Analytical study of wave propagation in micro polar elastic medium

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Abstract

In present investigation, propagation of wave in micropolar elastic medium at non-free surface is discussed. The amplitude ratio's of longitudinal displacement wave (LD wave), coupled transverse displacement (CD-I) and transverse rotational wave (CD-II) are obtained for incident waves.

Key Words: Micropolar elastic medium, non-free surface, wave propagation, amplitude ratio.

1 Introduction

Classical theories of elasticity are not able to examine the behaviour of materials having brous or course grain structure etc. When the microstructure of the material was considered to be rigid, it leads to the micropolar theory. This theory is more dependable for geological materials like solis and rocks as it accounts the intrinsic rotation and estimates the inner structure of the material. Eringen [1] introduced a new formulation of equations in thermoelasticity which was known as the equations for the micropolar elastic theory. Sharma [2] investigated impact of relaxation times and two temperatureson coe cients of re ection in a half-space of micropolar thermoelastic solid.

Fu and Wei [3] investigated the transmission and re ection problem at the im-perfect interface of the coupled transverse displacement and transverse rotational waves between two dissimilar micropolar solids. They discussed the impact of imperfect degree of interface on the transmission and the re ection coe cients. Khurana and Tomar [4] observed propagation of plane waves (two longitudinal waves and two sets of coupled transverse waves) for an nonlocal isotropic microp-olar solid and derived re ection coe cients and energy ratios when these waves incidents at stress-free boundary. Singh et.al [5] considered problem on Rayleigh wave for an rotating half-space in an orthotropic micropolar material and solved equations for the surface wave in the half space. They obtained the results to show the in uence of orthotropy, rotation and nondimensional frequency of the Rayleigh wave.

Zhang et.al [6] calculated the amplitude ratios of re ected waves for di erent incident waves and also, re ection coe cients in terms of energy ux ratios at non-free surface of a micropolar elastic half-space. Hassanpour and Heppler [7] reviewed the linear isotropic theory of micropolar elasticity with special attention on the notation, which are used for the representation in the micropolar elastic moduli and the experimental actions are taken to measure them. Videla and Atroshchenko [8] derived the analytical solution subjected to a remote uni-axial tension for the problem of a circular micropolar inhomogeneity in an in nite micropolar plate in homogeneous imperfect interface. They showed dependence of stress concentration factors on the micropolar material constants .

Gade and Ragunath [9] explored reduced micropolar theory to replicate ground motion during an earthquake. They calculated the expressions of ground displace-ment and rotational motions analytically for the case of buried seismic source. Singh [10] investigated a problem on Rayleigh surface wave in an isotropic mi-cropolar elastic solid half-space with impedance boundary conditions and derived a secular equation for non dimensional speed of the Rayleigh wave, which depends upon various parameters of material, frequency, micro-rotation and impedance parameters. Fan and Cheng [11] presented a elastic model set based on microme-chanics in the framework of micropolar theory having two-phase FGMs to study the impact of size on the e ective properties of the FGM and compared those results with experimental data.

2 Field Equations

Following Eringen [1], the basic equations and constitutive relations in micropolar elastic medium are:

(+k+2) (u)+k(!) (k+)(u) = @!u/@t 2 ; (1)

r r r r r

(++) (!) (u) 2! k +k(u) = j @!/@t 2 ; (2)

t_{ij} = $u_{r;r\ ij}$ + $(u_{j;i} + u_{i;j})$ + k(ijm m + $u_{j;i}$) (3)

m_{ij} = $r;r\ ij$ + $j;i$ + $i;j$ (4)

, - Lame’s constants, t-time, = (2 +k+3) t, t- coe cient of linear thermal expansion, C , - speci c heat and density, t_{ij} -components of stress tensor, ij- Kronecker delta, u - displacement vector, ,k, and are micropolar constants,

m_{ij} - couple stress tensor components, $_{ijm}$ is alternating tensor, $_{k}$ is microrotation vectors,.

$$r = \hat{i}\frac{@}{@x} + \hat{j}\frac{@}{@y} + \hat{k}\frac{@}{@z}\ ;\ r^2 = \frac{@^2}{@x^2} + \frac{@^2}{@y^2} + \frac{@^2}{@z^2}:$$

3 Formulation and the solution

We have taken a homogeneous, isotropic with micropolar in elastic half space on non free surface. The rectangular cartesian co-ordinate system (x_1, x_2, x_3) having origin at interface x_3 = 0 is considered along with x_3-axis pointing normally into medium. Plane waves in x_1,x_3-plane are considered in which wave front is parallel to x_2-axis, therfore all variables will depend on x_3,x_1 and t. Thus problem considered in two dimensional, so we take

$$\overset{!}{\ } = (0;\ _2;\ 0);\quad \overset{!}{u} = (u_1;\ 0;\ u_3) \tag{5}$$

To ease the solution, quantities having no dimesions are introduced as follows:

$$x_1^0 = \frac{!_1}{c_1}x_1;\ x_3^0 = \frac{!_1}{c_1}x_3;\ u_1^0 = \frac{!_1}{c_1}u_1;\ u_3^0 = \frac{!_1}{c_1}u_3;$$

$$t_{33}^0 = \frac{1}{}t_{33};\ t_{31}^0 = \frac{1}{}t_{31};\ {}_2^0 = \frac{2}{k}c_1^2\ ;\ m_{ij}^0 = \frac{m_{ij}!}{c_1}\ : \tag{6}$$

where

$$c_1^2 = \frac{2 + + k}{}\quad \text{and}\quad !_1 = \frac{k}{j}:$$

The expression related to components of displacement are expressed by using Helmholtz decomposition, therefore u_3 and u_1 are related to the and (scalar potential functions) having no dimensions are given by

$$u_1 = \frac{@}{@x_1} \frac{@}{@x_3}\ ;\ u_3 = \frac{@}{@x_3} + \frac{@}{@x_1}\ : \tag{7}$$

using equations (5)-(6) in (1)-(4) and assuming the motion to be harmonic and for solving the equations we assume solutions in the form

$$(\ ;\ ;\ _2) = (\ ^0;\ ^0;\ ^0_2)e^{(x_1 \sin\ \ x_3 \cos + t)};$$

where denoted as wave number, is known as iota, is angle of inclination and quanties such as ^{0}; ^{0}; $^0_2$ are arbitrary constants. Using the values of ; ; $_2$ we obtained following equations

$$(^4+A ^2+B)(_2;)=0 \quad (8)$$

$$(^2 \ 1) =0 \quad (9)$$

where (= !) represents the velocity of various waves; $_1$; $_2$; $_3$ are velocities of the longitudinal displacement (LD) wave, coupled transverse displacement (CD-I) wave and tansverse rotational (CD-II) wave respectivily and

$$a_1 = \frac{(+)}{_c \ ^2_1}; \ a_2 = \frac{+ k}{c \ ^2_1}; \ a_3 = \frac{k^2}{^2 c^4_1}; \ a_4 = \frac{}{^2_{c j} 1}; \ a_5 = \frac{c^2}{^2_{!j} 1} ;$$

$$a_6 = \frac{2k}{!^2 j_1}; \ a_7 = - ; \ a_8 = \frac{+ 2 + k}{}; \ a_9 = \frac{+ k}{}; \ a_{11} = \frac{k^2}{c^2_1} ;$$

$$a_{12} = \frac{!^2 k}{c_1^4}; \ A = \frac{a_2 a_6 \quad a_4 !^2 \quad a_3 a_5 \quad a_2 !^2}{!^2 \ a_6}; \ B = \frac{a_2 a_4 !^2}{!^2 \ a_6}; \ _3^2 = 1:$$

4 Boundary conditions

Appropriates conditions at surface x_3=0 are

$$\text{(i)} \ t_{33} = \ S_1 u_3; \quad (10)$$

$$\text{(ii)} \ t_{31} = \ S_2 u_1; \quad (11)$$

$$\text{(iii)} \ m_{32} = \ S_3 \ _2 \quad (12)$$

We assume that the values of , , $_2$,

$$= A_0 e^{k_0(x_1 \sin_0 \ x_3 \cos_0) + !t} + A_1 e^{k_3(x_1 \sin_3 \ x_3 \cos_3) + !t} \quad (13)$$

$$= \sum B_{0i} e^{k_0(x_1 \sin_0 \ x_3 \cos_0) + !t} + B_i e^{k_i(x_1 \sin_i \ x_3 \cos_i) + !t} \quad (14)$$

$$_2 = \sum d_i B_{0i} e^{k_0(x_1 \sin_0 \ x_3 \cos_0) + !t} + d_i B_i e^{k_i(x_1 \sin_i \ x_3 \cos_i) + !t} \quad (15)$$

where

$$d_i = \frac{a_2 k_i^2 \quad !^2}{a_3} ; \qquad (i = 1; 2)$$

where the values of d_i are coupling constants. B_{0i} are the amplitude of incident coupled transverse displacement (CD-I) and transverse ratational wave (CD-II)

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and A_0 is the amplitude of the incident L-D wave (Longitudinal Displacement wave). B_i are the amplitude of the re ected coupled waves i.e transverse rotational and transverse displacement wave and A_1 is the amplitude of the re ected L-D wave (Longitudinal Displacement wave). Using Snell's Law de ned as follows

$$\frac{\sin_0}{0} = \frac{\sin_1}{1} = \frac{\sin_2}{2} = \frac{\sin_3}{3} \quad (16)$$

where

$$_{11} = {}_{22} = {}_{33} = !; \quad \text{at} \quad x_3 = 0; \quad (17)$$

> 1; for incident LDWave

0 = 8 2; for incident CD IWave

3; for incident CD IIWave

:

Taking the phase for the re ected waves, one can write the equations (16)-(17)

$$\frac{\cos_j}{} = 2 \; \frac{0}{} \; 2 \quad \sin^2 \; 0 \; 3_2 \; : \quad (18)$$

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Following Schoenberg [12], if we write

$$\frac{\cos_j}{j} = \frac{\cos_j^0}{{}^0_j} + \frac{c_j}{2_0} \quad (j = 1; 2; 3);$$

$$\frac{\cos_j^0}{j} = \frac{1}{} \mathrm{Re} \; 8 \; \frac{0}{} \; 2 \; \sin^2 \; 0 \; {}^{2} 9 \; ; \quad c_j = 2\, I_m \; \frac{0}{} \; 2 \; \sin^2 \; 0 \; 2 \; ;$$

where $_j^0$, called real phase speed and $_j^0$, known as re ection angle and are given by

$$\frac{}{j} \quad \frac{}{j} \quad \sin^2 \; 0 + \mathrm{Re} \quad 2 \,\#\, \frac{1}{2}$$

and c_j, is knowns as attenuation in a depth and equals to $(2\ _0)=!$ i.e.wavelength of incident wave

Making use of the equation (7) in the conditions given by (10)-(12) and with the use of equations given by (13)-(15), a homogenous system equations is obtained as follows

$$\sum a_{ij} Z_j = Y_j; \; (i; j = 1; 2; 3);$$

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$$a_{1p} = (a_8 \quad a_7)\ p(\frac{p}{0})^2 \ [(\frac{0}{p})^2 \quad \sin^2 \ 0]^{\frac{1}{2}} \ \sin 0 \quad pS_1 \frac{p}{0} \sin 0 \qquad ;$$

$$a_{13} = \frac{2}{3}(a_7(\frac{3}{0})^2 \sin^2 0 + a_8(\frac{3}{0})^2 [(\frac{0}{3})^2 \quad \sin^2 0 \quad]) + {}_3S_1[(\frac{0}{3})^2 \quad \sin^2 0]^{\frac{1}{2}} ;$$

$$a_{2p} = {}_p[a_9 \quad 2 (\frac{p}{0})^2 [(\frac{0}{p})^2 \quad \sin^2 0] \quad (\frac{p}{0})^2 \sin^2 0] \quad a_{11}\alpha_p \quad {}_pS_2[(\frac{0}{p})^2 \quad \sin^2 0]^{\frac{1}{2}} ;$$

$$a_{23} = {}_3[(a_9 + 1)(\frac{3}{0})^2 \quad 2 \sin 0 [(\frac{0}{3})^2 \quad \sin^2 0]^{\frac{1}{2}}] \quad S_2 \quad {}_3\frac{3}{0} \sin 0 \qquad ;$$

$$a_{3p} = \alpha_p \lfloor a_{12p} \frac{p}{0} [(\frac{0}{p})^2 \quad \sin^2 0]^{\frac{1}{2}} + S_3]; \qquad a_{33} = 0 \qquad \text{where}(p = 1; 2):$$

5 Conclusion

In this investigation amplitude ratios are calculated numerically for non-free sur-face in homogenous isotropic micropolar elastic medium. The amplitude ratios are calculated for incident LD-Wave and Coupled waves, namely coupled transverse rotational wave and transverse displacement wave. The results of the problem can be useful to researcher working in the eld of seismology.

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