A comparative study of Several Continuous Distributions

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Abstract : A continuous probability distribution is a probability distribution with a cumulative distribution function that is absolutely continuous. Equivalently, it is a probability distribution on the real numbers that is absolutely continuous with respect to Lebesgue measure. Such distributions can be represented by their probability density functions.

Keywords : *continuous probability distribution , cumulative distribution and probability density functions*

1 Introduction

In [1] describes the importance of different types of continuous probability distribution according to the condition. The authors in [2] describes origin of probability concepts, random variables, the properties and examples of different distributions. The authors in [3] proposed to predict the continuous probability distribution of image emotions which are represented in dimensional valence-arousal space.

1.1 Definition

1.1.1 Continuous probability distribution:

A probability distribution in which the random variable X can take on any value (is continuous). Because there are infinite values that X could assume, the probability of X taking on any one specific value is zero.

2 Description

We will discuss some of the continuous probability distributions such as normal distribution, beta distribution, gamma distribution, Weibull distribution, exponential distribution.

2.1 Weibull distribution

In probability theory and statistics, the Weibull distribution is a continuous probability distribution. It is named after Swedish mathematician Waloddi Weibull, who described it in detail in 1951, although it was first identified by Fréchet (1927) and first applied by Rosin & Rammler (1933) to describe a particle size distribution.

2.1.1 Probability Density Function

The probability density function of a Weibull random variable is:

$$f(x;\lambda,k)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/k)^k}; x\geq 0\\ 0; x<0\end{cases}$$

where $k>0$ is the shape parameter and $\lambda>0$ is the scale parameter of the distribution. Its complementary cumulative distribution function is a stretched exponential function. The Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution $(k=1)$ and the Rayleigh distribution $(k=2, \lambda=\sqrt{2}\sigma)$.

If the quantity *X* is a "time-to-failure", the Weibull distribution gives a distribution for which the failure rate is proportional to a power of time. The shape parameter, *k*, is that power plus one, and so this parameter can be interpreted directly as follows:

- A value of $k<1$ indicates that the failure rate decreases over time (Lindy effect). This happens if there is significant "infant mortality", or defective items failing early and the failure rate decreasing over time as the defective items are weeded out of the population. In the context of the diffusion of innovations, this means negative word of mouth: the hazard function is a monotonically decreasing function of the proportion of adopters;

- A value of $k=1$ indicates that the failure rate is constant over time. This might suggest random external events are causing mortality, or failure. The Weibull distribution reduces to an exponential distribution.

- A value of $k>1$ indicates that the failure rate increases with time. This happens if there is an "aging" process, or parts that are more likely to fail as time goes on. In the context of the diffusion of innovations, this means positive word of mouth: the hazard function is a monotonically increasing function of the proportion of adopters. The function is first convex, then concave with an inflexion point at $(e^{1/k}-1)/e^{1/k}, k>1$.

In the field of materials science, the shape parameter *k* of a distribution of strengths is known as the Weibull modulus. In the context of diffusion of innovations, the Weibull distribution is a "pure" imitation/rejection model.

2.2 Exponential Distribution

In probability theory and statistics, the exponential distribution is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution, and it has the key property of being memoryless. In addition to being used for the analysis of Poisson point processes it is found in various other contexts.

The exponential distribution is not the same as the class of exponential families of distributions, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the normal distribution, binomial distribution, gamma distribution, Poisson, and many others.

2.2.1 Probability density function

The probability density function (pdf) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, x < 0 \end{cases}$$

Here $\lambda > 0$ is the parameter of the distribution, often called the rate parameter. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim \text{Exp}(\lambda)$.

The exponential distribution exhibits infinite divisibility.

2.3 Rayleigh distribution

In probability theory and statistics, the Rayleigh distribution is a continuous probability distribution for nonnegative-valued random variables. It is essentially a chi distribution with two degrees of freedom.

A Rayleigh distribution is often observed when the overall magnitude of a vector is related to its directional components. One example where the Rayleigh distribution naturally arises is when wind velocity is analyzed in two dimensions. Assuming that each component is uncorrelated, normally distributed with equal variance, and zero mean, then the overall wind speed (vector magnitude) will be characterized by a Rayleigh distribution. A second example of the distribution arises in the case of random complex numbers whose real and imaginary components are independently and identically distributed Gaussian with equal variance and zero mean. In that case, the absolute value of the complex number is Rayleigh-distributed.

2.3.1 Probability Density Function

The probability density function of the Rayleigh distribution is

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, x \geq 0$$

where σ is the scale parameter of the distribution.

2.4 Normal Distribution

In probability theory, a normal (or Gaussian or Gauss or Laplace–Gauss) distribution is a type of continuous probability distribution for a real-valued random variable. The general form of its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The parameter μ is the mean or expectation of the distribution (and also its median and mode); and σ is its standard deviation. The variance of the distribution is σ^2. A random variable with a Gaussian distribution is said to be normally distributed and is called a normal deviate. Normal distributions are important in statistics and are often used in the natural and social sciences to represent real-valued random variables whose distributions are not known.[1][2] Their importance is partly due to the central limit theorem. It states that, under some conditions, the average of many samples (observations) of a random variable with finite mean and variance is itself a random variable whose distribution converges to a normal distribution as the number of samples increases. Therefore, physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have distributions that are nearly normal. Moreover, Gaussian distributions have some unique properties that are valuable in analytic studies. For instance, any linear combination of a fixed collection of normal deviates is a normal deviate. Many results and methods (such as propagation of uncertainty and least squares parameter fitting) can be derived analytically in explicit form when the relevant variables are normally distributed. A normal distribution is sometimes informally called a bell curve.

3 Conclusion

On the other hand, a **continuous probability distribution** (applicable to the scenarios where the set of possible outcomes can take on values in a continuous range (e.g. real numbers), such as the temperature on a given day) is typically described by probability density functions (with the probability of any individual outcome actually being 0). The normal distribution is a commonly encountered continuous probability distribution. More complex experiments, such as those involving stochastic processes defined in continuous time, may demand the use of more general probability measures.

References

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