Optimization of slit profile in wavering condition through fuzzy linear programming

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Abstract

In this paper the demonstrating and optimization of dregs influx through *T*(b,i)number Fuzzy straight programming (FLP) issue is decided. Within the reasonable combination of circumstances for direct programming, there exists a certain increase or diminish within the confined values or the availability of the imperatives which leads to uncertainty within the decision-making prepare or in objective work of LPP.FLP may be a sales of fluffy framework and optimization in direct decision-making issues and the mistake can be moderated. In this paper, the approach of the optimized fluffy direct programming issue is connected to the bed profile information of the Bhakra dam. This bed profile information constitutes the records of water influx, outpouring, and precipitation and dregs influx from the year 1974-75 and after that from the year 1995-2003. The water influx, surge, and precipitation imperatives were focused on in arrange to optimize the dregs influx. The slightest and most prominent bound are computed for the dregs optimizes comes about for objective function and then optimized fuzzy LPP is obtained. The optimized FLP provides the membership function for the optimization of sediment inflow. This membership grade depicts that how the increase in the sediment inflow is much feasible for the system.

Keyword: *Fuzzy linear programming, modelling and optimization, membership function and sediment inflow*

1. Introduction

Operational research has been intently accredited as pertinent to the strengthening of optimization. Linear programming problem helps in attaining optimum use of productive factors. It provides better tools for meeting the changing condition. It deals with the problem of optimum output. Optimization is a mathematical technique which is used to optimize function of several variables subject to a set of constraints. It is a scientific approach for solving for executive management and appeared during 2nd world war when, the transportation problem of resources was formulated. Optimization is a mathematical tool of solving real and pragmatic problems by means of linear function where the variable involved subject to constraints (inequalities). The standard form of linear programming problem can be expressed as:

$$MaxZ = \sum_{i=1}^{n} C^{*} x_i \text{Subjectto-} \sum_{i=1,j=1}^{m,n} A^{*}{}_{ij} x_j \le b^{*}{}_{i} \quad \forall x_j \ge 0 \tag{1.1}$$

In the general LPP the values of $b^{*}{}_{i}{}'s$ remains constant or we can say that the availability of constraints remains constant but if the value of $b^{*}{}_{i}$ increase or decrease at a certain point of time then this general LPP is called the fuzzy LPP. The membership grade for the fluctuated value of $b^{*}{}_{i}$is introduced using FLPP models. This membership function fulfils the condition to obtain the optimized results which is in the range of least and greatest bounds for the LPP. Then our original problem will be converted into an equivalent crisp problem. There can be two situations for fuzziness in the LPP (a)The objective function may be fuzzy with crisp constraints (b)The constraints may be fuzzy with crisp objective function. Here the crisp deal with the term which is clearly defined and no fuzziness exists. Fuzzy means lacking of clarity or any concept which is vague in any manner. For example, the degree of satisfaction for the character "Young "could be different for different indusial. Generally, in the set theory there exists clarity about the belongingness of elements but fuzzy set theory deal with the partial belongingness of the elements to the set. For example, if any element belongs to the set, we assign it the membership grade 1 and if not then 0 but in fuzzy set theory we can also assign the membership grades between 0 and 1. The fuzzy logic concept was first introduced by Loft Zadeh. By using FLP, we can quantify changes in certain objective function if it constrains are changed. There are several real-life solicitations of FLP such as corporate and industrial efficiency evaluations. The idea of intuitionist fuzzy set was given by K. T. Atanassov [1]and then J. J. Lu and S. C. Fang[2]applied nonlinear optimization problems with in fuzzy decision-making problems. After that the duality results were expressed with nonlinear fuzzy to mathematical programming problems by H.-C. Wu [3,4 and 6] he also applied the duality results with nonlinear fuzzy.K. Maityet.al [5] concentrated on fuzzy possibility and necessity constraints and their de-fuzzification. With the existence of optimal control and the fuzzy linear programming many researchers introduced methods for the solution of these problems in [7-10]. Some new method using intuitionist fuzzy number and operations in the LPP was introduced in [12-15]. Y. R. Fan et.al [16] focused on various uncertainties in the optimization process and provides the ideal solution of the problem. Ebrahimnejad and M. Tavana[17] used the trapezoidal fuzzy number to obtain the feasible solution of the problem. Multi-objective reliability optimization was carried out by H. Garg et.al [18]. The fuzzy fractional approaches was used by Das et al [19] in which optimization was done under multi criteria decision making system and further generalization was provided in [20].In this paper we are using the triangular (right angle) fuzzy number LPP to deal with probabilistic increment (pi) in the basic availability (bi) of classical optimisation and analysing the result with targeted membership grade. Triangular fuzzy numbers are used to interpret the feasible uncertainty and incomplete information in decision-making, risk rating, and expert systems.

2. Preliminaries

2.1: Fuzzy Set and its Components

Definition 2.1.1: Let X^*be the universal spaceand a fuzzy set$\tilde{A}$, is a set in which each element of the set$X^*$is associated with a membership grade defined as:

$$\tilde{A} = \{(x^*, \mu_{\tilde{A}}(x^*)): x^* \epsilon X^*, \mu_{\tilde{A}}(x^*)) \to 0\ 1]\}$$

Definition2.1.2: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]for\alpha^* \in \mu_{\tilde{A}}(x^*), then\ \alpha^* -cut$ of$\tilde{A}$is defined as $\tilde{A}^{\alpha^*}$

$$\tilde{A}^{\alpha} = \{x^* | \mu_{\tilde{A}}(x^*) \geq \alpha^*, x^* \epsilon X^*\}$$

Definition 2.1.3: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*), then\ \ strong\ \alpha^* -cut$ of$\tilde{A}$ is defined as $\tilde{A}^{\alpha^{*+}}$

$$\tilde{A}^{\alpha^{*+}} = \{x^* | \mu_{\tilde{A}}(x^*) > \alpha^*, x^* \epsilon X^* \}$$

Definition 2.1.4: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*)$, then the support of $\tilde{A}$ is defined as

$$\tilde{A}^s = \{x^* | \mu_{\tilde{A}}(x^*) > 0, x^* \epsilon X^* \}$$

Definition2.1.5: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ for$\alpha^* \in \mu_{\tilde{A}}(x^*)$, then the height of $\tilde{A}$ is defined as $H(\tilde{A}) = max(\mu_{\tilde{A}}(x_i^*))$ and $\tilde{A}$ is said to be normal if $H(\tilde{A}) = 1$.

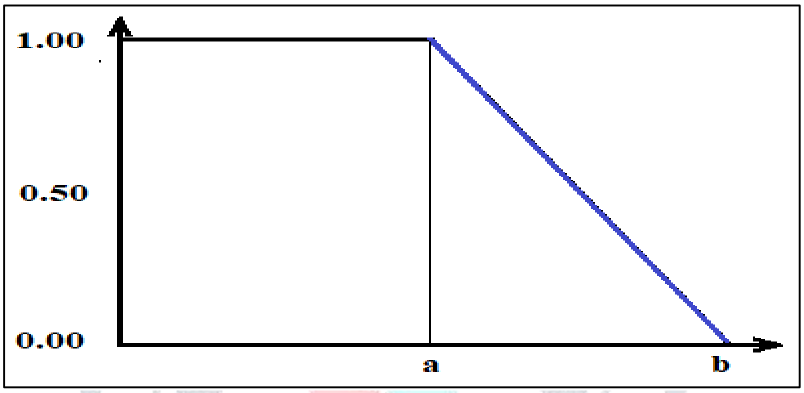
Definition 2.1.6: Let a fuzzy set$\tilde{A}$defined on the universal space$X^*$with $\mu_{\tilde{A}}(x^*)) \to [0\ 1]$ then it is called convex, if for any two $x_i^*, x_j^* \epsilon X^*$

$$\mu_{\tilde{A}}\{ \lambda x_i^* + (1-\lambda)x_j^* \} \geq \min\{\mu_{\tilde{A}}(x_i^*), \mu_{\tilde{A}}(x_j^*)\} \text{ ,Where } 0 \leq \lambda \leq 1.$$

Definition 2.1.8: LetX^*be the universal space of real number and$a , b\ and\ \ c\ \epsilon\ X^*$

$such\ that\ a < b < c$ then a triangular fuzzy number$\tau_{b,i}$ with membership grade $\tau_{b,i}(x)$ is defined as

$$\tau_{b,i}(x) = \left\{ \begin{matrix} 1 & x < a \\ \frac{b-x}{b-a} & a \leq x \leq b \\ 0 & x > b \end{matrix} \right\} \text{ where } i = b - a.$$



Graph1: Triangular fuzzy number $\tau_{b,i}$with its membership grade.

2.2: The mathematical representation of LPP with fuzzy coefficients: -

$$MaxZ = \sum_{j=1}^{n} \tilde{c}_j^* x_j \text{Subject to } \sum_{j=1}^{n} \tilde{a}_{ij}^* x_j \leq \tilde{b}_i^*, 1 \leq i \leq m, 1 \leq j \leq n \text{And } x_j \geq 0 \qquad (2.1)$$

Here the term $\tilde{a}_{ij}^*, \tilde{b}_i^* \tilde{c}_j^*$represents the fuzzy coefficients.

But some linear programming problem which contains the coefficients $\tilde{a}_{ij}^*, \tilde{b}_i^*$as acrisp value and can be modelled with $\tilde{b}_i^*$ as fuzzy value with some grade. For the both FLPPs, the problem is first converted into crisp problem. It is a problem of ruling a point which fulfils the constraints and the objective function. Thus, the important step in the FLPP is to transform it into the crisp value problem. Then this crisp problem can be simply resolved by any appropriate method of LP which will give the least and greatest bounds for the FLP. This paper deals with the optimization of sediment inflow using fuzzy approach when there is increase in the constraints in the bed profile data of Bhakra dam.

3. Methodology

Sediment is the material that settles to the bottom of the water reservoirs. It can be in the form of silt, sand, rocks and other matter carried and deposited by water. The constraints are those quantities which have an effect on the objective function. So this optimization will be with respect to the constraints of water inflow, outflow and rainfall.

Here the total availability with respect to constrains of water inflow, outflow and rainfall is in fluctuated mode so in system (2.1) the $\tilde{b}_i^*$ are $\tau_{b,i}$ type fuzzy numbers

$$MaxZ = \sum_{j=1}^{n} c_j x_j$$

Subject to $\sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b_\iota}^*, i = 1,2 \dots, m, j = 1,2 \dots, n, x_j \geq 0$ (3.1)

The linear function with Max Z is the objective function for optimization and the other equation represents the constraints. The $\widetilde{b_\iota}^*$ demonstrates the availability of constraints that the priority symbol implies that the quantity is fuzzy, which means that the volume is increased or decreased after some time.

$MaxZ = \sum_{j=1}^{n} c_j x_j$ Subjected to $\sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b_\iota}^* \sim \widetilde{b_\iota}^* + p_i, i = 1,2 \dots, m, j = 1,2 \dots, n$

$, x_j \geq 0$ (3.2)

The total availability of the constraints $\widetilde{b_\iota}^*$ increases probabilistically till $\widetilde{b_\iota}^* + p_i$. Hence the membership grades in-between increased and basic restrictions will be implemented. The membership grades for $\widetilde{b_\iota}^*$ are defined as

$$\widetilde{b_\iota}^* = \begin{cases} 1 & when\ x \leq \widetilde{b_\iota}^* \\ \frac{\widetilde{b_\iota}^* + p_i - x}{p_i} & when\ \ \widetilde{b_\iota}^* \leq x \leq \widetilde{b_\iota}^* + p_i \\ 0 & when\ \ \geq \widetilde{b_\iota}^* + p_i \end{cases} \quad (3.3)$$

After that the least and greatest bounds are obtained

The value for least bound **(Zl)** is as follow-

$$MaxZ = \sum_{j=1}^{n} c_j x_j$$

Subject to $\sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b_\iota}^*, i = 1,2 \dots, m, j = 1,2 \dots, n$ (3.4)

The greatest bound **(Zu)** is as follow-

$$MaxZ = \sum_{j=1}^{n} c_j x_j$$

Subject to $\sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b_\iota}^* + p_i, i = 1,2 \dots, m, j = 1,2 \dots, n, x_j \geq 0$ (3.5)

Both the LPP for greatest and least bound can be solved using the simplex approach and then to get the fuzzy function for membership grade the OFLPP structure is described as.

$$Max\ Z = \lambda^*$$

Subject to $\lambda^*(Zu - Zl) - cx \leq -Zl$

$\lambda^*(p_i) + \sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b_\iota}^* + p_i, x \geq 0, 0 \leq \lambda^* \leq 1$ (3.6)

This system will provide the all the decision variable and the membership grade using them the objective function of initial LPP can be optimized

4. Data source and technique

The data source for this paper work is secondary. The data sample for this paper contains the every years data from 1974 to 1979 and then from 1997 to 2003. This data is for water inflow, outflow, and rainfall and sediment inflow of Bhakra dam. The units of data are million m^3except for rainfall. The unit of rainfall is mm. The methodology explained above is a quantitative technique for solving these types of problems. The methodology for this paper work is based on the deductive approach that is the results derived from this work will be specific generalised from the given general result.

The results analysed for this work will be related to the effect of increase in the constraints i.e water inflow, outflow and rainfall on sediment inflow. The change in the sediment inflow will be analysed with respect to the change in the other quantities of constraints.

4.1: Problem Identification

The Bhakra dam was commissioned in 1963 and one of the oldest dams in India. This dam has brought many positive changes in north India like benefits to irrigation and power. Total water storage capacity of the dam is 2431.81 Mm^3. The Sutlej River canalizes massive quantity of sediment through water, which deteriorates the storage capacity of reservoir.

Owing to deforestation, overgrazing and improper unscientific agriculture techniques the slit is added every year in the basin. Some other natural factors also attributes to the high level of sediment transport. The data for this paper work is as follow:-

Year	Inflow(million m^3)	Outflow(million m^3)	Rainfall (mm)	Sediment inflow(millionm^3)
1974-75	10871	10871	858	22
1975-76	17625	17625	901	39
1976-77	14782	14782	780	17
1977-79	39607	33680	2188	92
1995-97	39422	30474	1850	73
1997-99	37744	29300	1806	71
1999-01	32625	27460	1523	90
2001-03	31589	24561	1608	68

Table: 4.1Shows the bed profile data of Bhakra dam

This chart contains the bed profile data of Bhakra dam. There is a data for every two years from 1974-79 and from 1995-2003. This chart contains the data for water inflow, outflow, and rainfall and sediment inflow. The units of inflow, outflow and sediment inflow are in million m^3 and the units of rainfall is mm.

From this data the inflow of sediment is taken as an objective function which is analysed to be as minimum nature with respect to the water inflow, outflow and rainfall. The total availability of water inflow and out flow and rain fall is not fixed but could be increased or decreased. As per the given data the total availability of water inflow from year 1974-03 is at least 82885 mm^3 but it may be extended till 141381 mm^3. In the same way the increase in water outflow is from 76958 to 111796 mm^3 and in rainfall it is from 4727 to 6786 mm^3. The task is to optimize sediment inflow in the situation of this increase in availability of constraints. These are the graphical representations of entries of chart acc. to data in different years.

5. Modelling and Result

As discussed above the objective is to know how minimum sediment inflow could be there using the available constraints. Let $x_1, x_2, x_3, ..., x_8$ be variables for different eight years given in the bed profile data and Z is total quantity of sediment inflow. The modelling of the data is shown by system (5.1) and structure of OFLPP is shown by the system (5.2) here the λ^* is taken as λ

$$MinZ = 22x_1 + 39x_2 + 17x_3 + 92x_4 + 73x_5 + 71x_6 + 90x_7 + 68x_8$$

Subjected to

$$10871x_1 + 17625x_2 + 14782x_3 + 39607x_4 + 39422x_5 + 37744x_6 + 32625x_7 + 31589x_8 \geq 82885 \sim 141381$$

$$10871x_1 + 17625x_2 + 14782x_3 + 33679x_4 + 30474x_5 + 29300x_6 + 27460x_7 + 24561x_8 \geq 76958 \sim 111796 .$$

$$858x_1 + 901x_2 + 780x_3 + 2188x_4 + 1850x_5 + 1806x_6 + 1523x_7 + 1608x_8 \geq 4727 \sim 6786$$

$$x_1, x_2\, x_3, x_4, x_{5,} x_6, x_7 x_8 \geq 0 \text{ and } 0 < \lambda < 1 \qquad (5.1)$$

Max λ subjected to

$$60.57\lambda - 22x_1 - 39x_2 - 17x_3 - 92x_4 - 73x_5 - 71x_6 - 90x_7 - 68x_8 \leq -103.02$$

$$58496\lambda + 10871x_1 + 17625x_2 + 14782x_3 + 39607x_4 + 39422x_5 + 37744x_6 + 32625x_7 + 31589x_8 \leq 141381$$

$$34838\lambda + 10871x_1 + 17625x_2 + 14782x_3 + 33679x_4 + 30474x_5 + 29300x_6 + 27460x_7 + 24561x_8 \leq 111796 .$$

$$2059\lambda + 858x_1 + 901x_2 + 780x_3 + 2188x_4 + 1850x_5 + 1806x_6 + 1523x_7 + 1608x_8 \leq 6786$$

$$x_1, x_2\, x_3, x_4, x_{5,} x_6, x_7 x_8 \geq 0 \text{ and } 0 < \lambda < 1 \qquad (5.2)$$

B_i Represents the basic availability of the constraint and P_i is the probabilistic change in terms of increment in the availability.

The membership function in term of grade for these constraints are calculated as -

Let β_1 be the membership grade for water inflow and it varies as

$$\beta_1 = \begin{cases} 1 & when\ x \leq 82885 \\ \dfrac{B_i + P_i - x}{P_i} & when\ 82885 \leq x \leq 141381 \\ 0 & when\ x \geq 141381 \end{cases} \qquad (5.3)$$

This is equivalent to

$$\begin{cases} 1 & when\ x \le 82885 \\ \dfrac{141381 - x}{58496} & when 82885 \le x \le 141381 \\ 0 & when\ x \ge 141381 \end{cases} \tag{5.4}$$

Let β_2 be the membership grade for water outflow and it varies as

$$\beta_2 = \begin{cases} 1 & when\ x \le 76958 \\ \dfrac{111796 - x}{34838} & when 76958 \le x \le 111796 \\ 0 & when\ x \ge 111796 \end{cases} \tag{5.5}$$

Let β_3 be the membership grade for rainfall and it varies as

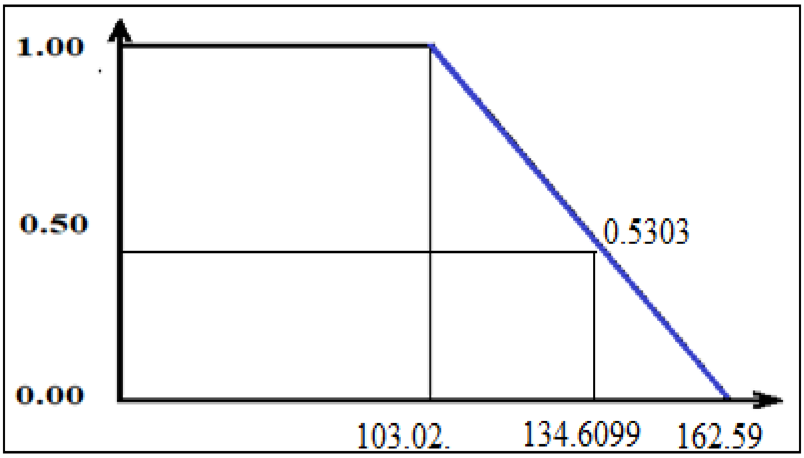
$$\beta_3 = \begin{cases} 1 & when\ x \le 4727 \\ \dfrac{6786 - x}{2059} & when 4727 \le x \le 6786 \\ 0 & when\ x \ge 6786 \end{cases} \tag{5.6}$$

.

Using described methodology, the least and greatest bounds for sediment inflow have calculated. It was observed that if there is an increase or probabilistic extension in the input constraints of this LPP the minimum sediment inflow is 103.02 unit and it can be extended till 162.59 units or the value of sediment inflow will lie between these least and greatest bounds.

$$\lambda = \begin{cases} 1 & when\ x \le 103.02 \\ \frac{162.59 - x}{59.57} & when\ 103.02 \le x \le 162.59 \\ 0 & when\ x \ge 162.59 \end{cases} \tag{5.7}$$

These least and greatest bound are used to find the optimized fuzzy linear programming problem (OFLPP) which will give the membership grade for the objective function that is the minimization of sediment inflow. This solution gives the optimum value of sediment inflow from initial objective function. Graph 2 explained the how the sediment inflow illustrated using the membership function and the following remarks are observed



Graph 2: The optimized membership grade and sediment inflow

- From year 1974 to 2003 when there is increase in the availability of constraints the minimum sediment inflow is 103.02millionm^3 and it can be reached till 162.59millionm^3
- The solution of optimized fuzzy LPP depicts membership grade $\lambda = 0.5303$ hencethe respective optimized sediment inflow is 134.6099 millionm^3.

Conclusion and Future Applications

In this paper the modelling and optimization of bed profile data of Bhakra dam is performed. The first attempt of fuzzy linear programming is made to optimize the sediment inflow. For this the sediment inflow is taken as an objective function and water inflow, outflow and rainfall are taken as the constraints. The performance parameters are least and greatest bounds and the membership grade. The overall results are satisfactory. The results will differ for any other change in the constraints. It can also be concluded that fuzzy linear programming is an application of fuzzy set theory in linear decision making. This technique of fuzzy linear programming is of great significance for the future analysis of any system or organisations. This technique can be applied to analyse the future performance of water reservoirs.

This approach can also be applied by factories and organisations to analyse the change in their future profits or costs when they are going to change their input constraints. They can estimate their future profits and loss by using the fuzzy linear programming problem. This way this technique can improve the performance of factories and organisations. This technique is easily applicable to real world like in banking and finance, agricultural economics and manufacturing and production. In near future this technique may be in more trends for decision processes of organizations.

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