Revisiting Chua's and MLC Electronic Circuits: A MATLAB Approach to Solve the Circuit Equations

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Abstract

Chua's and MLC (Murali-Lakshmanan-Chua) circuits are revisited via MATLAB coding. The paper intends to show the chaotic oscillators via MATLAB coding. The codes are attached with this paper. Non-linear electronic circuits such as Chua's diode and MLC circuit are capable of exhibiting the phenomenon chaos in electronic circuits. The non-linearity in the electronic circuits arise because of the non-linear resistor component which may be termed as the Chua's diode. The governing dynamical equations, if parameterized properly and solved, then the circuit behaves as a chaotic oscillator.

I. INTRODUCTION

The study of non-linear dynamics has found importance since 1960 with the discovery of soliton, spatio-temporal patterns, chaos and other such novel concepts. The field however has a genesis of basic ideas which are the works done long back in the past. In the last few decades the field has seen significant developments both from mathematical and physical standpoints [1]-[5]. The schematic of the two circuits mentioned above are shown in figure-1 [1]. A circuit is said to be non-linear if it has at least one non-linear component. However, out of all non-linear components, non-linear resistor is the easiest to build. The following circuits are based on the non-linear resistor as the non-linear component. The non-linear resistor attached in both the circuits is responsible for the oscillatory chaotic behavior of the circuits. In general, a linear resistor has straight I-V characteristics. Whereas, the non-linear resistor has piece-wise linear or as a whole non-linear I-V curve. The electronic circuits are easy to design and analyze.

In this article we show the MATLAB based method of solving the governing equations of the circuits. The codes are written using the in-built MATLAB function ode-45. The governing equations of the circuit are ordinary differential equations and can be solved by any standard method such as Runge-Kutta 4th order method. As shown in the figure, the Chua's circuit consists of a single non-linear resistor and two capacitors along with an inductor.

This is a modified LCR circuit. On the other hand the MLC circuit consists of a single capacitor along with the non-linear resistor component and the inductor.
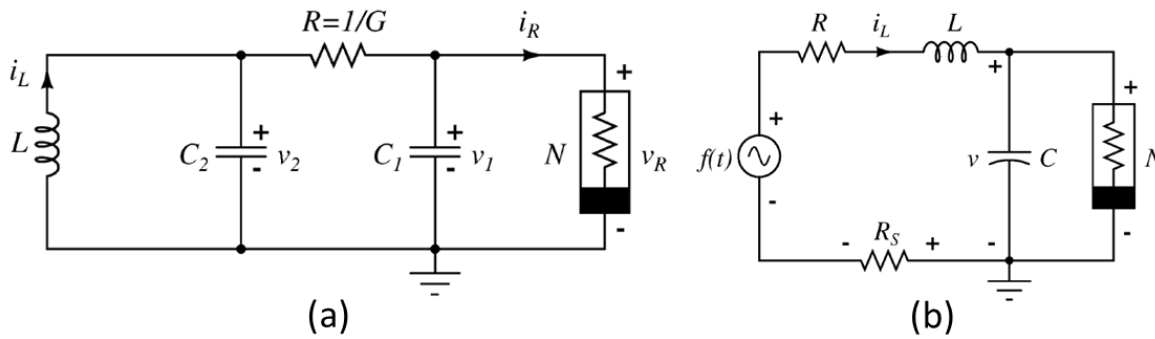

Figure 1: The circuit diagrams of (a) Chua's and (b) MLC circuit

II.   BASIC EQUATIONS FOR MODELLING

With reference to the book [1], the basic equations in non-dimensional form for the chua's circuit are written as:

$$\dot{x} = \alpha(y - x - h(x))$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\beta y$$

Here, $h(x) = bx + 0.5(a - b)[|x + 1| - |x - 1|]$, $\alpha = C_2/C_1$ and $\beta = C_2 R^2/L$. Rest of the parameters are as described in the code.

Similarly, for the MLC circuit the non-dimensional equations are written as:

$$\dot{x} = y - h(x)$$
$$\dot{y} = -\beta(1 + v)y - \beta x + f\sin(\omega t)$$

Here, $\beta = \frac{C}{LG^2}$, $v = GR_s$ and $f = F\beta/B_p$. The function h(x) is represented by

$$h(x) = \begin{matrix} bx + a - b, x \geq 1 \\ ax, \ |x| \leq 1 \\ bx - a + b, x \leq -1 \end{matrix}$$

The symbols have their meaning as given in [1].

III.   THE MATLAB CODES

The MATLAB codes for the above governing equations corresponding to the Chua's circuit and MLC circuit are shown below in figure 2 and 3 respectively.

```
1
2    % drv_chua
3    clc; clearvars;
4
5    global R C1 C2 L
6
7    R = 1.79*(1E3);
8    C1 = 10*(1E-9);
9    C2 = 100*(1E-9);
10   L = 18*(1E-3);
11
12   [x,y]=ode45('sub_chua',[0 100],[0.1 0.1 0.1]);
13
14   subplot(121), plot(y(:,1),y(:,2)), grid on
15   xlabel('x'), ylabel('y')
16   subplot(122), plot(x,y(:,1)), grid on
17   xlabel('Time'), ylabel('x')
18
19   title('Chuas Chaotic oscillator')
20
```

```
1    % sub_chua
2    function dy = sub_chua(~,y)
3    global R C1 C2 L
4    dy = zeros(3,1);
5    alpha = C2/C1;
6    beta = C2*(R*R/L);
7
8    Ga = -0.758*(1E-3);
9    Gb = -0.409*(1E-3);
10   a = R*Ga;
11   b = R*Gb;
12   h = (b*y(1))+0.5*(a-b)*(abs(y(1)+1)-abs(y(1)-1));
13
14   % Main part of writing the differential equations
15   dy(1) = alpha*(y(2)-y(1)-h);
16   dy(2) = y(1)-y(2)+y(3);
17   dy(3) = -beta*(y(2));
18
19   end
```

Figure 2: The MATLAB code for the Chua's circuit

```
% drv_MLC
clc, clearvars;
global BETA NEW a b w f
% Define Parameters
BETA=1;
NEW=0.015;
a=-1.02;
b=-0.55;
w=0.75;
f=0.1;
%------------------------------------------------
% Solve the set of ODEs
trange = 0:0.1:300;
[t,y]=ode45('subMLC',trange,[1e-1 1e-2]);
%------------------------------------------------

figure(1)
subplot(121), plot(y(:,1),y(:,2)), grid on, hold on
xlabel('x'), ylabel('y')
subplot(122), plot(t,y(:,1)), grid on, hold on
xlabel('Time'), ylabel('x')
```

```
% sub_MLC
function dy=subMLC(t,y)
global BETA NEW a b w f
dy=zeros(2,1);
%------------------------------------------------
L=mod(y(1),0);
if  y(1)>=1
  m=(b*y(1)+a-b);
elseif  y(1)<=-1
  m=(b*y(1)-a+b);
elseif  L<=1
  m=a*y(1);
end
%------------------------------------------------
% Define the Differential Equations
dy(1)=(y(2)-m);
dy(2)=(-BETA.*(1+NEW).*y(2))-(BETA.*y(1))+(f.*sin(w*t));
end
```

Figure 3: The MATLAB code for the MLC circuit

IV.   RESULTS OF THE CODE EXECUTION

While executing the codes it must be remembered that the driver file is to be run and both the driver file and the sub files should be in the same folder. The results of the code execution are shown in the figure-4 and 5 respectively.
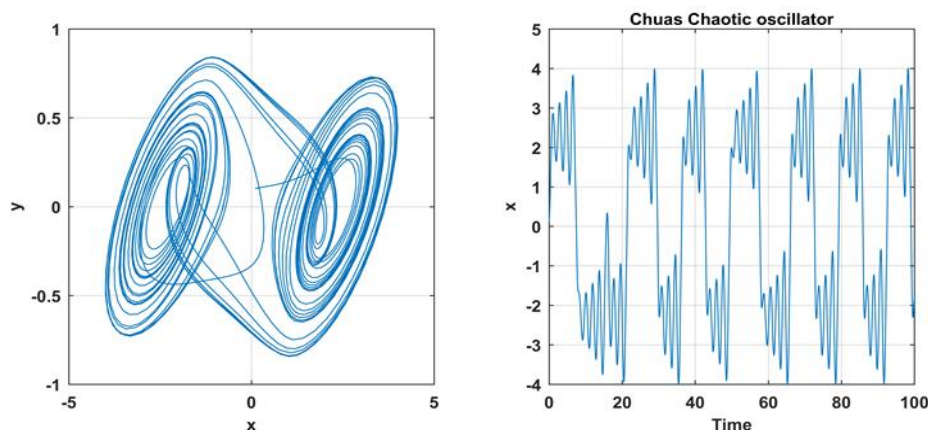

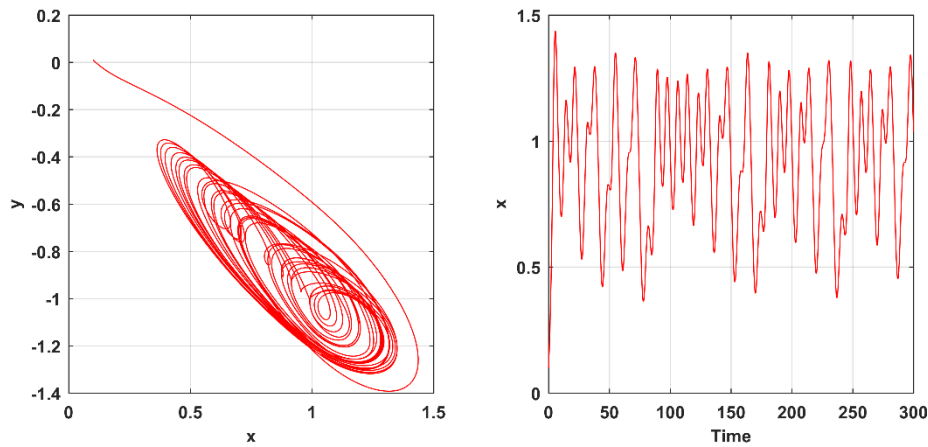Figure 4: Chaotic oscillation of Chua's circuit.

Figure 5: Chaotic oscillation of Chua's circuit.

V.　　CONCLUSIONS

In this article we have shown the MATLAB code for executing the governing equations for the well-known Chua's and MLC circuits. Both the circuits are shown to exhibit chaotic oscillations under suitable choice of parameters.

References:

[1] M. Lakshmanan and S. Rajasekar, Non Linear Dynamics: Integrability Chaos, and and Patterns, Springer, ISBN 978-3-642-62872-6, (2003).

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