A review on the theoretical models to predict the dynamic viscosity of Nanofluids

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Abstract

The dynamic viscosity of mixture, having the uniform suspension of solid particles of nanosized and based fluid as a component, depends on the various parameters such as hydrodynamic interaction between the particle-particle and particle-base fluid, electroviscous effect, Brownian motion, etc. Numerous theoretical models reported to investigate the effects of various parameters on the viscosity of nanofluids. The present study reviewed these models and the variation in the dynamic viscosity of nanofluid with respect to the particle concentration, particle size, thickness of surrounding boundary layer and temperature is also investigated. Few newly developed models based on the artificial neural network, thermodynamic mechanisms are also discussed.

Keywords: *Nanofluid, viscosity, Brownian motion, electroviscous effect, hydrodynamic interaction.*

INTRODUCTION

With the advancement of nanotechnology, the suspension of nanoparticles in the based fluid is an innovation to regulate the properties of prepared mixture as per the requirement of the applications. The primary advantage of the nanoparticle suspension is the enhancement in the thermal conductivity, which improves the heating or cooling processes. Numerous studies have been conducted on the characterization of nanofluids, including the measurement of thermo-physical properties of nanofluids.

In spite of increment in the thermal conductivity, the suspension of nanoparticles increases the viscosity of the mixture also, which affect adversely the cooling or heating processes. An increment in the viscosity of mixture due to suspended nanoparticles is comparatively less than the micron sized particles and affects less to fluid flow processes (e.g. forced convection) due to very smaller particle size, less than 100 nm. But some processes, those not involving the fluid flow, like natural convection, are affected by the increment in the viscosity in case of nanoparticle suspensions. Therefore, the viscosity plays a vital role to investigate the heating or cooling processes. Abundant studies, theoretical and experimental, have been directed on the prediction or measurement of viscosity variation due to nanoparticle dispersion and are summarized herein .

In the early age of investigations on the viscosity of the suspension, large number of mathematical models were presented to correlate the different parameters affected the viscosity of suspension and also have genuine significance in present consequence. In the present study, primarily the effect of Brownian motion, hydrodynamic interaction, electroviscous effect, and few newly developed mathematical models are investigated and the effect of these parameters on the dynamic viscosity of nanofluid is also discussed. The presented mathematical models are given in table 1.

BROWNIAN MOTION

In 1827, Robert Brown, a Scottish biologist, notified the zigzag motion of pollen grains in water but did not recognise the reason behind the motion. In 1906, Einstein formulated a relation between the Brownian motion and size of suspended spherical solid particle based on the kinetic theory and calculated the diffusion coefficient for the spherical particle. Einstein (1906) was the first to investigate the influence of the motion of the spherical particles on the viscosity of base liquid by using hydrodynamic equations. Einstein's correlation investigated the effect of Brownian motion of solid spherical nanoparticles in the base fluid and was valid for only dilute solutions. Einstein also introduced a shape factor, which depended on the shape, rigidity and Brownian motion of particles, in his expression and resulted that the coefficient of internal friction for a low particle concentration of suspended spherical particles is increased by a fraction which is equal to 2.5 times the total volume of the suspended spherical particles in a unit volume. But Kuntiz (1926) was not agreed with the Einstein's equation and set a new expression to compute the viscosity of solution of suspended particles having a higher concentration as 50% of solutions of such substances as sugars, glycogen, casein and rubber. To prove the accuracy of his expression; he computed the specific volume of solute and found that specific volume remains approximately constant for various concentrations of solute. Both Einstein and Kuntiz did not consider the interaction between the particles while estimating the viscosity of suspended particles.

Robinson (1949) extended the Einstein's equation to determine the viscosity of higher concentration of suspended spheres and experimentally investigated the viscosity of suspended glass spheres having a diameter of 10-20 microns and different types of base fluids such as motor oil, castor oil, polyethylene glycol, corn syrup and sucrose solution. Through experiments, Robinson (1949) observed the similar viscosity of solution to the base fluid at low concentrations of glass spheres similar to Einstein's equation.

Hinch and Leal (1972) investigated the rheology of a mixture of suspended spherical solid particles of low volume concentration in to the base fluid and considered the effect of rotary Brownian motion in shear flow for different cases of aspect ratio and dimensionless shear rate λ/D. A decrement in the effective viscosity was observed with increasing the shear strength (D/γ) in the transition region. Batchelor (1977) investigated the bulk stress in a suspension of interacting rigid spherical particles and derived an explicit expression to evaluate the role of Brownian motion for statistically homogeneous suspension.

HYDRODYNAMIC INTERACTION

Two type of interactions occurred in the colloidal dispersion, one is Brownian motion between the particle-particle, and other is hydrodynamic interaction between the dispersed and dispersion medium. The effect of hydrodynamic interaction on the viscosity of the colloidal dispersion was investigated by many researchers. Vand (1948) derived the mathematical expression to determine the viscosity of suspended spheres with and without considering the interaction between the suspended spheres. Derivation to calculate the viscosity of suspended particles without considering the interaction between the particles was similar to the Arrhenius formula. Vand (1948) also derived an expression to estimate the viscosity by considering the collisions between the suspended spheres caused by shearing motion of liquid with unequal velocity of different lamina, which primarily depended upon the concentration of spheres.

Table 1: Theoretical models for calculating the dynamic viscosity of solid-particle suspension.

Reference	Theoretical models	Outcome
Einstein (1906)	$\frac{\mu}{\mu_0} = (1 + 2.5\phi)$	Model considered the effect of Brownian motion and was only valid for low concentration of solid particles
Kuntiz (1926)	$\frac{\mu}{\mu_0} = \left(\frac{1 + 0.5\phi}{(1-\phi)^4}\right)$	Model was used for high particle concentration (up to 50%)
Robinson (1949)	$\frac{\mu}{\mu_0} = 1 + \frac{2.5\phi}{(1 - S'\phi)}$	Extension of Einstein's equation; for higher concentration
Hinch and Leal (1972)	$\frac{\mu}{\mu_0} = \left[1 + \phi\left\{\frac{5}{2} + \epsilon^2\left(\frac{78}{441} + \frac{3}{5}\left(\frac{(6D/\gamma)^2}{1 + (6D/\gamma)^2}\right)\right)\right\} + \cdots\right]$	Model considered the effect of Brownian motion and was only valid for low concentration of solid particles
Batchelor (1977)	$\frac{\mu}{\mu_0} = [1 + 2.5\phi + 6.2\phi^2]$	Model considered the effect of Brownian motion and was only valid uniform solution;
Vand (1948)	$\log_e \frac{\mu}{\mu_0} = 2.5\phi$ (No interaction b/w particles) $\log_e \frac{\mu}{\mu_0} = \frac{2.5\phi}{(1-Q\phi)}$ (Interaction b/w particles) $\frac{\mu}{\mu_0} = 1 + 2.5\phi + 7.349\phi^2 + ..$ (Collision b/w particles)	Consider slip mechanism at walls for higher concentrations and included the average fraction of time spent in collisions
Simha (1952)	$\frac{\mu}{\mu_0} = 1 + 2.5\phi + \left[\frac{125}{64\phi_{max}}\right]\phi^2 + \cdots$	Based on cage model; More adequate at higher concentrations
Graham (1981)	$\frac{\mu}{\mu_0} = \frac{9}{4}\left[1 + (\frac{h}{2r})\right]^{-1}\left[\frac{1}{\left(\frac{h}{r}\right)} - \frac{1}{\left[1 + \left(\frac{h}{r}\right)\right]} - \frac{1}{\left[1 + \left(\frac{h}{r}\right)\right]^2}\right] + (1 + 2.5\phi)$	Based on Einstein work and used cell theory to calculate the viscosity, valid for entire range of concentration;
Brinkman (1952)	$\frac{\mu}{\mu_0} = \frac{1}{(1-\phi)^{2.5}}$	Assumed the mixture as continuum
Cohen et al. (1997)	$\frac{\mu}{\mu_0} = (1 + 2.5\phi + 4.59\phi^2)$	Visco-elastic behavior of particles was calculated

Avsec and Oblak (2007)	$\frac{\mu}{\mu_0} = 1 + (2.5\phi_e) + (2.5\phi_e)^2 + (2.5\phi_e)^3 + (2.5\phi_e)^4 + \cdots$	Based on statistical approach at molecular level and assume a liquid layer on nanoparticles
Masoumi et al. (2009)	$\frac{\mu}{\mu_0} = 1 + \frac{1}{\mu_0} \times \frac{\rho_p V_B d_p^2}{72C\left(\sqrt[3]{\frac{\pi}{6\phi}}\right) d_p}$ $C = \mu_0^{-1}\left[(0.09d_p - 0.393) - (1.133d_p + 2.771)\phi\right]$	Model considered the effect of Brownian motion and function of size and concentration of suspended particles

The effect of the average fraction of time, particles spent in collision, on the shear rate was also considered. The expressions were valid for a wide range of concentrations of solid particles. Frankel and Acrivos (1967), through their asymptotic analysis, resulted in an expression to calculate the maximum attainable concentration of the suspended particles in the solution and expressed the high influence of the hydrodynamic interaction between the suspended particles on the viscosity rise in comparison of collisions, aggregations and inertial effects occurred. Simha (1952) investigated the effect of the spherical shape arrangement of solid particles around the central particle, called cells, to the hydrodynamic interaction between the suspended solid spherical particles. The viscosity of suspension was increased proportionally with the radius of the cell which was a function of particle concentration. Krieger and Dougherty (1959) investigated the non-Newtonian behaviour of the suspended rigid spherical particles. A flow equation was formulated based on the mechanism of interactions between the suspended neighbouring spherical particles by applying shear rate and a relation was found between the viscosity and shear rate. On the basis of work of Einstein (1906) and Frankel and Acrivos (1967), Graham (1981) neglected the effect of inertia, Brownian, London-van der Walls and electroviscous effect on the viscosity. For the entire range of particle concentrations, an expression to predict the dynamic viscosity was developed using the cell theory. For the very dilute and concentrated suspension, the formulated expression is reduced to Einstein's equation (1906) and Frankel and Acrivos's equation (1967), respectively.

Brinkman (1952) formulated the expression to evaluate the viscosity of spherical solid particles suspended in the fluid by assuming the mixture as a continuum and found increase in viscosity with particle concentration. Cohen et al. (1997) investigated the Newtonian viscosity and visco-elastic behaviour of concentrated neutral hard-sphere colloidal suspension. An increased effective viscosity was the fraction of colloidal particle pairs in interaction when particles collide and exchange the momentum and increases the dissipation. Cheng and Law (2003) proposed two expressions to evaluate the effective viscosity of suspended particles in fluid. First, without considering the dynamic effects between the particles and fluid, and then extended former to exponent formula to evaluate the effective viscosity for high particle concentration up to 35% after considering the inter-particle collisions and random motion of particles, i.e. Brownian motion. Barthelmes et al. (2003) theoretically examined the effect of concentrated suspension (up to 20 %) of non-spherical solid particles and shear rate on the transient behaviour due to the size distribution of the nanoparticles and viscosity. The population balance modal was considered for the coagulation and fragmentation of suspended particles.

ELECTROVISCOUS EFFECT

When a solid particle is brought in the contact with liquid, particle's surface acquired a charge due to ionization, ion adsorption or ion dissolution. Due to charged particles, counterions experience the attraction force towards the surface of the particle, while the co-ions experience the repulsive force away from the surface. The occurred charge difference at the surface of the particle and the surrounded liquid leads to the formation of double layers of ions, identified as electric double layer (EDL). The EDL consists two layers of ions, an inner layer of counterions, known as compact layer or stern layer and outer layer of co-ion. The ions are distributed due to the influence of electrical forces and random thermal motion in outer layer, known as diffuse layer. Due to the charge difference at the boundary of compact layer and diffuse layer, an electrical potential, called zeta-potential, is occurred. Debye length, thickness of the EDL, depends on the inverse of the square root of the ion concentration in the liquid and surface potential of the flow boundary.

Booth (1950) considered the effect of surface charge and electrical double layer on the effective viscosity of solid suspension and modified the Einstein's equation for very dilute suspension and thickness of double layer. The effective viscosity of solid suspension increased with the thickness of electrical double layer and the same effect tvanished when the radius of the solid particle was large as compared to the thickness of electrical double layer. Russel (1976) neglected the effect of hydrodynamic interactions and double layer distortion and considered the viscous force on individual particles and Brownian motion of individual suspended particles. He estimated the viscosity of suspension by using the secondary electroviscous effect. Natraj and Chen (2002) found numerically that viscosity was decreased as the double layer distortion was enhanced due to the additional stresses produced by the electrical interaction between the distorted ions and charge on the particle, and modification in fluid flow due to electrical body forces associated with charged spheres. Ruiz-Reina et al. (2005) developed a theoretical model to investigate the electroviscous effect including hydrodynamic interactions between the suspended particles and overlapping of the electric double layers. For doing so, the cell model theory was used by considering the thickness of electric double layer comparable with the inter-particle distance. Due to higher number of ions in the electric double layer for low zeta potential, more distortion of the flow around the particle was occurred and resulted in the increased dissipation energy as well as viscosity of suspension and opposite occurred for the high value of zeta potential and distortion of the flow reduced. Ohshima (2006) derived an expression for the effective viscosity of the dilute suspension of charged mercury drops. The effective viscosity of suspension was estimated to equal to that of uncharged rigid spheres at very high zeta potential, and the phenomenon was called as solidification effect.

Particularly for the nanofluids, the classical and statistical approaches were used to evaluate the physical properties of nanofluids in literature. Whereas classical mechanics was not proved a better option for the insight in to the microstructure, on the foundation of better insight in the intermolecular and intramolecular interaction between the particles, statistical mechanics calculated the physical and thermal properties of nanofluids with a good agreement compared to experimental data.

Avsec and Oblak (2007) formulated an expression to calculate the dynamic viscosity of nanofluids by considering molecular level layering of the liquid on the particle interface and clustering in nanoparticles. An effective volume concentration was used based on the thickness liquid layers on the particle interface to calculate the viscosity and it was found very good agreement with the experimental results for nanofluids of TiO_2 and Al_2O_3 nanoparticles having 27 nm and 13 nm mean diameter, respectively. Masoumi et al. (2009) formulated an equation to evaluate the dynamic viscosity of nanofluids as a function of temperature, particle diameter, Brownian motion and relative distance between the particles in suspension. The effective viscosity of suspension was decreased with temperature for constant particle concentration.

NEWLY DEVELOPED MODELS

In addition to the above discussed viscous models based on the above parameters, few models were also presented in the literature based on the different mechanisms and methods. Wei et al. established a model to calculate the dynamic viscosity of CuO/ethanol nanofluid using locally weighted moving regression (LWMR) method, based on the k-nearest neighbour algorithm. This model predicted precise value of dynamic viscosity using the measured values of the viscosity having the uncertainty values. By applying the friction theory, Bardool et al. (2019) developed a numerical model to predict the dynamic viscosity of the nanofluids by considering the dilute gas viscosity μ_{KE}, which depends on the kinetic theory of gases, and the residual viscosity $\Delta\mu$, used to consider the effect of deviation of the effective viscosity from dilute gas condition.

$$\mu = \mu_{KE} + \Delta\mu$$

The residula term of viscosity was estimated as a function of the attraction pressure (P_a) and repulsive pressure (P_r), given as

$$\Delta\mu = K_r P_r + K_a P_a + K_{rr} P_r^2$$

Here, K_r, K_a, and K_{rr} are the friction coefficients based on the temperature. The pressure terms were calculated by using the equation of state given by Peng - Robinson (1976) and Esmaeilzadeh – Roshanfekr (2006). Using these equations of state, an absolute relative deviations of 2.38 % and 2.46% were observed for the equation of state given by Peng - Robinson and Esmaeilzadeh – Roshanfekr, respectively. Gholami, Vaferi, and Ariana (2018) used the artificial intelligence (AI) based various models such as multi-layer perception, radial basis function neural network, cascade feedforward neural network, and least square support vector machines, to predict the accurate values of dynamics viscosity of nanofluids. Out of all the studied models, the multi-layer perception model was found the most accurate to predict the viscosity. Another advantage of the former model was the ability of incorporation of mathematical models of fluid dynamics to estimate the pressure drop and pumping power.

Based on the Maxwell – type constitutive equation, a thermodynamic model was presented by the Lebon and Machrafi (2018) to predict the viscosity of nanofluid as a function of particle concentration (ϕ), length of mean free path (l), radius of the particle (r) and thickness of surrounding layer (h), given as

$$\mu = \mu_0 \frac{1 + 2.5\phi(1 + h/r)^3}{1 + 4\pi^2\phi^2(1 + h/r)^4(l^2/r^2)}$$

Based on the above relation, it was found that the dynamic viscosity increased with the particle loading and decreased with the particle size.

CONCLUSIONS

In the present study, a detailed review has been conducted on the various mathematical or numerical models presented to predict the effective viscosity of the nanofluids. The effect of Brownian motion, electrical double layer, particle concentration, shear strength, temperature, hydrodynamic interactions, etc. on the viscosity have been discussed in the present work. The effects of various parameters on the viscosity of nanofluids, from theoretical studies, are summarized in following.

- An increment in the viscosity of nanofluid was reported with the concentration of nanoparticles.
- The effective viscosity of solid suspension was increased with the thickness of electrical double layer and the same effect was vanished when the radius of the solid particle was large as compared to the thickness of electrical double layer.
- The viscosity of nanofluids was decreased as the size distribution of dispersed solid particles was increased.
- The decrement in the viscosity of nanofluids was observed with the temperature due to increment in the Brownian motion, for constant particle concentration.
- The viscosity of nanofluids was decreased with increasing the shear strength.
- The viscosity of nanofluids was decreased as the double layer distortion was enhanced due to the additional stresses was produced by the electrical interaction between the distorted ions and charge on the particle.

In present, various mathematical modelling based on the different approaches, such as molecular dynamic simulation, etc. is also utilized to predict the viscosity. The advancement in the high-speed computing systems support the modelling with the more accurate processes. In future, these approaches with the high-speed computing systems can be more helpful to predict the viscosity with more accuracy.

NOMENCLATURE:

μ	Dynamic viscosity of nanofluid
μ_0	Dynamic viscosity of base fluid
Φ	Particle concentration
d_p	Diameter of nanoparticles

P	Density of nanoparticles
V_B	Brownian velocity
H	Thickness of the electric double layer
R	Radius of the nanoparticles
γ/D	Dimensionless shear rate

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