r_{ch} if the radius of the plasma channel. The factor $e^{-\beta AA^*}$is associated with the ponderomotive force experienced by the plasma electrons due to intensity gradient of the laser beam. Hence, the coefficient β is termed as coefficient of ponderomotive nonlinearity.

Due to its nonlinear nature eq. (1) does not possess any exact analytical solution. Hence, in order to have physical insight into the propagation dynamics of the laser beam we have used a semi analytical method known as Hamiltonian formulation. According to Virial theory evolution of a laser beam through a nonlinear medium is a variational problem characterized by the Hamiltonian

$$H=\int \mathscr{H} r dr$$

with

$$\mathscr{H}=\frac{1}{2k_0^2}(|\nabla_\perp A|^2 \quad -\frac{1}{c^2}\int[\omega_{p0}^2 e^{-\beta AA^*|_{r=0}}-\left(\omega_{p0}^2+\frac{r^2}{r_{ch}^2}\omega_{pch}^2\right)e^{-\beta AA^*}]d(AA^*))$$

The basic idea of Hamiltonian formulation is then the selection of a trial function containing the physical parameters of interest. This trial function characterizes the actual solution of the problem as close as possible. The Hamiltonian formulation then recasts the original problem of solving a PDE into that of solving a set of ODEs governing the evolution of these parameters. In the present analysis we assume A(r; z) takes the form of the function given by[10]

$$A(r,z)=\frac{E_{00}}{f}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{-\frac{q}{2}}e^{i\theta} \quad (2)$$

where, the parameter *f*(z) can be referred to as dimensionless beam width parameter. The phenomenological parameter q is related to the deviation of irradiance over the beam cross section from ideal Gaussian profile and hence, it is termed as deviation parameter. The function $\theta(z)$is known as axial phase of the laser beam.

By using Hamilton's equations of motion by treating (f, $\frac{df}{dz},\theta,\frac{d\theta}{dz}$) as generalized coordinates we get following set of coupled equations for beam width and axial phase of the laser beam

$$\frac{d^2f}{d\xi^2}=\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3}-\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)J-\frac{1}{f}\left(\frac{df}{d\xi}\right)^2 \quad (3)$$

$$\frac{d\theta_p}{dz}=\frac{1}{f^2}\left(1-\frac{1}{q}\right)-\frac{1}{2f^2}\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}-\frac{1}{2}\left(1-\frac{1}{q}\right)I_4 \quad (4)$$

where

$$J=\frac{\beta E_{00}^2}{f^3}\left(\frac{\omega_{p0}^2r^2}{c^2}I_1+f^2\frac{\omega_{pch}^2r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2}I_2\right)+\frac{\omega_{p0}^2r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2}f^2I_3$$

$$I_1=\int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1}e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}}dt$$

$$I_2=\int_0^\infty t^2\left(1+\frac{t}{q}\right)^{-2q-1}e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}}dt$$

$$I_3=\int_0^\infty t\left(1+\frac{t}{q}\right)^{-q}e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}}dt$$

$$I_4 = \int_0^\infty \left(1+\frac{1}{q}\right)\left\{\frac{\omega_{p0}^2 r_0^2}{c^2} e^{-\frac{\beta E_{00}^2}{f^2}} - \left[\frac{\omega_{p0}^2 r_0^2}{c^2} + \frac{\omega_{pch}^2 r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2} f^2 t\right] e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}}\right\} dt$$

$$t = \frac{r^2}{r_0^2 f^2}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

Results and Discussion

In solving eqs.(3) and (4) it has been assumed that while entering into the channel the laser beam is collimated and is having plane wave front. Mathematically these conditions are defined as f=1, $\frac{df}{d\xi}$ =$\theta = 0$ at ξ=0. In the present study eqs.(3) and (4) have been solved for a typical set of parameters: $\omega_0 = 1.78 \times 10^{14} rad\ sec^{-1}$, $r_0 = 15\mu m, n_0 = 1.5 \times 10^{18} cm^{-3}$,$\Delta n = (0, 10, 15) cm^{-3}$,$E_{00} = (3 \times 10^9, 6 \times 10^9, 9 \times 10^9) Vm^{-1}$ and q=(3,4, ∞). The corresponding evolutions of beam with and axial phase of the beam are shown in the form of graphs in figs1-3.

It can be seen that during the journy of the laser beam through the plasma channel, its axial decreases monotonically with distance, showing step like behaviour. This is due to the periodical self-focusing/defocusing of the laser beam. As the laser beam undergo self-focusing, its intensity increases and hence, the laser phase fronts start experiencing larger refractive indices. This results in decreased phase velocity of the phase fronts that leads to decreased spacing between the phase fronts. This fact can be explained in another way. The axial phase shift of the laser beam occurs due to the transverse momentum gained by the photons due to reduction in the volume of space available for their propagation. As the transverse spatial confinement of the laser beam occurs due to self-focusing, the photons gain additional transverse momentum (k_x; k_y) due to position momentum uncertainty $\Delta k_x \Delta x = constant$ and $\Delta k_y \Delta y = constant$. As the over all momentum of the laser beam is conserved, the increase in the transverse momentum reduces the longitudinal momentum. Thus, during the propagation of laser beam its longitudinal momentum reduces as the beam keep on focusing. This results in monotonic decrease in its axial phase.

Step like behaviour of the axial phase can also be seen from figs. (1)-(3). This indicates that while moving from one focal position to another the axial phase of the laser beam remains almost constant while at the position of its focus, the axial phase takes an abrupt jump. This behaviour of the axial phase can be explained through its manifestation to Berry's phase (the phase that adiabaticaly evolving systems acquire after a complete cycle in their parameter space). In case of axial phase of an optical beam the parameter that is cycled is their wave front curvature. During the focusing of the beam, the radius of curvature of its wave fronts decreases as the wave fronts become more and more convex due to self-focusing. Hence, the axial phase of the laser beam takes a jump the focus. As the laser beam gets maximum possible intensity, the nonlinear terms in beam width equation (eq.3) vanishes. Hence, the laser beam propagates as if it is propagating through vacuum i.e., it starts diverging and hence, the wave front curvature changes its sign i.e., it becomes convex from concave. Now till the wave fronts again become plane when the nonlinearity of the medium comes into picture, the axial phase of the beam remains almost constant. Hence, as the wave front curvature changes periodically, the axial phase of the laser beam shows step like behaviour with abrupt jumps at the focal positions.

It has been seen from fig. (1) that with the increase in the value of deviation parameter q there is a reduction in the rate of change of axial phase of the laser beam with distance. This is due to the one to one correspondence between the transverse extension of the laser beam and its axial phase. The axial phase of the laser beam varies as inverse of the effective cross sectional area of an optical beam. As with increasing q the focusing of the laser beam gets reduced, hence, the rate of change of axial phase also reduces with increase of deviation parameter q.

The curves in figs. (2)and(3) indicate that with increase in either of channel depth and laser intensity, the rate of decrease of axial phase increases. This is due to enhancement of focusing of the laser beam with increase in these parameters.

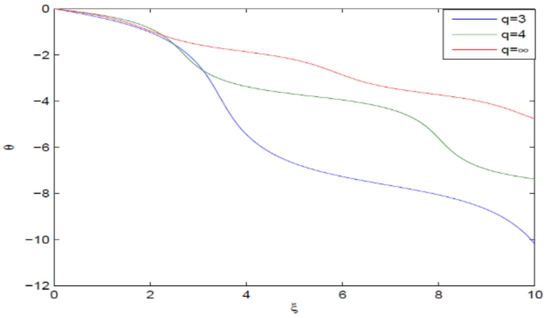

Figure 1. Effect of deviation parameter q on axial phase shift of laser beam.

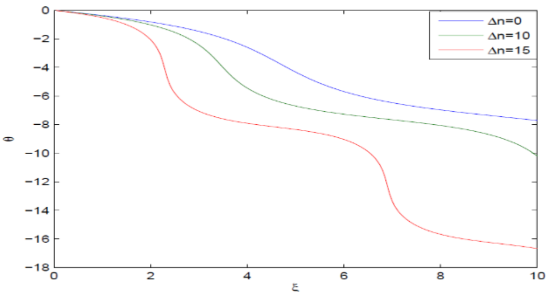

Figure 2. Effect of channel depth on axial phase shift of laser beam.

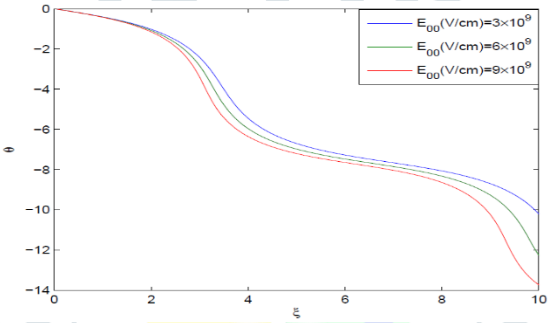

Figure 3. Effect of laser field amplitude on its axial phase shift.

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