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On r g-continuous functions in topological spaces

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Abstract. In this paper, we study r g-irresolute and r g-continuous functions through the idea of r g-closed sets along with the ideas related to the classes of r g-compact spaces and r g-connected spaces.

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Keywords: r g-closed set, continuous map, connectedness and com-pactness

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1 Introduction

Levine in 1970 developed the concept of g-closed sets , de ned as a set X (X;) is g-closed if cl(X) is contained into each open superset of X [10]. The thought has been used widely as of late by numerous topologist since g-closed sets remain not just characteristic speculation of closed sets. They likewise proposed a few fundamental characteristics of topological spaces. The investigation of g-closed sets would give the conceivable usage in PC illustrations [[7]- [9]] and their properties had been observed to be valuable in software engineering and digital topology (see [[6]-[9]], for instance). Due to speculations of pre closed sets, gp- closed were presented and examined in [14]. The same creators [11] utilized gp-closed sets to get a few portrayals of prenormal spaces in [12]. This thought was further contemplated in the work of [3],[1], [11] and [13]. Further [15] characterized and concentrated upon the idea of gpr-closed sets using gp-closed sets and presented the ideas of preregular $T_{\frac{1}{2}}$ - space and rgp-continuity. Authors

[5] have proceeded with the investigation of characteristics of gpr-closed sets and gpr-continuous functions. As of late, [4] characterized the idea of gp-closed sets and utilized this thought to get hypothesis for quasi normal spaces. All the more as of late, Bhardwaj et. al. [2] had presented and examined the idea of r g-closed sets as well as ^g -closed sets. In this paper, we will proceed with the investigation of r ^g-closed sets, along with presenting and describing r ^g-continuous and r ^g-irresolute functions. We also presented the ideas of r g-compactness and r g-connectedness, and investigated their conduct under r g-continuous functions.

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2 De nition of Regular - Generalized Weakly Closed Function and its Properties

In this section, we consider the functions by involving r ^g-closed sets and present

a new class of regular ^-generalized weakly (brie y r ^g-continuous) continuous

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mapping, concept of quasi r g-open functions and discuss their characterization
and basic properties.

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De nition 1. A function : (A;) ! (B;) known as regular -generalized

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closed (in brief r g- closed) map if closed set F of space (A;), (F) is regular
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-generalized closed set in space (B;).

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Theorem 1. Each r-closed map is a r -generalized closed map.

Proof. Consider : (A;) ! (B;) is r-closed map i.e. (X) is r-closed set for

each closed set X of A. Since each r-closed set is r ^g-closed set so (X) also

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satis es the de nition of r g- closed set. Thus we have (X) is r g-closed set for

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each closed set X of A. Hence is r g-closed. The opposite of this hypothesis does not remain constant as appeared by taking after case.

Example 1. Let us consider A = fl; m; n; og and topologies = fA; ; fl; mg; fn; ogg,
f f g f g f g f gg ^
= A; ; l ; m ; l; m ; l; m; n . Here we have collections of r g-closed sets

in (A;) and (A;) are fA; ; fl; mg; fn; ogg and f ; A; fng; fog; fn; og; fl; n; og; fm; n; ogg and collection of (A;) r-closed sets is fA; ; fl; mg; fn; ogg. De ne : (A;) ! (B;) by (l) = n, (m) = n, (n) = l, (o) = m. Now (fl; mg) = n where
fng is r ^g-closed set but fng is not r-closed. Thus is r ^g-closed map but not r-closed map.

Theorem 2. If : A ! B is a closed map and : B ! C , a r ^g-closed map
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then : A C is r g-closed
! ! ^
Proof. Consider : A B be closed and: B C be r g-closed. To show

: A ! C is r ^g-closed. Suppose a closed set H from space A. Then by
de nition of closed map (H) is closed and (H) = ((H)) is r ^g-closed as

is r ^g-closed. Thus, is r ^g-closed.

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3 r g-continuous and r g-irresolute functions

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De nition 2. A function :(A;)! (B;) known as r g-continuous if
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(V) is r g-closed in space (A;) for each closed set in space (B;).

Example 2. Let A = fl; mg, = fA; ; flg; fmgg also B = fl; m; ng, =

f ; A; fl; mgg de ne : (A;) ! (B;) with (l) = l and (m) = n. Since
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every subset of (A;) is r g-closed thus is r g-continuous.

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De nition 3. A function : (A;) ! (B;) known as r g-irresolute if (V)
^ ^

is r g-closed within space (A;) 8 r g-closed set V in (B;).

Example 3. Let A = fl; m; ng, = f ; A; fl; mgg and = f ; A; fl; mg; fngg de ne : (A;) ! (A;) by (l) = n, (m) = m and (n) = l. At that point
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inverse image of each r g-closed set is r g-closed under . Therefore, be r g-irresolute.

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Theorem 3. Every r g-irresolute function be r g-continuous.

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Proof. Let : (A;) ! (B;) be rgw-irresolute function, i.e. (V) be r g-
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closed in (A;) for each r g-closed set V in (B;). Now since, each closed set
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be r g-closed set. Thus (V) be r g-closed in (A;) for each closed set V in (B;).

Theorem 4. If : (A;) ! (B;) and : (B;) ! (C;) are two functions then
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(i) o be r g-continuous, if is continuous and be r g-continuous.
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(ii) o be r g-irresolute, if be r g-irresolute and be r g-irresolute.
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(iii) o be r g-continuous, if be r g-continuous and be r g-irresolute.

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Proof. (i) Consider and be r g-continuous and continuous respectively. Let V be closed in (Z;), then by de nition of continuity 1(V) is closed in (B;)
^ 1 1 ^
and r g-continuity of implies ((T)) is r g-closed in (A;). That is
1 ^ ^
o (V) is r g-closed in (A;) where V is closed in (C;). Hence, o is r g-continuous.

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(ii) Let and as de ned above be r g-irresolute. Let V be r g-closed in (C;),
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then by de nition of r g-irresolute function (V) is r g-closed in (B;). As
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is r g-irresolutes ((V)) is r g-closed in (A;). That is (o) (V) is
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r g-irresolute in (A;) where V is r g-irresolute in (C;). Therefore, o is
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r g-irresolute.

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(iii) Let and as de ned above be r g-irresolute and r g-continuous respec-
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tively. Let V be closed in (C;), then by de nition of r g-continuity (V) is
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r g-closed in (B;). As is r g-irresolute so ((V)) is r g-closed in (A;).
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That is (o) (V) is r g-continuous where V is closed in (C;). Therefore, o
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is r g-continuous.

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4 Quasi r g-open functions

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De nition 4. The map : (A;) (B;) known as r g-open map if the image
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(X) is r g-open in (B;) for all open set X in (A;).
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De nition 5. A function : (A;) (B;) known as Quasi r g-open if the
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image of all r g-open set in A be open in B.

Example 4. Let A = fl; m; ng, = f ; A; flg; fmg; fl; mgg and = f ; A; fn g; fl; mgg de ne :
(A;) ! (B;) by (l) = n, (m) = n and (n) = n. We have the col-
^ f f g f g f g f g f g f gg lection of r g-open set in (A;) are ; A; l ; m ; n ;
l; m ; l; n ; m; n .

Now we see the image of each r g-open subset maps into open set of (B;).

Finally, by de nition 5 is Quasi r g -open.

Theorem 5. A function : (A;) ! (B;) known as Quasi r g-open i for

all subset U of A, (r g int(U)) int((U)).

Proof. Let be the Quasi r g-open function and U be a subset of A. Now,

we have int(U) U and r g int(U) be a r g-open set. Thus we get that

(r g int(U)) (U). As (r g int(U)) is open (r g int(U)) int((U)).

On the other hand, expect that U be a r g-open set in A then (U) = (r g int(U)) int((U)) but int((U)) (U). Consequently, (U) = int((U)) and

hence is Quasi r g-open.

Theorem 6. If a function : (A;) ! (B;) be Quasi r g-open then r g int(1 (G)) 1 (int(G)) for all subset G of B.

Proof. Consider G to be any arbitrary subset of B. At that point, r g int(1 (G))

is a r g-open set in A and B be a Quasi r g-open, then, (r g int(1 (G)))

int((1 (G))) int(G). Thus, r g int(1 (G)) 1 (int(G)). Review that

a subset S is known as a r g-neighbourhood of a point a of A, if 9 a r g-open

set U s.t. a 2 U S.

De nition 6. A map : (A;) ! (B;) is known as a r g-closed map if the

image (A) is r g-closed in (B;) for each closed set A in (A;).

De nition 7. A function : A ! B is known as Quasi r g-closed if the image

of each r g-closed set in A is closed in B. i.e. each Quasi r g-closed function

is closed as well as r g-closed.

De nition 8. A function : (A;) ! (B;) is known as r ^g -closed if 8 r ^g-closed set F of A, (F) be r g-closed in B.

Example 5. Let A = B = fl; m; ng, = f ; A; fl; mgg and = f ; B; fng; fl; mgg. De ne a function : (A;) ! (B;) by (l) = m, (m) = n and (n) =

l. Here, we have the collections of r g-closed sets in (A;) and (B;) are f ; A; flg; fmg; fng; fl; mg; fl; ng; fm; ngg and f ; B; flg; fmg; fng; fl; mg; fl; ng; fm; ngg

respectively. Now, for each r g-closed set in (A;) function maps into r g-

closed set in (B;). Therefore, is r ^g -closed.

Theorem 7. If : (A;) ! (B;) and : (B;) ! (C;) are two functions. Then

(i) o is closed if is r g-closed and is Quasi r g-closed.
(ii) o is r ^g -closed if be Quasi r ^g-closed and is r ^g-closed.

(iii) o is Quasi r ^g-closed if is r ^g -closed and is Quasi r ^g-closed.

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Proof. (i) Here be r g-closed and be Quasi r g-closed. To prove o be

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closed. Let F be arbitrary closed in (A;). If, be r g-closed so (F) be r g-

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closed in (B;). Also be Quasi r g-closed so ((F)) is closed in (C;). i.e.

o (F) is closed in (C;) where F be closed set in (A;). Thus, o is closed.

^ ^ ^ (ii) Here be Quasi r g-closed and be r g-closed. To prove o be r g -

closed. Let F be arbitrary r ^g-closed in (A;). If, be Quasi r ^g-closed so (F)

be closed in (B;) also, is r ^g-closed so ((F)) is r ^g-closed in (C;). i.e.

o (F) is r ^g-closed in (C;) where F be r ^g-closed set in (A;). Thus o is

r ^g -closed.

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(iii) Here be r g -closed and be Quasi r g-closed. To prove o is Quasi

r ^g-closed. Let F be arbitrary r ^g-closed in (A;). Since is r ^g -closed so

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(F) is r g-closed in (B;) also is Quasi r g-closed so ((F)) is closed in

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(C;). i.e. o (F) is closed in (C;) where F be r g-closed set in (A;). Thus,

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o is Quasi r g-closed.

Theorem 8. If : (A;) ! (B;) and : (B;) ! (C;) are two functions

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such that o : (A;) ! (C;) be Quasi r g-closed. Then

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(i) If be r g-irresolute surjective, then be Quasi r g-closed.

(ii) If be r ^g-continuous injective, then be r ^g -closed.

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Proof. (i) Here : (A;) ! (B;) and o : (A;) ! (C;) are r g-irresolute

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surjective and Quasi r g-closed respectively. To show is Quasi r g-closed. Let

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F be r g-closed in (B;) as is r g-irresolute, so (F) is r g-closed in (A;).

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As o is Quasi r g-closed and is surjective. Thus, o ((F)) = (F) and

^
closed in (C;), that is, (F) be closed in (C;) where F is r g-closed in (A;).

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Therefore, is Quasi r g-closed.

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(ii) Here : (B;) ! (C;) and o : (A;) ! (C;) are r g-continuous

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injective and Quasi r g-closed respectively. To show is r g -closed. Let F be

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r g-closed in (A;) as o is Quasi r g-closed, so o (F) be closed in (C;).

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Again be r g-continuous and injective function. Thus, (o (F)) = (F),

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which is r g-closed in (B;), that is, (F) is r g-closed in (B;), where F is

r ^g-closed in (A;). Thus is r ^g -closed.

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5 On Regular generalized (r g)-Compactness in Topological Space

In this section we extend the concept of open cover and compactness in the form

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of r g-closed sets to introduce r g-open cover and r g-compactness and discuss their properties and characterization.

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De nition 9. The collection fG$_j$: j 2 g of r g-open sets in a topological
^ S$_f$ 2 g
space A is known as r g-open cover of a subset S is S G$_j$: j holds.
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De nition 10. The topological space (A;) is known as r g-compact if r g-open cover of A has a nite sub cover.

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De nition 11. A subset S of a topological space (A;) is known as r g-Compact
8 f 2 g ^
relative to A if collection G$_j$: j of r g-open subsets of A such that
S S
S fG$_j$: j 2 g 9 nite subset $_0$ of such that S fG$_j$: j 2 $_0$g.

De nition 12. A subset S of a topological space A is known as $\hat{r}$g-Compact if S is $\hat{r}$g-Compact as a subspace of A.

Theorem 9. Let : (A;) ! (B;) is a surjective and $\hat{r}$g-continuous map. If A is $\hat{r}$g-compact, then B is compact.

Proof. Here, we have : (A;) ! (B;) a surjective and $\hat{r}$g-continuous map and A be $\hat{r}$g-compact. To prove B be compact. Let fGj : j 2 g is an open cover of B. Then f ^{1}(Gj) : j 2 g be a $\hat{r}$g-open cover of space A. As, A is $\hat{r}$g-compact, it has a nite sub cover, say f ^{1}(G1); ^{1}(G2); ^{1}(G3);:::; ^{1}(Gn)g. Surjectiveness of implies fG1; G2; G3; : : : ; Gng is an open cover of B. Therefore, B be compact.

Theorem 10. For a map : (A;) ! (B;) is $\hat{r}$g-irresolute and a subset S of A is $\hat{r}$g-compact relative to B, then the image (S) is $\hat{r}$g-compact relative to B.

Proof. Here, we have : (A;) ! (B;) is $\hat{r}$g-irresolute also S subset of A is $\hat{r}$g-compact relative to A. To prove (S) is $\hat{r}$g-compact relative to B. Let fGj : j 2 g be a collection of $\hat{r}$g-open sets in B such that S fGj : j 2 g. Then S f ^{1}(Gj) : j 2 g where ^{1}(Gi) is $\hat{r}$g-open in A for each j. Since S is $\hat{r}$g-compact relative to A, there exist a nite subcollection fG1; G2; G3; : : : ; Gng such that S f ^{1})(Gi) : i 2 g i.e. (S) fGj : j 2 g. Hence, (S) is $\hat{r}$g-compact relative to B.

Theorem 11. A $\hat{r}$g-closed subset of a $\hat{r}$g-compact space A is $\hat{r}$g-compact relative to A.

Proof. Consider M is a $\hat{r}$g-closed subset of a $\hat{r}$g-compact space A. Therefore, AnM be $\hat{r}$g-open in A. Now to prove M is $\hat{r}$g-compact relative to A. Let be a $\hat{r}$g-open cover for M. Then f ; AnMg be a $\hat{r}$g-open cover for A. As, A be $\hat{r}$g-compact, by de nition 12, X has a nite subcover, say fA1; A2; A3; : : : Ang = 1. If AnM does not belong, then 1 n(AnM) is a subcover of M. Thus by de nition 12 M is $\hat{r}$g-compact relative to A.

6 $\hat{r}$g-Closure

De nition 13. For a subset X of (A;), we de ne the r g-closure of X as follows:

r ^g cl(X) = T fF : F isar ^g closedinA; X F g ^
Theorem 12. Let X be a subset of (A;) and a 2 A. Then a 2 r gcl(X) i
^
V T X 6= 8 r g open ^
set V containing a.
X A n V , r g cl(X) A n V and then a 2= r g cl(X), T
Proof. Let there be a r g-open set V containing a so that V X = . Since
^ ^
a contradiction.

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Conversely, let a 2= r g cl(X); then 9 a r g-closed set F containing X s.t
^
a 2= F . Since a 2 A n F and A n F is r g-open, (A n F) X = , which is a

contradiction. T
Theorem 13. Let X and Y be subsets of (A;). Then
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(a)r g cl( ) = and r g cl(A) = A
^ ^
(b)If X Y , then r g cl(X) r g cl(Y)
^ ^ ^
(c)r g cl(X) = r g cl(r g cl(X))
^ ^ ^
(e)r ^g cl(X S Y) r ^g cl(X) S r ^g cl(Y)
(d)r g cl(X Y) r g cl(X) r g cl(Y)

T T ^ ^
A,F g = . Hence, proved. Tf
Proof. (a) By using the de nition 13 r g cl( ) = F : F is a r g-closed in
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r g -closed in A, Y F g. As X Y implies X F where F is a rT
(b) Here X Y and by using de nition 13 we have r g cl(Y) = fF : F is a
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g-closed in
^ ^
A. i.e. r g cl(X) r g cl(Y). Hence, proved.
^ ^
in A, X F g, So, r ^g cl(r ^g cl(X)) = r ^g cl(T fF : F is a r ^g-closed
(c) By using the de nition 13, we have r g cl(X) = fF : F is a r g-closed

in A, X F g). Hence, proved. T
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is a r g-closed S T gg f f S T gg
(d) r g cl(X Y) = fF : F is a r g-closed in A, X Y F g ffF : F
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r g cl(X) r g cl(Y). S T
in A, X F F : F is a r g-closed in A, Y F =
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Hence, proved
to part (d).

(e) Similar S

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