A Prey-Predator Model with Beddington-De Angelis Holling type IV functional response

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Abstract:
The proposed work objective is to study systematically dynamical behaviour of one prey one predator system with Beddington-De Angelis Holling type IV functional response. The dynamical behaviour of the model using boundedness and local stability have been carried out. The numerical simulation is also performed to support analytical results.
Keywords: Prey predator populations , functional response, model, stability

1 Introduction

The aim of ecology is known as the dynamical relationship between predation and prey.
The foremost significant factor of the prey predator ‘s rate of feeding upon prey referred to as
Predator’s functional response which is average number of prey killed per individual predator prey unit of time[1]. In 1965, Holling gave the different types of functional response for various sorts of species to model the phenomenon of predation pointed out by the Lotka[2]and Volterra[3]. Further, Beddigton and DeAnglies independently gave a functional response which is analogous to Holling II and contained an additional term describing mutual interference by predators. The dynamical relationship between predators and their prey has long been in continuation and will become the dominant in mathematical ecology, because of universal existence and importance[4], investigated by Berrymann[5].The consumption between prey and predators is simple correspondence having two differential equations which is one of the significant method for mathematical modelling. In this proposed work the dynamical behaviour of prey predator model has been studied[6]. The dynamical behaviour of prey predator has been also studied by Beddington and DeAngelis . The modelling of ecosystem in which there is mutual interference among the predators established with the help of Beddington and DeAngelis[7]. Beddington and DeAngelis[8] proposed a functional response named as B-D functional response. In most of the times , predators have to find or face challenge or to do partitons for food incorporating B-D functional response the stage structured prey predator model come into picture Chen et.al[9] studied the prey predator model for non-autonomous system which gives the stability on boundary value solutions which reflect the dynamical relationship of competing the prey and predators in the changing atmosphere. Xia et.al[10] assumed the Beddington and DeAngelis functional response for stability and travelling waves with stage structured prey predatorreaction diffusion system with harvesting and nonlocal delays[11]. Chen et al. also studied how the amount of two species will become less during the time of competition. Many authors[12],[13] have also discussed the prey predator system that three predator depend on functional response and will gave the best result of predator feeding in the area of prey predator abundance. In most of the cases, B-D type functional response studied theoretically to show that the predator-dependent functional response can have different properties from prey-dependent functional response[14][15].

2 Mathematical Model

In this paper a mathematical model on prey predator food chain has beeninvestigated. The model variable are considered as prey and predator populationdenoted by $p(t)$ and $q(t)$.Further the interaction of prey predator with the predation process using BeddingtonDeAnglies functional response with Holling type IV has been studied.

$$\frac{dp}{dt} = gp\left(1-\frac{p}{k}\right) - \frac{\gamma pq}{1+hp^2+mq} \quad (1)$$

$$\frac{dq}{dt} = \frac{\alpha\gamma pq}{1+hp^2+mq} - \mu q \quad (2)$$

$$p(0) \geq 0, q(0) \geq 0$$

Where p=prey population
q = Predator population
g= Logistic growth of prey
γ= Consumption rate
μ =Death rate of predator in the absence of prey
m = Interaction between prey and predator
h = Coefficients of intraspecific competition between prey and predator
α = Search rate of predator for its food (prey)

3 Dynamical behaviour of model

3.1 Boundedness of the model

Theorem : Every solution of the system is bounded for all $t \geq 0$ and solutions are ultimately bounded.
Proof: Let $V(t) = \alpha p(t) + q(t)$ and after calculating the derivative of $V(t)$ with respect to t, along the Positive solution we have,

$$\dot{V(t)} = \alpha \dot{p(t)} + \dot{q(t)}$$

$$\dot{V(t)} = \alpha g p\left(1-\frac{p}{k}\right) - \frac{\alpha\beta pq}{1+hp^2+mq} + \frac{\alpha\beta pq}{1+hp^2+mq} - \mu q$$

$$\dot{V(t)} = \alpha g p\left(1-\frac{p}{k}\right) - \mu q$$

Now,

$$\dot{V(t)} + \eta V(t) = \alpha g p\left(1-\frac{p}{k}\right) - \mu q + \eta V(t)$$
$$= \alpha g p\left(1-\frac{p}{k}\right) - \mu q + \eta[\alpha p(t) + q(t)]$$
$$= (\eta-\mu)q + \alpha p\left[g - \frac{gp}{k} + \eta\right]$$
$$\leq \alpha p\left(\eta + g - \frac{gp}{k}\right)$$

Where $\eta - \mu \leq 0$
$\Rightarrow \eta \leq \mu$,

$$\dot{V(t)} + \eta V(t) \leq M$$

Where$M \leq \frac{\alpha(\eta+g)^2 k}{4g}$,

$$V(t) \leq \left(V(0) - \frac{M}{\eta}\right)e^{-\eta t} + \frac{M}{\eta}$$

3.2Equilibrium points

In this section, equilibrium points are discussed with their feasible conditions. The system of equation has three equilibrium points which are statedwith their feasible conditions given as:

- $E_0 = (0,0)$the trivial equilibrium points always exists.
- $E_1 = (k,0)$>0 always exists on boundary of first octant.
  In the absence of prey, predator can't survive, so there will not be any equilibrium point in this plane.
- $E_2 = (p,q) > 0 \ \ when\ p = \frac{\alpha\gamma}{h\eta} \pm \sqrt{\left(\frac{\alpha\gamma}{h\eta}\right)^2 + \frac{4}{h}(1+mq)}$(3)
  and$q = \frac{g\left(1-\frac{p}{k}\right)(1+hp^2)}{\gamma - gm\left(1-\frac{p}{k}\right)}$ (4)

$$\text{provided}\frac{\alpha\gamma}{h\eta} > \sqrt{\frac{4}{h}(1+mq)}, \quad (5)$$

$$\text{and} p < k(6)$$

3.3 Local Stability

In the pervious section, we have studied there are three non-negative equilibrium points named as E_0, E_1, E_2.Now, we will study dynamical behaviour using local stability for model equations using above equilibrium points.The variational matrix of given model variable equations has been calculated and its characteristic equation at each equilibrium point is determined.

- For $E_0 = (0,0)$ the variational matrix is

$$M_0 = \begin{bmatrix} g & 0 \\ 0 & -\mu \end{bmatrix}$$

- For $E_1 = (k,0) > 0$the variational matrix is

$$M_1 = \begin{bmatrix} -g & 0 \\ 0 & -\mu \end{bmatrix}$$

- For $E_2 = (p^*, q^*) > 0$; the variational matrix is

$$M_2 = \begin{bmatrix} g\left(1-\frac{2p}{k}\right) - \frac{\gamma q Q}{P} & \frac{\alpha\gamma q Q}{P} \\ \frac{-\gamma p(1+hp^2)}{P} & \frac{\alpha\gamma p(1+hp^2)}{P} - \mu \end{bmatrix}$$

From the nature of roots of characteristics equation, it has been observed that:

- $E_0 = (0,0)$is showing unstability along p-direction.
- From M_1 variational matrix,stability of $E_1 = (k,0) > 0$ has been observed along $p - q$ directions i.e., $x - y$ direction as all the Eigen values are negative which promoted the stability.
- From equilibrium point $E_2 = (p^*, q^*) > 0$, let the characteristics equation of variational matrix M_2 by Routh-Hurwitz be

$$a_0\lambda^2 + a_1\lambda + a_2 = 0$$

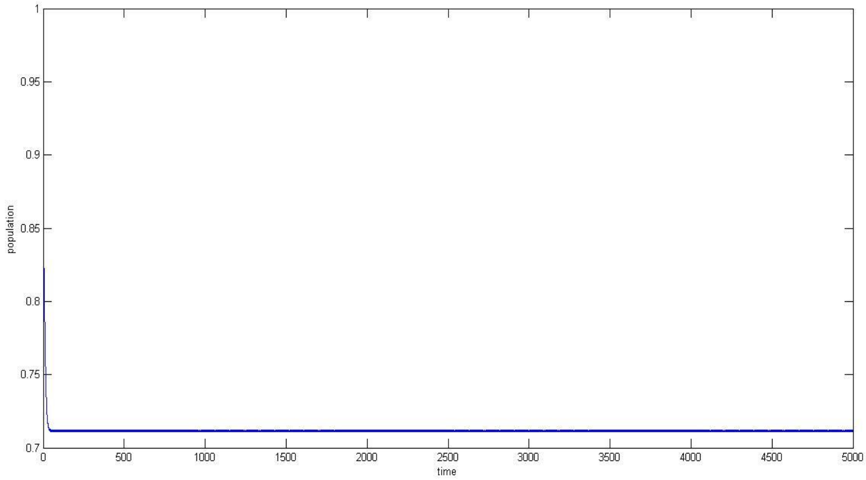
Here necessary condition for stability is
All the coefficients of $a_i > 0$ and $a_0 a_1 - a_2 > 0$
From the calculation, it has been observed that third equilibrium point is stable if it holds the following conditions:

- $g > \frac{\gamma q Q}{P}\left(\frac{k}{k-2p}\right)(7)$
- $p < \frac{P\mu}{\alpha\gamma(1+hp^2)}(8)$

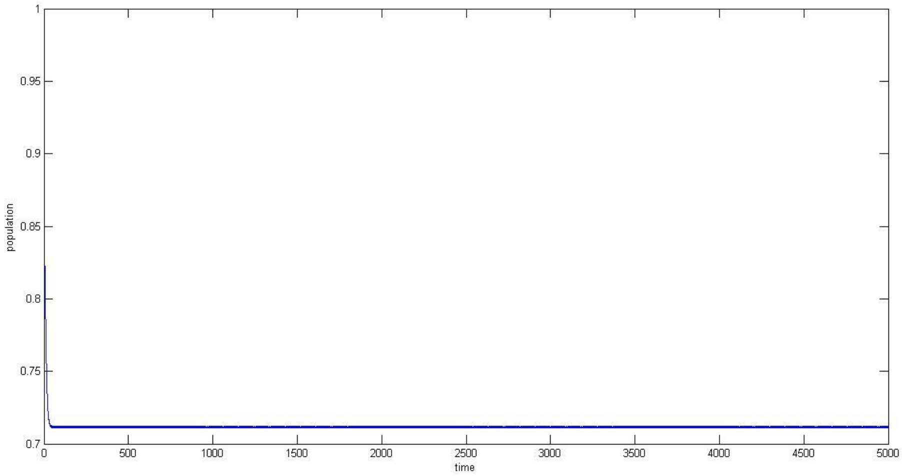
4 Numerical Simulation:

The two species prey-predator food chain system has been solved out numerically by taking some parameters:

- $g = 8.0; k = 1; \alpha = 3; h = 2; m = 0.2; \mu = 0.7; \gamma = 0.78;$



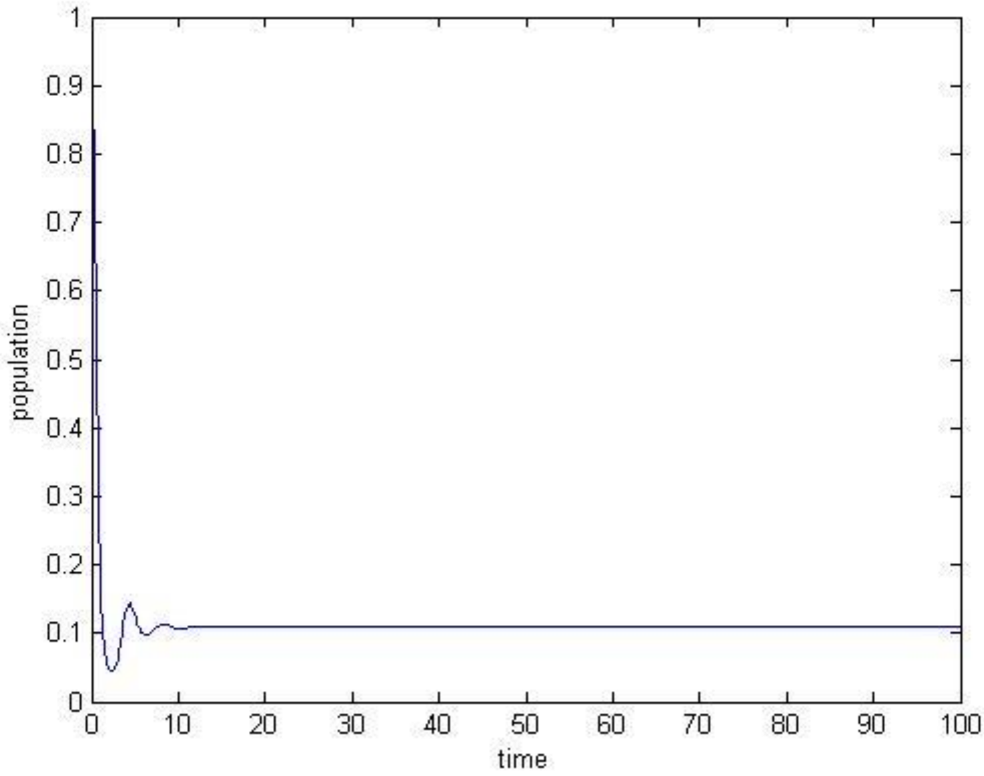
$Figure$ 1: Stability graph of prey$p(t)$ with above mentioned parameters.



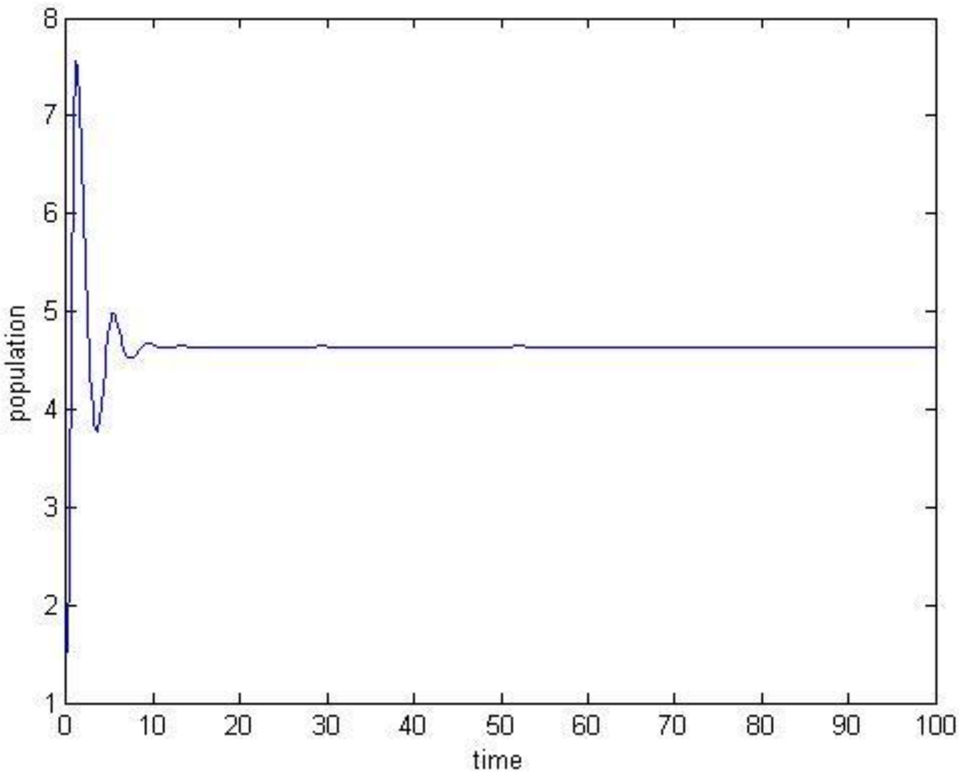
$Figure$ 2: Stability graph of predator $q(t)$ with above mentioned parameters.

By changing the parameter i.e., of , further more dynamical behaviour has been seen which become stable afterwards.

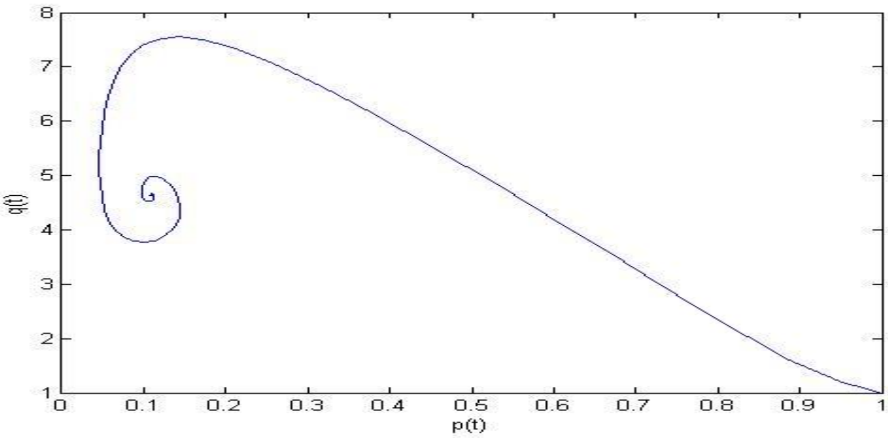
- $g = 8.0; k = 1;\ \alpha = 3;\ h = 2;\ m = 0.2;\ \mu = 0.7;\ \gamma = 4.2$



$Figure$ 3:Stability of prey $p(t)$ after changing the value of γ from 0.78 to 4.2



$Figure$ 4:Stability of predator$q(t)$, after changing the value of γ from 0.78 to 4.2



$Figure$ 5: Phase plane graph of prey-predator stability

Conclusion:

In this paper, a prey-predator model with Beddington-De Angelis Holling type IV functional response has been studied. The structure of all the equilibrium points and their linear stability (local stability) is discussed. The boundary equilibrium point E_1 is locally asymptotical stable. The interior positive equilibrium point E_2 is locally stable when the conditions (7)and (8) are satisfied. The stability behaviour is shown in figure(1) and figure(2). Further, it is also observed that as the interaction coefficient value changes, the prey population decreases and predator population increases and showing the stability behaviour as shown in figure(3) , figure (4) and figure(5).

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