EFFECTS OF HALL CURRENT ON THE PERISTALTIC FLOW OF A CONDUCTING NEWTONIAN FLUID IN AN INCLINED ASYMMETRIC CHANNEL

J.Murali kumar[1], Dr.Y.V.K.Ravi kumar[2],Dr.M.N.Rajasheker[3], Dr.P.Vinod kumar[4]

[1]*Vivekananda Institute of Technology & Science, Karimnagar, India*

[2]*Practice School Division, Birla Institute of Technology and Science (BITS), Pilani, Hyderabad, India*

[3&4] *Department of Mathematics, JNTUH College of Engineering , Nachupally, Jagtial, India.*

ABSTRACT: In this paper attempt is made to show the effect of hall on the peristaltic flow is fluid in bi-dimensional channel under the long wave length. A closed form solution has been seen at axial velocity and pressure grapiest. Attempt is made to graphical analysis of time-averaged volume flow with various emerging parameters.

INTRODUCTION

The word peristaltic stem derived from the Greek word peristaltic. It means clasping and compressing. In peristaltic pumping process fluid transport in distensible tube is simultaneously occur as the progressive waves travel among wall of the tube. When stimulated at any point contractile rings emerge and spread in the circular muscle tubes. In this way, peristaltic occurs in the gastrointestinal tract, the bile ducts, other glandular ducts throughout the body, the ureters, and many other smooth muscle tubes of the body, Guyton and Hal (2003).

Peristalsis mechanism has recently become a very important subject of scientific research in both mechanical and physiological situations the first research work carried out by Latham in 1966 in this field.

Peristalsis is a pumping mechanism used in biology as well as in industry for pumping and propelling fluid. This pumping is done through contraction and expansion of progressive wave moving along the wall of the tube. Peristaltic movement in biology performs different functions. By contraction and expansion of muscles of tubular organs food and liquids can be propelled forward. In some particular organs it does the work of mixing the content log ether to amalgamate them (1-4).

In industry it has numerous applications. In sanitary fluid transportation and in transportation of corrosive fluid peristaltic mechanism plays an important role. The same mechanism can be found in nuclear industry for transportation of toxic liquid. After the first investigation of Latham a host of scientists and research scholars jumped upon it to carry out experiments Shapiro at al investigated a mathematical model for peristaltic flow. A great number of scholars take Shapiro’s investigations as the base for their research in the area of peristaltic mechanism. Most of the studies and research work on peristaltic flow deals with Newtonian fluid. In the biological point of view this mechanism motivated investigations in different non-Newtonian fluid also.

There is a great number of engineering processes in peristaltic pumping mechanism. In chemical and pharmaceutical industries this mechanism is used to handle a wide range of fluids.

In physiology peristaltic process is found in blood pumping in heart and lungs. The biological fluids behave like non-Newtonian fluids. Therefore interest is focused on non-Newtonian fluid process.

Srivastava studied Peristaltic transport of blood: Casson model. Mekheimer carried out his investigations in the peristaltic transport of HMD flow in an inclined planar channel.

Hence several researchers and investigators studied the flow behavior of non-Newtonian fluid in physiological system of living bodies. Among them hyperbolic tangent system explains in detail. In this paper the graphical result are presented to discuss the physical behavior of various parameters of interest.

MATHEMATICAL FORMULATION

Let us consider the peristaltic transport of an incompressible Newtonian Fluid flow in an inclined asymmetric channel of half-width. The angle of inclination is α. A sinusoidal wave propagating with constant speed c on the channel walls induces the flow. The geometry of the wall surface is defined as

$$Y = H = a + b\sin\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\bar{t}\right)\right] \tag{1}$$

Where λ is the wave length, c is the velocity of propagation, $\bar{t}$ is the time and b is the amplitude

Under the assumption that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame moving with velocity c away from the fixed frame. The transformation between these two frames is defined by

$$\bar{x}=\bar{X}-c\bar{t} \quad , \bar{y}=\bar{Y} \quad , \bar{u}=\bar{U}-c, \bar{v}=\bar{V}\ , and \quad \bar{p}(x)=\bar{P}(\bar{X},t). \tag{2}$$

Where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{3}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{x}}+\mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\frac{\sigma}{\ }-\frac{\partial \bar{\tau}_{\overline{XY}}}{\partial \bar{Y}}+\eta \sin\alpha, \tag{4}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B_0^2}{1+m^2}\left(m(u+c)+v\right)-\rho g\cos\alpha \tag{5}$$

Where ρ is the density, σ is the electrical conductivity, B_0 is the magnetic field strength and m is the Hall parameter.

The dimensional boundary conditions are

$$u=-c \quad \text{at} \quad y=H \tag{6}$$

$$\frac{\partial u}{\partial v}=0 \quad \text{at} \quad y=0 \tag{7}$$

Introducing the non-dimensional quantities

$$\bar{x}=\frac{x}{\lambda},\ \bar{y}=\frac{y}{a},\ \bar{u}=\frac{u}{c},\ \bar{v}=\frac{v}{c\delta},\ \delta=\frac{a}{\lambda},\ \bar{p}=\frac{a^2}{\mu c\lambda},\ \bar{t}=\frac{ct}{\lambda},\ h=\frac{H}{a},\ \phi=\frac{b}{a},\ \bar{q}=\frac{q}{ac},\ M^2=\frac{\sigma a^2 B_0^2}{\mu} \tag{8}$$

From the equations (3) to (5), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{M^2}{1+m^2}\left(m\delta v-(u+1)\right)+\frac{\mathrm{Re}}{Fr}\sin\alpha \tag{10}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\delta M^2}{1+m^2}\left(m(u+1)+\delta v\right)+\frac{\delta\,\mathrm{Re}}{Fr}\cos\alpha \tag{11}$$

Where $Fr=\frac{c^2}{ag}$ is the Froude number, M is the Hartmann number and Re is the Reynolds number. Using long wavelength i.e. δ is less than 1, approximation, the equations (10) and (11) become

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{\partial p}{\partial x}-\frac{\mathrm{Re}}{Fr}\sin\alpha+\frac{M^2}{1+m^2} \tag{12}$$

$$\frac{\partial p}{\partial y}=0 \tag{13}$$

From equation (13), it is clear that p is independent of y .Therefore Eq. (12) can be rewritten as

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha+\frac{M^2}{1+m^2} \tag{14}$$

The corresponding non-dimensional boundary conditions are given as

u=-1 at y=H (15)

$\frac{\partial u}{\partial y}=0$ at y=0 (16)

Knowing the velocity, the volume flow rate q in a wave frame of reference is given by

$$Q=\int_0^h u\,dy \tag{17}$$

The instantaneous flow $Q(X,t)$ in the laboratory frame is

$$Q(X,t)\int_0^h U\,dY=\int_0^h (u+1)\,dy=q+h \tag{18}$$

The time averaged volume flow rate $\overline{Q}$ over one period $T=\frac{\lambda}{c}$ of the peristaltic wave is given by

$$\overline{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{19}$$

SOLUTION OF THE PROBLEM

Solving equation (14) together with the boundary conditions (15) and (16), we get

$$u = \frac{1}{\beta^2}\left(\frac{dp}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left[\frac{\cosh\beta y}{\cosh\beta h} - 1\right] - 1 \quad (20)$$

where $\beta = M/\sqrt{1+m^2}$

The volume flow rate q in a wave frame of reference is given by

$$q = \frac{1}{\beta^3}\left(\frac{dp}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left[\frac{\sinh\beta y - \beta h\cosh\beta h}{\cosh\beta h}\right] - h \quad (21)$$

From equation (21) we write

$$u = \frac{1}{\beta^2}\left(\frac{dp}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left[\frac{\cosh\beta y}{\cosh\beta h} - 1\right] - 1 \quad (22)$$

$$\frac{dp}{dx} = \frac{(q+h)\beta^3\cosh\beta h}{\sinh\beta h - \alpha h\cos\beta h} + \frac{\mathrm{Re}}{Fr}\sin\alpha \quad (23)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx}dx \quad (24)$$

Note that, as $\alpha \to 0, M \to 0$ and $m \to 0$ our results coincide with the results of Shapiro et al. (1969).

RESULTS AND DISCUSSION

The variation of pressure rise Δp with time -averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5, \mathrm{Re} = 5, \alpha = \frac{\pi}{4}, Fr = 2$ and $m = 0.2$ is depicted in Fig. 1. It is found that, the time-averaged flow rate $\overline{Q}$ increases in the pumping region Δp is positive with increasing M, while it decreases in both the free-pumping Δp is equal to 0 and co-pumping Δp is negative regions with increasing M.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Froude number Fr with $\phi = 0.5, \mathrm{Re} = 5, \alpha = \frac{\pi}{4}, m = 0.2$ and $M = 1$ is shown in Fig. 2. It is observed that as increase in Fr decreases the time averaged flow rate $\overline{Q}$ in all the pumping, free -pumping and co-pumping regions.

Fig. 3 shows the variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Reynolds number Re with $\phi = 0.5, Fr = 2, \alpha = \frac{\pi}{4}, m = 0.2$ and $M = 1$. It is found that, on increasing increases the time averaged flow rate $\overline{Q}$ in all the pumping, free-pumping and co-pumping regions.

Fig. 4 lustrates The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5, \mathrm{Re} = 5, \alpha = \frac{\pi}{4}, Fr = 2$ and $M = 1$. It is observed that, the time-averaged flow rate $\overline{Q}$ decreases in the pumping region with an increase in m, while it increases in both the free -pumping and co-pumping regions with increasing m.

Fig. 5 depicts the variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $\alpha = \frac{\pi}{4}, \mathrm{Re} = 5, \mathrm{Re} = 5, m = 0.2$ and $M = 1$ and . It is observed that, the time-averaged flow rate $\overline{Q}$ increases with increasing amplitude ratio ϕ in both the pumping and free pumping regions, while it decreases with increasing amplitude ratio ϕ in the co-pumping region for chosen.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of inclination angle α with $\phi = 0.5, \mathrm{Re} = 5, \mathrm{Re} = 5, m = 0.2$ and $M = 1$ and is presented in Fig. 6. It is noticed that, the time averaged flow rate $\overline{Q}$ increases with increasing α in all the pumping, free-pumping and co-pumping regions.

CONCLUSION

In this chapter, the effect of Hall on the peristaltic flow of a conducting fluid in an inclined channel under the assumption of long wavelength approximation is investigated. The expressions for the velocity and pressure gradient are obtained analytically. It is found that, the time-averaged flow rate in the pumping region is increases with increasing Hartmann number Reynolds number, angle of inclination or amplitude ratio, whereas it decreases with increasing hall parameter or Froude number.

Further it is observed that, the pumping is more for vertical channel. When the horizontal channel $\alpha = 0$ and $\alpha = \frac{\pi}{4}$.

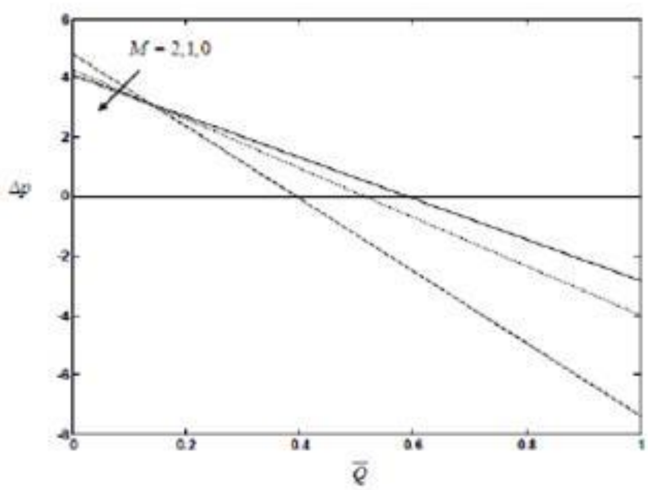

Fig.1

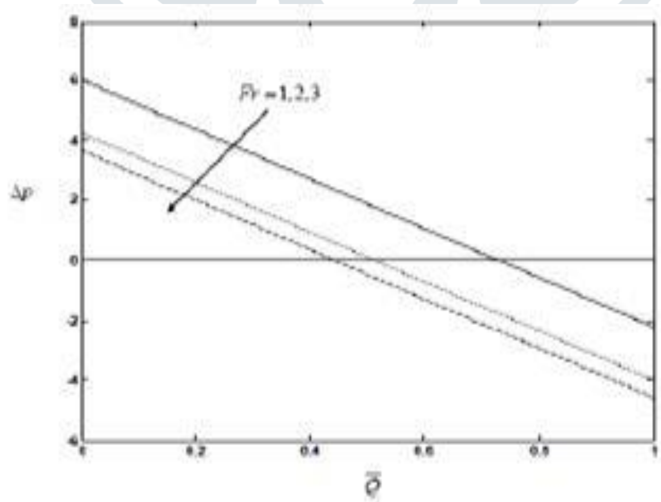

Fig.2

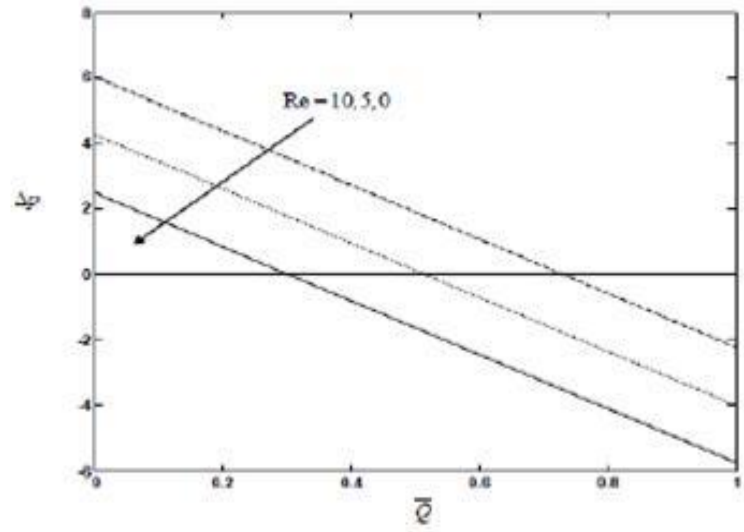

Fig.3

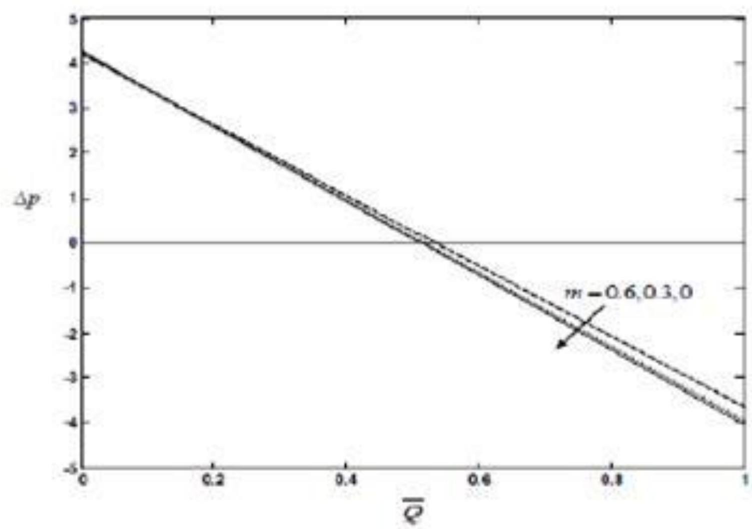

Fig.4

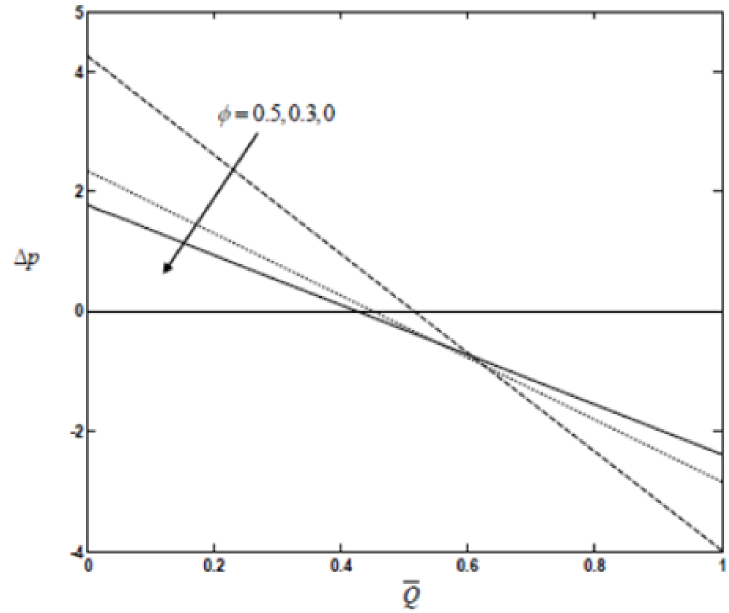
$\phi = 0.5, 0.3, 0$
Δp
$\overline{Q}$

Fig.5

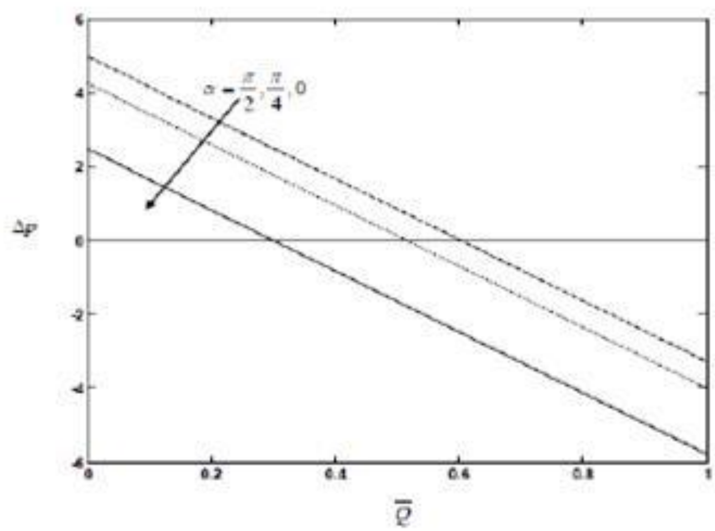
$\alpha = \frac{\pi}{2}, \frac{\pi}{4}, 0$
Δp
$\overline{Q}$

Fig.6

REFERENCES

[1]. Agrawal, H.L and Anwaruddin, B. Peristaltic flow of blood in a branch, Ranchi University Math. J. 15(1984),111-121.

[2]. Eldabe, N.T.M., Ahmed Y. Ghaly, A.Y., Sallam,S.N., Elagamy, K. and Younis, Y.M. Hall effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer, Journal of Applied Mathematics and Physics, 2015, 3, 1138-1150.

[3]. Hayat, T., Ali, N, and Asghar, S. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, Phys. Letters A, 363(2007), 397-403.

[4]. Jaffrin, M. Y. and Shapiro, A. H. Peristaltic Pumping, Ann. Rev. Fluid Mech., 3(1971), 13-36.

[5]. Guyton, A.C., and Hall, J. E. Text book of medical physiology, Elsevier: Saunders Co., 2003. Tenth edition.

[6]. Latham, T. W. Fluid motions in peristaltic pump, M.S. Thesis, MIT, Cambridge, Massachusetts, 1966.

[7]. Mekheimer, Kh.S. Peristaltic flow of blood under effect of a magnetic field in a non uniform channel, Appl.Math. Comput, 153 (2004), 763-777.

[8]. Shapiro, A.H., Jaffrin, M.Y. and Weinberg, S.L. Peristaltic pumping with long wavelengths at low Reynolds number, J. Fluid Mech. 37(1969), 799-825.

[9]. Srivastava, L. M. and Srivastava, V. P. Peristaltic transport of blood: Casson model II, J. Biomech., 17(1984), 821-829.