A REVIEW ON MATHEMATICAL MODELS AVAILABLE FOR SOLAR RADIATION ESTIMATION

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Abstract

Solar energy is the cleanest form of energy sources which can be utilized for purposes such as power generation and heating applications. However the availability of this energy varies across the globe. Therefore the assessment of solar radiation intensity becomes crucial for the development of solar applications. However, many developing countries are facing the problem of non availability of measuring devices and poor handling. Hence mathematical prediction of solar radiation intensity helps the mankind in the sustained energy development resources. The present paper reviews some of the mathematical models derived by researchers across the globe for estimation of global solar radiations and it was concluded that global solar radiation intensity depends upon many geological and meteorological factors.

1 Introduction

Energy is the vital source for overall development of a nation and its economy. It not only opens the path of financial development but in parallel also open up different opportunities for future research as well. Till 20th century, non renewable energy sources have been utilized to a greater extent such as fossil fuels. The use of these non renewable sources of energies, which can not replenish by their own, have serious impacts on environment and human life. Therefore, efforts for advancements in renewable sources of energies increased. Solar energy appeared to be the popular form of energy because of ease of availability and environment friendliness.

Solar energy holds a significant place as an alternative for non renewable source of energy [1]. However, the correct estimation of global solar radiations in some of the world is still facing plethora of problems. Problems such as non availability of appropriate measuring devices and improper handling of these devices in developing countries are the major reasons. Due to this, development of many solar devices is still in infancy in those part of the world.

Hence the solar radiation assessment is the primary step in developing solar energy based devices. Moreover for the product marketing also companies require the exact data of solar intensities [2,3]. Researchers also require the information of solar radiation data for estimating thermal load of buildings which is further used in developing energy efficient buildings. Intensity information is also helpful in formulating crop growth models and irrigation systems.[4]

Hence it is imperative to predict or to measure the intensity of solar radiations for development of many systems. However when precise measurement becomes a problem, prediction plays a vital role. Information of solar radiation data can be predicted mathematically which matches with the exact measured data. The present work encompasses few such mathematical models which are formulated to predict the solar radiation across many parts of the world.

2 Mathematical Models for Solar Radiation Estimation

Different correlations have been developed that utilizes various meteorological factors to calculate the value of irradiance (H) on the earth. Factors such as cloud cover, extra terrestrial radiations, latitude, humidity, solar declination angle etc have been used to develop mathematical models. One such model is Angstrom-Prescott model [5] which relates solar radiation with relative sunshine hours given as-

$$K_T = \frac{H}{H_o} = a + b\frac{\bar{n}}{N}$$ Eq. 1

where , K_T= clearness index, H_0= extra terrestrial radiations, n= monthly average sunshine hours, N= monthly average sunshine duration or day length, a and b are constants and n/N= relative sunshine.

However this formula suffers from a problem of being dependent on the location. Similar work was carried out by Fagbenle et al [6] in which the author selected specific regions of Nigeria. They formulated a quadratic model using a two year data to predict solar radiation for the entire nation.

$$K_T = 0.376 - 0.138\left(\frac{\bar{n}}{N}\right) + 0.660\left(\frac{\bar{n}}{N}\right)^2$$ Eq.2

This mathematical model suffers from the problem of less geometrical spread and less number of locations. Hence accuracy of this model was questionable. Also the number of years for data collection creates issue. In order to have an accurate prediction, the data needs to be collected from a wider range of places. However this model is restricted to selected region.

Ajayi et.al.[7] formulated a better model by including various factors such as changes in location, changes in weather and seasonal fluctuations in humid tropical regions. Authors formulated a system of five equations or mathematical models to estimate the value of solar incident radiation.

Model 1

$$H = a\cos\phi + b\cos n + cT_{\max} + d\left(\frac{\bar{n}}{N}\right) + e\left(\frac{T_{\max}}{R.H}\right) + f\left(\frac{T_{\max}}{R.H}\right)^2 + g\cos\phi\cdot\cos n + h$$ Eq.3

Model 2

$$H = a\cos\phi + b\cos n + cT_{\max} + d\left(\frac{\bar{n}}{N}\right) + e\left(\frac{T_{\max}}{R.H}\right) + f\left(\frac{T_{\max}}{R.H}\right)^4 + g\cos\phi\cdot\cos n + h\frac{T_{\max}}{\cos\phi} + i$$

Eq. 4

Model 3

$$H = a\cos\phi + b\cos n + cT_{\max} + d\left(\frac{\bar{n}}{N}\right) + e\left(\frac{\bar{n}}{N}\right)^3 + f\left(\frac{T_{\max}}{R.H}\right) + g\left(\frac{T_{\max}}{R.H}\right)^2 + h\left(\frac{T_{\max}}{R.H}\right)^3 + i\cos\phi\cdot\cos n + j\left(\frac{T_{\max}}{\cos\phi}\right) + k\cos^2 n + l$$

Eq.5

Model 4

$$H = a\cos\phi + b\cos n + cT_{\max} + d\left(\frac{\bar{n}}{N}\right) + e\left(\frac{\bar{n}}{N}\right)^3 + f\left(\frac{T_{\max}}{R.H}\right) + g\left(\frac{T_{\max}}{R.H}\right)^2 + h\left(\frac{T_{\max}}{R.H}\right)^3 + i\left(\frac{T_{\max}}{R.H}\right)^4 + j\cos\phi\cdot\cos n + k\left(\frac{T_{\max}}{\cos\phi}\right) + l\cos^2 n + m$$

Eq.6

Model 5

$$H = a\cos\phi + b\cos n + cT_{\max} + d\left(\frac{\bar{n}}{N}\right) + e\left(\frac{T_{\max}}{R.H}\right) + f(R.H) + g\cos\phi\cdot\cos n + h\left(\frac{T_{\max}}{\cos\phi}\right) + i\left(\frac{T_{\max}}{R.H}\right)^2 + j\left(\frac{\bar{n}}{N}\right)^2 + k\cos^2 n + l$$

Eq.7

Here $\bar{n}$ = daily sunshine hours, N= day length, T_{max}= maximum daily temperature , n= number of days in a year, a,b,c,d,e,f,g,h,i,j,k,l and m are constants , ø= latitude of the location and R.H= relative humidity.

Another research by Khorasanizadeh et.al. [8] chose six empirical models from the previous work from the literature to estimate daily global radiation in 4 Iranian cities of Isfahan, Kerman and Tabass and Bandarabass. Statistical Non linear Regression technique was used and coefficients were developed. Models formulated as as follows-

Model 1

$$H = a + b\cos\left(\frac{2\pi}{364}n_{day} + c\right)$$

Eq.8

Model 2

$$H = a + b\left|\sin\left[\frac{\pi}{365}(n_{day} + 5)\right]\right|^{1.5}$$

Eq.9

Model 3

$$H = a + b\sin\left(\frac{2\pi}{c}n_{day} + d\right)$$
Eq.10

Model 4

$$H = a + b\sin\left(\frac{2\pi c}{365}n_{day} + d\right) + e\cos\left(\frac{2\pi f}{365}n_{day} + g\right)$$
Eq.11

Model 5

$$H = a\exp\left[-0.5\left(\frac{n_{day} - b}{c}\right)^2\right]$$
Eq.12

Model 6

$$H = a + bn_{day} + cn_{day}^2 + dn_{day}^3 + en_{day}^4$$
Eq.13

Different statistical parameters such as mean absolute bias error(MABE), mean absolute percentage error(MAPE), root mean square error(RSME) were selected to test the validity of the model. It was confirmed by the author that model 4 was best suited for Kerman, Tabass and Bandarabass whereas model 6 showed accurate results for Isfahan.

R.J Stone[9] proposed a statistical indicator that can evaluate and compare different mathematical models for prediction of solar data. Author formulated an indicator named as t-statistic which is used in addition with root mean square error(RMSE) and mean bias error (MBE) to assess a model's performance. According to the author-

$$t = \left(\frac{(n-1)\mathrm{MBE}^2}{\mathrm{RMSE}^2 - \mathrm{MBE}^2}\right)^{1/2}$$
Eq. 14

To determine whether the values from a model are valid, it is required to know the value f critical value of t from standard statistical table. If the t value obtained comes out less than the critical value, values of the model are significant. This criteria not only provided the means of evaluating and comparing the various models but also provided the basis to predict the values at some confidence level also.

Mghouchi et al [10] stated that total global radiation flux consists of direct solar flux (I_h) and diffuse solar flux (D_h) for which they formulated a model to estimate them numerically which is as follows

$$I_h = I_0.C_t.\Gamma.\exp\left(-\frac{0.13}{\sin(h)}\right).\sin(h) \quad \text{Eq. 15}$$

Here Γ is the turbidity atmospheric factor for clear skies, C_t is the correction factor for earth-sun distance which further depends upon j i.e. number of day and h is the height of the sun.

$$D_h = 120.\Gamma.\exp\left(-\frac{1}{(0.4511+\sin(h))}\right) \quad \text{Eq.16}$$

Equation 16 shows the mathematical formula for diffuse solar flux.. Hence global solar flux $G_h = I_h + D_h$.

Fan et al [11] evaluated and compared the performance of six newly formulated mathematical correlations to estimate the global incident solar radiation in tropical and subtropical regions of China. The proposed models are-

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right) \quad \text{Eq.17}$$

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right) + eT \quad \text{Eq.18}$$

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right)[1 + ef(P)] + fT \quad \text{Eq.19}$$

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right)[1 + ef(P_{j-1}) + ff(P_j) + gf(P_{j+1})] + hT \quad \text{Eq.20}$$

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right)[1 + ef(P)] + fT + gH_r \quad \text{Eq.21}$$

$$R_s = R_a\left(a + b\Delta T + c\Delta T^{0.25} + d\Delta T^{0.5}\right)[1 + ef(P_{j-1}) + ff(P_j) + gf(P_{j+1})] + hT + iH_r \quad \text{Eq.22}$$

Here f(P)=ln(P+1),a,b,c,d,e,f,g,h,i are coefficients calculated by non linear least square method, R_s is the global incident radiation in W/m^2

Authors reported that second model was accurately predicting the global radiations at the sites where only air temperature is available for prediction. However, when precipitation and humidity are also involved in prediction, then model 5th and 6th showed better results.

3 Conclusion

Solar radiation intensity plays significant role in development of solar energy based devices. Hence, prediction of solar intensity becomes an inevitable process. The present paper encompasses some of the mathematical models available in the literature which are used to estimate the value of solar incident radiation on a certain part of globe. It is observed that in predicting the solar radiation intensity, factors such as air temperature, humidity, height of the sun, distance between earth and sun, day of the year, maximum daily temperature, latitude of the location, monthly average sunshine hours and day length are critical.

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