Allocation of Nitrogen in Root and Shoot Compartments of a Plant Under the Effect of Toxicant

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Abstract: This article pertains to the distribution and allocation of nitrogen in the root and shoot compartments of a plant, where it is assumed that the plant is already under the effect of toxicant. The variables under consideration are: concentrations of nitrogen in root N_r, concentrations of nitrogen in shoot N_s and concentrations of toxic metal in soil M. The model is analysed by studying its boundedness, positivity and equilibrium points. The analytical results show the adverse effect of toxicity on concentrations of nitrogen in both the compartments. Numerical simulation is used to support these analytical results.

Introduction

Nutrient influence an individual plant growth, thereby affecting the plant population dynamics. It is evident that individual plants and plant population are under different types of stresses, such as low nutrient availability, acidification causing lowering of pH, temperature fluctuations, drought, and presence of many salts, aluminium, heavy metals and other toxic chemicals in the soil. All these stresses affect plant growth and yield. In real world, the growth rates of an individual plant or plant population will often not respond immediately to change in its own population, but rather will do so after a time lag. The factors that introduce time delay may include toxic metals in soil, utilization coefficient and nutrient use efficiency in plant growth dynamics. Mathematical models are a true replica of the amalgamation of environmental and ecological information. In mathematical model of natural phenomenon, it is not possible to consider all uncountable variables and factors, but only the pertinent information is considered to have a logical understanding of the nature of the dynamics. Boros and Micle [1] showed the effect of copper on seed germination and growth of sunflower. Cu [2] showed that growth of the crop decreases if it is cultivated in a contaminated field. Geng et al [3] studied the arsenic sensitivity in relation to phosphorus. Kalra and Kumar [4,5,6,7] studied the impact of delay parameter in plant growth dynamics effected by toxicant using different models. Liao et al [8] studied root distribution and elemental accumulation of Chinese brake from arsenic contaminated soils where forty hectares of arsenic polluted land was irrigated by local farmers. The decrease in plant biomass and oscillatory behaviour for a value of delay under the effect of toxicant was shown by Naresh et al [9]. Pavel et al [10] used Lepidium sativum as a test plant to assess the phytotoxic effects of chromium and cadmium. It was found that root development and dry biomass were adversely affected by toxic stress. Pigna et al [11] reported that applying phosphorus fertilizers enhances the arsenic level in soil. Ruan [12] gave a detailed analysis of various kinds of stabilities like Absolute stability, conditional stability and bifurcation in predator-prey system with discrete delays. Ruan and Wei [13] analysed the nature of zeros of exponential characteristic equation. The stability analysis of equilibrium points involving a non-linear system of delay differential equations is carried out by Ruan [14]. The extent of plant injury by elevated zinc concentration was assessed by Tsonev and Lidon [15], considering its specific and strong dependence on the environmental conditions and availability of other heavy metals. Tu and Ma [16] showed that natural formation and anthropogenic activities are the main reasons of entrance of arsenic into terrestrial and aquatic environment.

Proposed Model

The model is divided into two compartments- root and shoot for study of allocation of nitrogen in plants under the effect of toxic metal. Three state variables are concentrations of nitrogen in root N_r, concentrations of nitrogen in shoot $N_s$and concentrations of toxic metal in soil M. Let the model governing this plant soil dynamics is given by:

$$\frac{dN_s}{dt} = \frac{\beta}{R_N}(N_r - N_s) - \Delta_1 N_s \qquad (1)$$

$$\frac{dN_r}{dt} = (K_N - \alpha_1 M) - \frac{\beta}{R_N}(N_r - N_s) - \Delta_2 N_r \qquad (2)$$

$$\frac{dM}{dt} = I - \alpha M N_r - \Delta M \qquad (3)$$

With initial conditions: $N_s(0) > 0, N_r(0) > 0,\ M(0) > 0$.

The system parameters are defined as:

R_N is resistance to nitrogen transport, β is scaling factor which is included to allow for the way in which resistance change with plant size. $\Delta_1$is natural decay of N_S and Δ_2 is natural decay ofN_R. K_N is the uptake rate of nitrogen by root. $I$is the input rate of heavy metals.Δ is the first order decay rate of M. α is depletion rate of M due to interaction between M and N_R. $(K_N - \alpha_1 M)$ is the uptake rate of plant inhibited due to presence of toxic metal in root compartment of plant. Here all the parameters$R_N, \beta, \Delta_1, \Delta_2, \Delta, K_N, \alpha, \alpha_1$ and I are taken as positive.

Analysis of the Model

Boundedness:

Consider the following function:$W(t) = N_s(t) + N_r(t)$

$$\frac{dW(t)}{dt} = \frac{d}{dt}[N_s(t) + N_r(t)]$$

Using Equations (1) -(2) and $\varphi = \min(\Delta_1, \Delta_2)$, we get

$$\frac{dW(t)}{dt} \leq K_N - \varphi W(t).$$

Applying the comparison theorem, we get as$t \to \infty$:

Therefore: $W(t) \leq \frac{K_N}{\varphi}$

$N_s(t) + N_r(t) \leq \frac{K_N}{\varphi}$.

Also $W(t) \geq 0$

So, $0 \leq N_s(t) + N_r(t) \leq \frac{K_N}{\varphi}$

From equation (3): $\frac{dM}{dt} = I - \alpha M N_r - \Delta M$

$\frac{dM}{dt} \leq I - \Delta M$, then by usual comparison theorem, we get when $t \to \infty$:

$M \leq \frac{I}{\Delta} = M_u$

Again, from equation (3), we get $\frac{dM}{dt} = I - \alpha M N_r - \Delta M$

$\frac{dM}{dt} \geq I - \alpha M \frac{K_N}{\varphi} - \Delta M$

$\frac{dM}{dt} \geq I - \vartheta_3 M$ Where $\vartheta_3 = \left(\frac{\alpha K_N}{\varphi} + \Delta\right)$

By usual comparison theorem, when $t \to \infty$: $M \geq \frac{I}{\vartheta_3} = M_l$ So $M_l \leq M \leq M_u$.

Thus, the model has all its solution lying in the region $C = \left[(N_s, N_r, M) \in {R_+}^3 : 0 \leq \leq N_s + N_r \leq \frac{K_N}{\varphi}, M_l \leq M \leq M_u\right], as\ t \to \infty$, for all positive initial values $\{N_s(0), N_r(0), M(0)\} \in C \subset {R_+}^3$, where $\varphi = \min(\Delta_1, \Delta_2)$.

Positivity of Solutions:

From equation (1): $\frac{dN_s}{dt} = \frac{\beta}{R_N}(N_r - N_s) - \Delta_1 N_s$

$\frac{dN_s}{dt} \geq -\left(\frac{\beta}{R_N} + \Delta_1\right) N_s$, which means $N_s \geq c_1 e^{-\left(\frac{\beta}{R_N}+\Delta_1\right)t}$

From equation (2): $\frac{dN_r}{dt} = (K_N - \alpha_1 M) - \frac{\beta}{R_N}(N_r - N_s) - \Delta_2 N_r$

$\frac{dN_r}{dt} \geq -\left(\frac{\beta}{R_N} + \Delta_2\right) N_r$, which means $N_r \geq c_2 e^{-\left(\frac{\beta}{R_N}+\Delta_2\right)t}$

From equation (3): $\frac{dM}{dt} = I - \alpha M N_r - \Delta M$

$\frac{dM}{dt} \geq -\left(\frac{\alpha K_N + \Delta\emptyset}{\emptyset}\right) M$, which means $M \geq c_3 e^{-\left(\frac{\alpha K_N+\Delta\emptyset}{\emptyset}\right)t}$

Hence, $N_s(0) > 0, N_r(0) > 0, M(0) > 0\ \ for\ all\ t > 0$

Interior Equilibrium:

We calculate two equilibriums: uniform equilibrium E_1 and a feasible interior equilibrium E_2 of model. The system of equations (1) -(3) has uniform equilibrium $E_1(\overline{N_s} \neq 0, \overline{N_r} \neq 0, \overline{M} = 0)$:

Where $\overline{N_s} = \dfrac{K_N \frac{\beta}{R_N}}{\left((\Delta_1+\Delta_2)\frac{\beta}{R_N}+\Delta_1\Delta_2\right)}$ and $\overline{N_r} = \dfrac{K_N\left(\frac{\beta}{R_N}+\Delta_1\right)}{\left((\Delta_1+\Delta_2)\frac{\beta}{R_N}+\Delta_1\Delta_2\right)}$

The system of equations (1) -(3) has one feasible interior equilibrium

$E_2(N^*{}_s \neq 0, N^*{}_r \neq 0, M^* \neq 0)$:

where $N^*{}_r = \frac{I-\Delta M^*}{\alpha M^*}$, provided $I > \Delta M^*$

$N^*{}_s = \frac{\beta N^*{}_R}{\beta+\Delta_1 R_N}$

$M^* = \frac{R_N(K_N-\Delta_2 N^*{}_r)+\beta N^*{}_s - A\beta}{\alpha_1 R_N}$, provided $K_N > \Delta_2 N^*{}_r$

Numerical Example:

In this section, we have performed numerical simulation for the system governed by equations (1) -(3). We take set of parametric values as:

$$\frac{\beta}{R_N} = 0.7, \Delta_1 = 0.925, K_N = 0.8, \alpha_1 = 1.6, \Delta_2 = 0.7, I = 1.5, \alpha = 0.18, \Delta = 0.1$$

The behaviour of the system for above set of values is as follows:

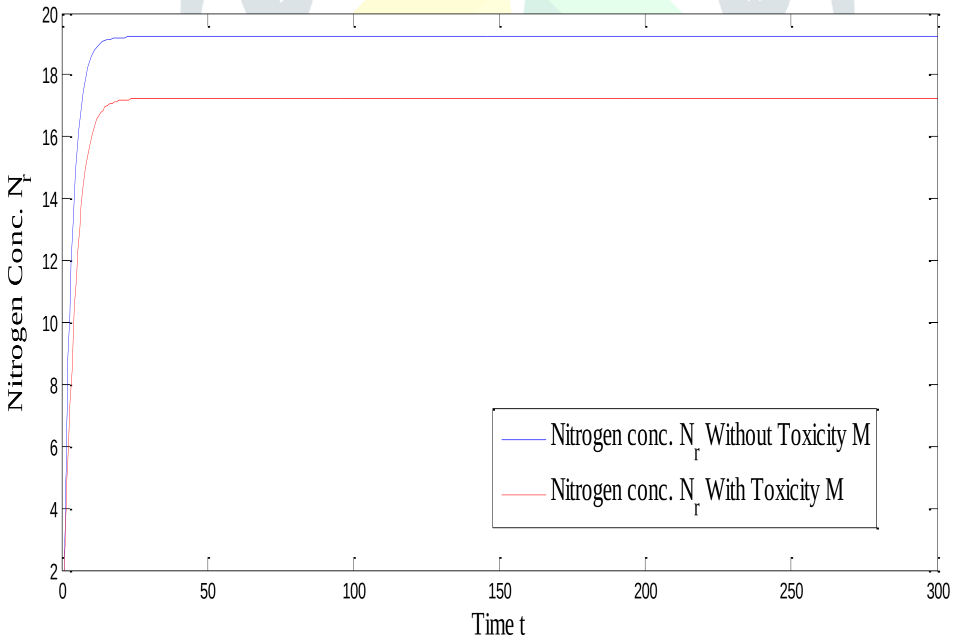

Figure1: Time series graph showing the adverse effect of toxicity on nitrogen concentration in root compartment.

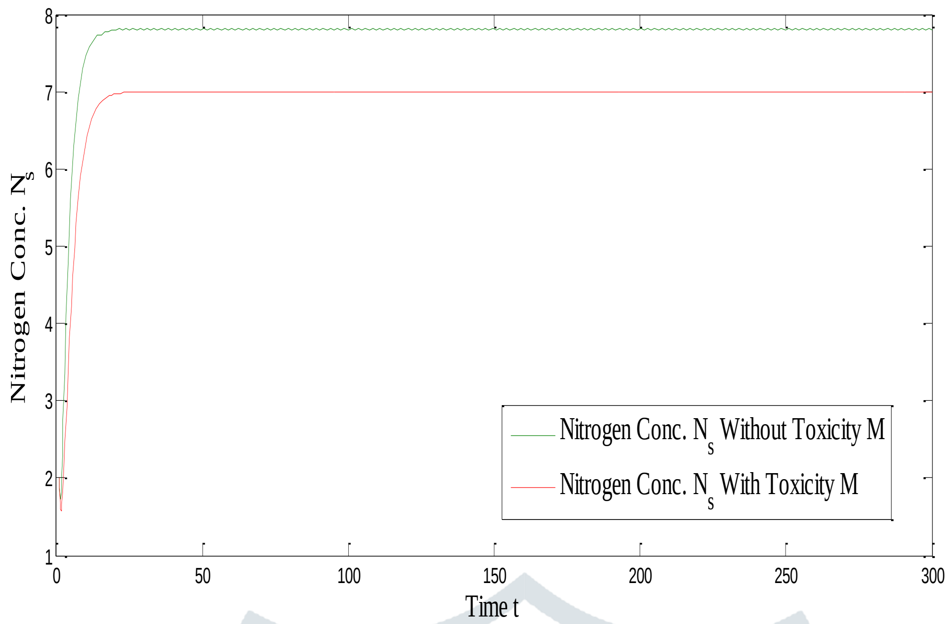

Figure2: Time series graph showing the adverse effect of toxicity on nitrogen concentration in shoot compartment.

Conclusion

In this paper, the adverse effect of toxicity on allocation of nitrogen in root and shoot compartment is shown. The value of nitrogen concentration in root compartment in absence of toxicity is 19.2340. This value decreases to 17.2379 with the introduction of toxic metal. This phenomenon is shown by Figure1. The value of nitrogen concentration in shoot compartment in absence of toxicity is 7.8038. This value decreases to 6.9550 with the introduction of toxic metal. This phenomenon is shown by Figure2. The boundedness of system using usual comparison theorem and the positivity of solutions show that the system persists.

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