Nonlinear Interaction of Ultra Intense Laser Beams with Collisional Plasma with Ramped Electron Density

Naveen Gupta

School of Chemical Engineering and Physical Sciences, Lovely Professional University

Phagwara, India

Abstract

Nonlinear interaction of ultra intense laser beams with collisional plasma having ramp shaped electron density has been investigated theoretically. Due to the non uniform heating of the carriers produced by the laser beam, the plasma responds nonlinearly to the laser beam. This results in copious nonlinear optical effects like self focusing, self trapping, self modulation of the spatial frequency of the laser beam etc. In the present investigation emphasis is put on investigation of self focusing of the laser beam. Variational theory based on Rayleigh Ritz optimization has been applied to find semi analytical solution of the wave equation for the pump beam. The effect of cavity imperfections of the laser system on irradiance profile of the laser beam has been incorporated through q-Gaussian distribution function for the amplitude structure over the wave fronts of the laser beam.

Introduction

The availability of ultra intense laser systems[1] has extended research interest on light matter interactions. Laser matter interactions are bursting with myriad nonlinear phenomena those are absent during the interaction of ordinary light with matter. These nonlinear effects are extremely complex but being rich in physics, have the potential to engage the interest of researchers for several upcoming years. Hence, since the invention of laser, in order to improve upon the understanding of light matter interactions, several researchers are making their conscious efforts. The major impetus has been built by the proposal of initiating fusion reaction[2], a process for the production of cleanest energy as that occurring in sun as well as all the stars, with the help of laser beams. This will be like capturing a star to run machines on earth without worsening global climate.

In most of the laser matter interaction based applications laser intensity is of major concern. Currently, the laser power has gotten into bottleneck at the order of few PW. Hence, secondary approaches like tight focusing and optical modulation of the laser beams are being targeted to achieve higher power densities. Being highly nonlinear and already in an ionized state plasmas offer a promising tool to produce tight focusing of the laser beams through the phenomenon of self focusing. Hence, during past few decades continuous efforts are being made to produce tight focusing of the laser beam by using under dense plasma targets.

Laser beams differing in irradiance profile behave differently in plasmas. In other words the nonlinear effects invoked by laser beams in plasmas depend critically on the intensity distribution over the cross section of the laser beam. However, from literature review it has been seen that most of the earlier investigations on nonlinear interaction of lasers with dielectrics, semiconductors and even plasmas have been carried out under the assumption that the irradiance profile of the optical beam emitted by the laser system whose cavity is operating in its fundamental mode i.e., TEM_{00}mode can be exactly modeled by ideal Gaussian distribution. Under this assumption, the previous investigations on laser plasma interactions have been carried out under the framework of paraxial theory. The paraxial theory over simplifies the analysis by Taylor expanding the index of refraction of the plasma only up to r^2or $r^4$where, r is the radial coordinate from the axis of the laser beam. By contrast, from the experimental investigations on the irradiance profile of ultra intense lasers like Vulcan laser at Rutherford Appelton laboratory[3], it has been found that the actual distribution of intensity over the laser beam cross section differs significantly from the ideal Gaussian profile. A significant amount of energy was observed to be lying outside the full width half maxima of the distribution. By fitting into the experimental data[4] it was suggested that the actual distribution of intensity over the laser beam cross section can be modeled by a class of distribution functions known as Tsalli's q-Gaussian distribution[5]. As, for the laser beams whose intensity distribution is described by q-Gaussian distribution, the neglect of the higher power terms of the Taylor expansion of index of refraction is not justified, the paraxial theory is not applicable to these beams. Thus, the aim of this paper is to give first investigation on self focusing of q-Gaussian laser beams in collisional plasmas with upward density ramp by using Variational theory which is free from the limitations of paraxial theory.

Mathematical Model

The evolution of amplitude A_0 of a laser beam interacting nonlinearly with collisional plasma with axially increasing plasma density n(z) is governed by NLSE

$$2ik_0\frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_p^2(z)}{c^2}\left\{1-\left(1+\frac{1}{2}\beta A_0 A_0^*\right)^{-\frac{5}{2}}\right\} A_0 \qquad \dots\dots(1)$$

where, $\omega_p^2(\mathrm{z})$ is the plasma frequency in the presence of laser beam, $\beta = \frac{e^2 M}{6K_0 T_0 m^2 \omega_0^2}$ is the coefficient representing the strength of collisional nonlinearity. As, here the potential itself if dependent on filed amplitude A_0 eq.(1) is nonlinear in nature. Hence, super position principle does not apply to eq.(1) which mathematically means that common methods of solving partial differential equations (PDEs) don't apply to eq.(1). Thus, eq.(1) can be solved either by numerical simulations or by some semi analytical method. Computer simulations are having drawbacks of convergence and large amount of time involved. Hence, in the present investigation we have used a semi analytical method known as variational technique[6] based on Rayleigh's optimization. This method converts a nonlinear PDE to a set of coupled ODEs for the parameters of interest involved in the actual problem.

The basic idea of variational technique is the selection of a trial function which is as close to the actual solution as possible. In the present investigation we have taken the trial function as[7]

$$A_0(r,z) = \frac{E_{00}}{f}\left(1 + \frac{r^2}{q r_0^2 f^2}\right)^{-\frac{q}{2}} \ldots\ldots (2)$$

where, the phenomenological parameter q is associated with the deviation of intensity profile from ideal Gaussian profile. The vlue of q varies from laser to laser and is associated with the inherent properties of a given laser system. The function $f(z)$ is termed as beam width parameter that gives instantaneous value of the spot size of the laser beam.

Following Variational theory we get:

$$\frac{d^2 f}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3} - \left(\frac{\omega_p(\xi) r_0}{c}\right)^2 \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{s}{2}-1\right)\frac{\beta E_{00}^2}{f^3}\int_0^\infty x\left(1+\frac{x}{q}\right)^{-2q-1}\left\{\left(1+\frac{1}{2}\frac{\beta E_{00}^2}{f^2}\right)^{\frac{s}{2}-1}\left(1+\frac{x}{q}\right)^{-q}\right\}^{-\frac{7}{2}} dx \ldots (3)$$

where, $\xi = \frac{z}{k_0 r_0^2}$ which is termed as normalized distance of propagation where the normalization has been performed with respect to the Rayleigh length $k_0 r_0^2$. Also, it has been assumed that the equilibrium electron density of the plasma varies with distance as $\omega_p(\xi) = \omega_{p0}(1 + \tan(d\xi))$. From eq.(3) it clear that variational technique has reduced PDE i.e., eq.(1) to an ODE resembling the equation of motion for a driven oscillator.

Results and Discussion

In solving eq.(3) it has been assumed that while entering into the plasma the laser beam is collimated and is having plane wave front. Mathematically these conditions are represented by the boundary conditions $f = 1$, $\frac{df}{d\xi}$=0 at $\xi = 0$. In the present study eq.(3) has been solved for following set of parameters

$$\omega_0 = 1.78 \times 10^{14}\text{rad/sec},\ r_0 = 15\mu m,\ \frac{\omega_{p0} r_0}{c}=3,\ \beta E_{00}^2 = 3 \text{ and } q = (3,4,\infty)$$

Fig.1 illustrates the effect of deviation parameter q of the evolution of beam width of the laser beam through plasma. It can be seen that during its propagation through the plasma the beam width of the laser beam varies harmonically with distance. This behaviour of the beam width of the laser beam can be explained by observing the role and origin of various terms contained in the equation of motion of beam width. The first term on the right hand side (R.H.S) that varies as f^{-3}originates due to the light's natural wave property to diffract. Hence, this term is also known as diffractive term. As the magnitude of this term is inversly proportional to the cube of instantaneous beam width of the laser beam, the narrower beams will have higher diffraction divergence. This is an obvious result similar to the diffraction of light by a single slit where, the diffraction divergence of the incident wavefront is inversly proportional to the width of the slit. The second term on the R.H.S of eq.(3) is having a complex dependence on the instantaneous beam width and it originates as a consequence of the nonlinear response of plasma to the incident beam and is responsible for the nonlinear refraction of the laser beam. Hence, this term is referred to as nonlinear refractive term. Due to its –ve sign this term tends to reduce the effect of second term. Hence, during the propagation of laser beam through the plasma there starts a fight between the two phenomena of diffraction and nonlinear refraction. The winning phenomenon decides the ultimate behaviour of the beam i.e., weather the beam will converge, diverge of will maintain its original value. Thus, there exists a critical intensity of the laser beam (that can be obtained by equating R.H.S of eq.3 with zero) above which the beam will converge. In the present study we have taken the initial intensity of the laser beam above the critical value that is why the laser beam is converging initially. As the laser beam converges due to the dominance of nonlinear refrction, its spot size decreases. As the laser beam width decreasses, the magnitude of diffraction term increases making the diffraction effect stronger but not enough to dominate the effect of nonlinear refraction. Also due to decrease in the beam width, the intensity of laser beam increases. When the laser intensity becomes too high, the illuminated portion of the plasma gets almost completely evacuated from plasma electrons and thus the nonlinear refraction vanishes, leaving only the diffraction effects to dominate. Hence, after focusing to minimum, the beam width bounces back to its original value. As the beam width of the laser beam starts increasing, the competition between diffraction and nonlinear refraction starts again. Now, this competition lasts till maximum value of f is obtained. These processes go on repeating themselves and thus give breather like behavior to the beam envelope.

Further it has been observed that after every focal spot the maximum as well as the minimum of the beam width shift downwards. This is owing to the fact that the equilibrium electron density is an increasing function of longitudinal distance. Hence, the plasma index of refraction keeps on decreasing with the penetration of laser beam into the plasma. Consequently, the self focusing effect gets enhanced and the maximum as well as minimum of the beam width gets shifted downwards after every focal spot. It is also seen that the frequency of oscillations of beam width increases with distance. The physics behind this fact is that denser is the plasma, higher will be the phase velocity of laser beam through it. Hence, in denser plasma laser beam takes less duration to get self focused.

Reduction in focusing of the laser beams with their irradiance closer to Gaussian profile can also be seen from fig.1. The reason behind this effect is that for laser beams with higher value of q most of the intensity is concentrated in a narrow region around the axis of the beam. Hence, these beams get a get a very less contribution from the off axial part in order to produce nonlinearity in the medium. As the self focusing is a homeostasis of nonlinear refraction of the laser beam due to optical nonlinearity of the medium, increase in the value of q reduces the extent of self focusing.

The plots in fig.1 also indicate that instead of their reduced focusing, the laser beams with higher value of q possess faster focusing character. This is due to the faster focusing character of axial rays. Being away from the axis, off axial rays take more duration to get self focused. As there are more number of off axial rays in laser beams with lower q values, hence by increasing the value of deviation parameter q, the focusing of the laser beam becomes faster. This result of faster focusing of laser beams with lower q value is contrary to that reported by Sharma and Kourakis[8] where it was shown that laser beams with lower value of q possess slower focusing. This difference between the two results is mainly due to the reason that in their analysis Sharma and Kourakis have expanded the dielectric function of the plasma only up to r^4. However, by analyzing the q-Gaussian distribution it can be seen that change in the value of q is having hardly any effect on the irradiance in the regions closer to the axis. The change in the value of q affects the irradiance only in the regions away from the beam axis that has been eliminated in the analysis of Sharma and Kourakis. However, in our study the dielectric function of the plasma has been considered as a whole. Thus, it can be concluded that by optimizing the value of q, one can control focusing as well as the location of the focal spot of a laser beam.

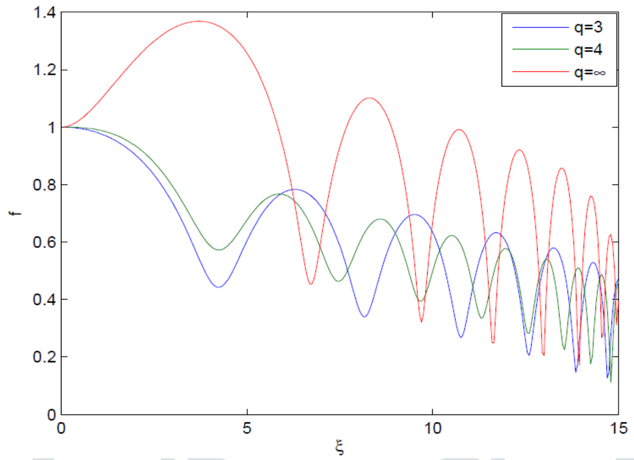

Fig1: Evolution of spot size of the laser beam with longitudinal distance.

Conclusion

In conclusion authors have investigated the non linear optical phenomenon of self focusing of the laser beams in collisional plasmas with density ramp. The effect of deviation of the intensity profile of the laser beam from Gaussian profile on the evolution of its beam width has been investigated. It has been bound that the laser beams having lower q value possess enhanced focusing effect.

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