STUDY ON STRESS RELAXATION BEHAVIOR OF CANTILEVER BEAM USING ABAQUS AND VALIDATION THROUGH ANALYTICAL CALCULATION

[1]Anusha J [2]Dr. J Sharana Basavaraja , [3]Vivek Govindan

[1]PG Student, [2]Professor, [3]Senior Engineer

[1,2] Department of Mechanical, BMSCE, Bangalore, India-560019

[3]Molex India Business and services Pvt. Ltd., Bangalore, India-560066

Abstract: The purpose of this study was to correlate the analytical calculation with FEA to validate the cause of failure and the impact of the viscoelastic property. It aims to understand the linear viscoelastic behavior of Copper-Nickel alloy due to the axial and bending load acting on the cantilever beam. The time-dependent behavior of metals and polymers is due to time, stress, strain, and temperature. This behavior of the material is assumed as linear viscoelastic behavior when there is a small amount of strain. Viscoelastic phenomena such as creep and stress relaxation are common in polymers. A cantilever beam is used here to analyze the viscoelastic behavior, as it is the most common structural part of the connectors such as latch or terminals. The usual method of determining the values in most of the studies is using the Prony series. However, understanding the behavior using basic models like Maxwell is necessary to start with analytical and simulation results correlation and can be applied to the Generalized Maxwell model. Prony series constants are obtained by fitting the test data to the constitutive equation. The correlation is observed in results when a simpler model, cantilever beam under loads like axial and bending loads are applied. Indicating the study can be extended on the assembly level components as well.

IndexTerms – Viscoelasticity, Creep, Stress relaxation, Prony parameters

I. INTRODUCTION

Viscoelasticity of a material consists of the viscous and elastic behavior where the Dynamic concept as forces is applied first and the response is observed later. there is a lot of research involving the concept of viscoelastic effects in continuum mechanics.

This phenomenon is mostly not considered by the engineers during their simulations on the failure of the components. although the effect of this property does not seem to be significant it has its effect on failure. Necessary design criteria should be considered while designing metals as well as polymers.

The term stress relaxation is observed as the decrease of the internal stress in response to plastic strain generated in the structure due to localized flow, under constant strain for some finite interval of time. This phenomenon was explained using few basic models developed by early scientists like Maxwell and Kelvin-Voigt. The Mechanical behavior is represented using springs and dampers in various combinations possible [2]. The generalized Maxwell model gives a clear overview of the viscoelastic behavior like stress relaxation and creep. Due to the dislocations in the atomic level, the non-equilibrium is caused. The atoms in the stressed regions are in high energy and tend to return to a lower energy equilibrium position by diffusion with time and temperature which lowers the energy in the system and cause the mechanism of stress relaxation in the material in turn, the loss of contact force and decrease in the mechanical stability of the contact interference with micromotions. Similarly, creep occurs in the component with an increased strain rate and will become constant over time due to the hardening of the material.

II. THEORETICAL CONCEPTS AND MATHEMATICAL FORMULATION

2.1. Maxwell model

The Maxwell model is a simple model which consists of spring and dashpot assembled in series. This model has only one DOF. In this model, the applied stress remains the same for spring as well as a damper. The Initial strain is applied to the model and kept constant. Since the structure is under constant strain conditions for some finite interval of time, the structure retains a certain amount of plastic strain, this model is an elastic steady-state creep mode. Here the spring and dashpot or the damper represent the elastic and plastic strain respectively at time t=0, an initial strain is applied to the component which causes the motion of the damper representing the plastic strain, this property of the material is influenced by the Viscosity of the material, η. This model can be applied to any structure when a small strain is applied. As the value of strain increases the model used is not preferred as the behavior captured is linear.

2.2. Kelvin- Voigt Model for creep

The KV model Consists of a spring and dashpot in parallel. The strain experienced by the spring is the same as that experienced by the dashpot. When Sudden load is applied to the Kelvin model, the spring will want to stretch immediately but held by the dashpot as represented in figure1. Stress is taken up completely by dashpot initially. The creep curve thus starts with an initial slope. The stress starts to decrease in the dashpot and increases in the spring.

$$\frac{\sigma_0}{\eta}$$

The stress starts to decrease in the dash pot and increase in the spring.

Figure1: Kelvin Voigt response to the applied load

- When placed in parallel representation the response showed that it was suitable for visco-elastic solid but maxwell with series shows its suitable for viscoelastic fluid (no restriction in displacement, but in KV model the displacement is to be shared by both and gives a more realistic and intuitive understanding).
- The response time scale looks similar $\tau = {}^{\eta}/_{E}$ in both the cases.
- The behavior for stress relaxation in Maxwell gives $G(t) = G_0 e^{-t/\tau}$ exponential decay.

2.3. Generalized Maxwell model

The behavior of a structure is analyzed effectively as the number of elements in the model increases. Prony series using the Generalized Maxwell model for correlation of analytical calculations with the Abaqus simulation is preferred in this study.
• The Generalized Maxwell model, also known as the Weichert model, figure 2.
• It considers that the relaxation does not occur in a single time but at a distribution of times.
• Due to molecular segments of different lengths with shorter ones contributing less than longer ones, there is a varying time distribution.
• The Weichert model shows this by having as many spring–dashpot Maxwell elements as necessary to accurately represent the distribution.[3]

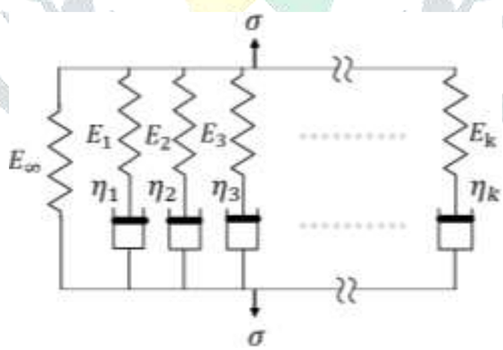

Figure 2: Weichert model

In first element there is no damper hence η = 0, therefore E_∞ is the only term remaining
At time t=0 no dampers are working hence only the springs are in parallel.
Therefore,

$$E_0 = E_\infty + \Sigma E_k$$

$$\sigma(t) = \varepsilon_0 [E[1 - \sum g_i (1 - e^{t/\tau_i})]]$$

where, g_k, k_k and τ_k are the Prony pairs.

Relaxation time is given by $\tau_k = \frac{\eta_k}{E_k}$ and $g_k = \frac{G_k}{G_0}$, $k_k = \frac{K_k}{K_0}$

III. MATERIALS AND METHOD

Table 1: Material data

Youngs modulus	Poisson ratio
110000	0.3

The study on linear viscoelastic property in Copper-Nickle material and the material properties are taken from the material data available (table 1). The viscoelastic effect on the component failure is analyzed initially using the Generalized Maxwell model. The material has a linear response as instantaneous properties such as cross-sectional dimensions are not considered.

For analytical calculations, the Generalized Maxwell equation is used. The behavior of this is represented on stress or strain v/s time graph by calculating for the time duration of 10 seconds. The results are compared with the simulation obtained when the axial or bending load is applied on the cantilever beam in Abaqus [5] with Prony constants as input parameters from (table 2).

Figure 3 represents the FEA model with dimensions which is considered for analytical as well as simulation. Cantilever beam is used for both axial and bending correlations.

Table 2: Prony constants

gi	ki	τr
0.20846	0	5.03E-06
8.46E-02	0	7.62E-03
4.07E-01	0	88.427

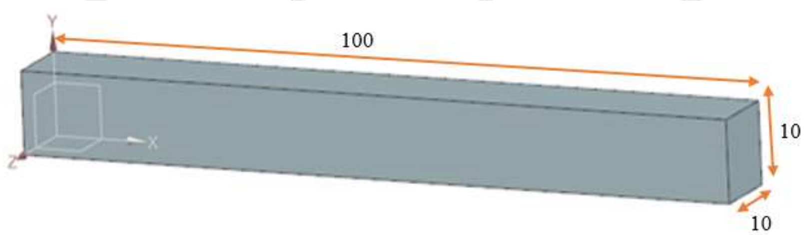

Figure 3: Cantilever beam model

IV. RESULTS AND DISCUSSION

1. Cantilever beam with axial loading

$$\sum_{i=1}^{N} \alpha_i \cdot e^{-t/\tau_i}$$

The analytical calculation for the cantilever beam, under viscoelastic conditions, was calculated using Equation [1]. The output simulation behavior was observed to follow a trend similar to that seen in the analytical calculation, shown in figure 4.1 and 4.2.

As per the analytical calculations done for the cantilever beam under creep, the initial and Final strain on the model, before and after creep loading, was observed to be 8.55e-6 and 1.23e-5 respectively. In comparison, the FEA simulation predicts the initial and final strain to be 8.8e-6 and 1.27e-5. A variation of 3% observed

between analytical calculations and simulations is attributed to approximations made in FEA, figure5.1 and 5.2. However, the trend of output observed between analytical, and simulations were seen to be similar.

For stress relaxation, though the graph had a similar pattern in the case of analytical as well as for the simulation, the error was almost 2.7% with the final values. Since the trend observed is similar and the error is mostly due to the calculation being ideal whereas, in the simulation, all aspects are considered during the interpretation of the results.

COMPARISON OF THE GRAPH BETWEEN HAND CALCULATION AND SIMULATION RESULTS

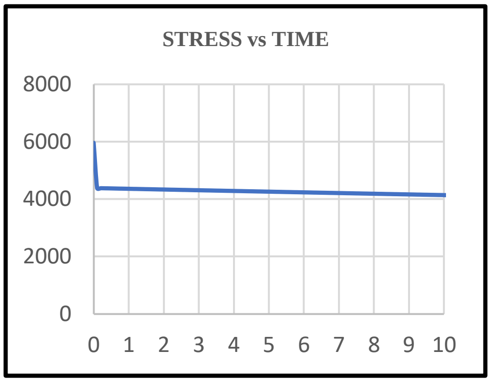

Figure 4.1: Stress relaxation analytical behavior for axial load

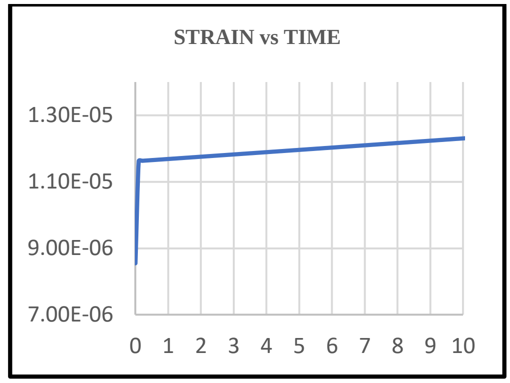

Figure 4.2: Creep behavior for analytical calculation axial load

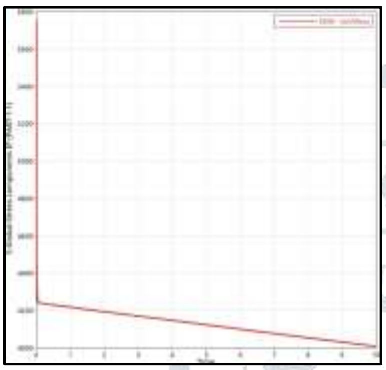
Figure 5.1: Stress relaxation simulation behavior for axial load

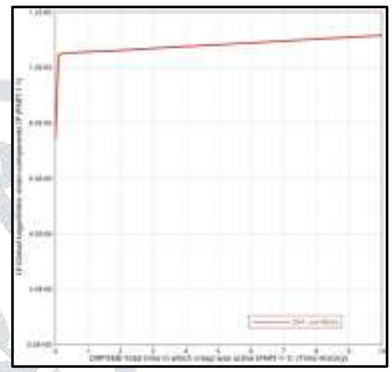
Figure 5.2: Creep behavior for Simulation axial load

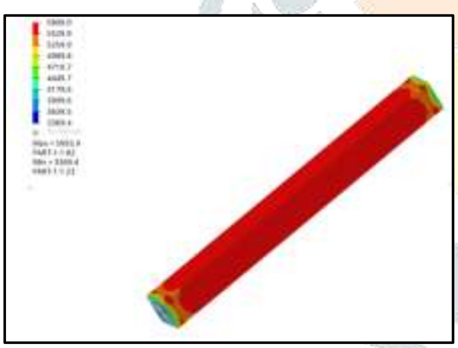
Figure 6.1: Stress relaxation simulation for axial load

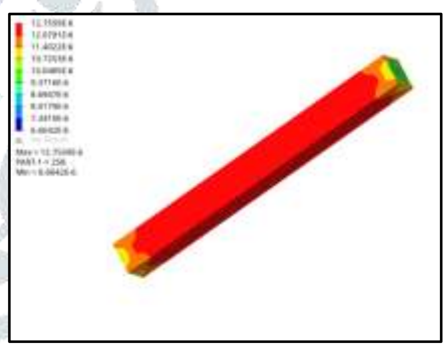
Figure 6.2: Creep Simulation for axial load

Figure 6.1 and 6.2 represents the stress relaxation and creep simulation using the ABAQUS. The range of values shows the distribution of the stress and strain across the cantilever beam respectively

2. Cantilever beam with bending load

As per the analytical calculations done for the cantilever beam under creep, the initial and Final strain on the model, before and after creep loading, was observed to be 0.000502848 and 0.000489 respectively. In comparison, the FEA simulation predicts the initial and final strain to be 0.001519714 and 0.001396. A variation of 3% observed between analytical calculations and simulations is attributed to approximations made in FEA. However, the trend of output served between analytical, and simulations were seen to be similar in figure 7.1 and 7.2 as well as 8.1 and 8.2 respectively.

Similar to axial load the stress relaxation with analytical and simulation had an error of 2% The error is not considered as the behavior observed is similar in terms of both cases and this error is mainly due to the equation used in the analytical case which is too ideal.

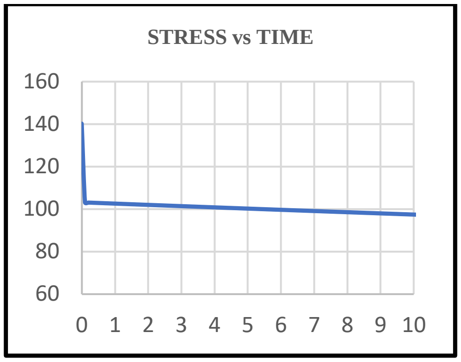

Figure 7.1: Stress relaxation analytical behavior for bending load

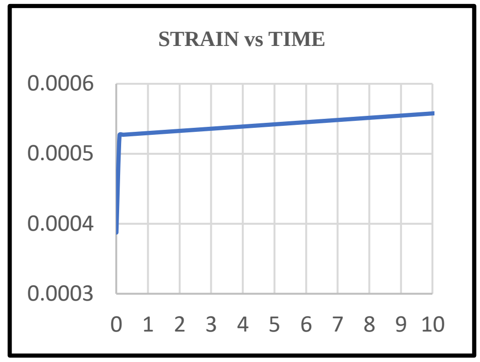

Figure 7.2: Creep behavior Analytical calculation for bending load

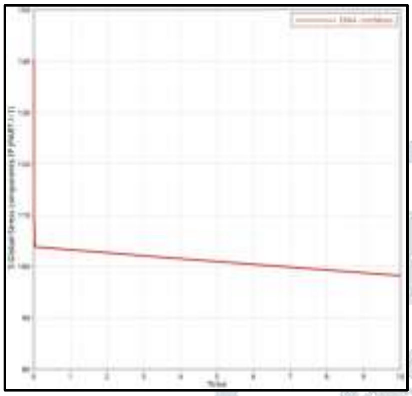

Figure 8.1: Stress relaxation simulation behavior for bending load

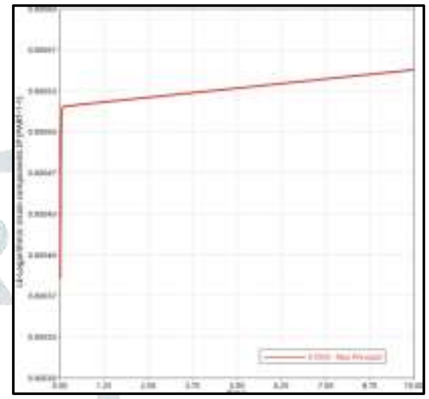

Figure 8.2: Creep behavior for Simulation bending load

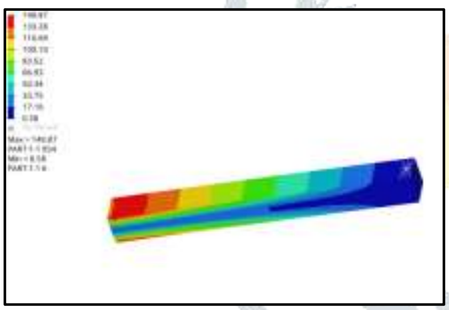

Figure 9.1: Stress relaxation simulation for bending load

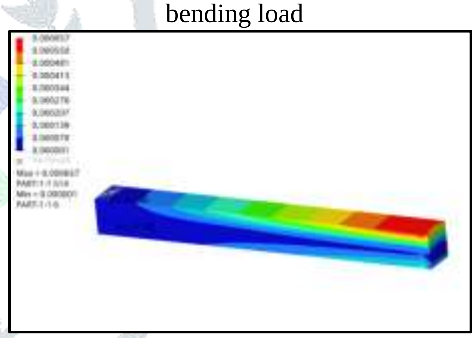

Figure 9.2: Creep Simulation for bending load

Simulation of FEA of the stress relaxation as well as for creep is represented in the figures 9.1 and 9.2.

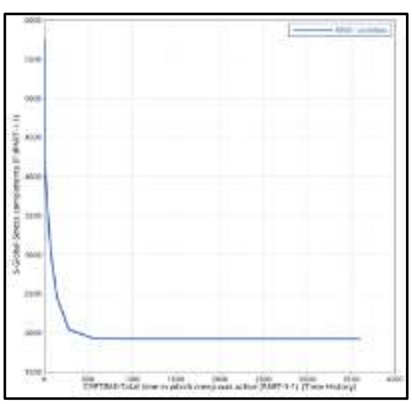

Figure 10.1: stress relaxation simulation behavior for axial load

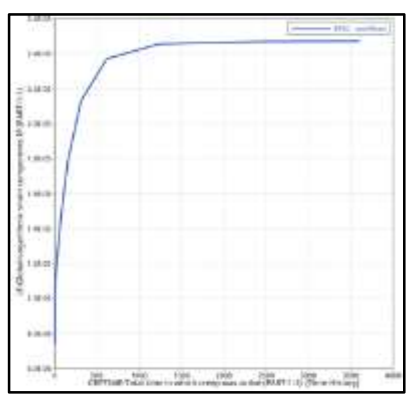

Figure 10.2: Creep simulation behavior for axial load

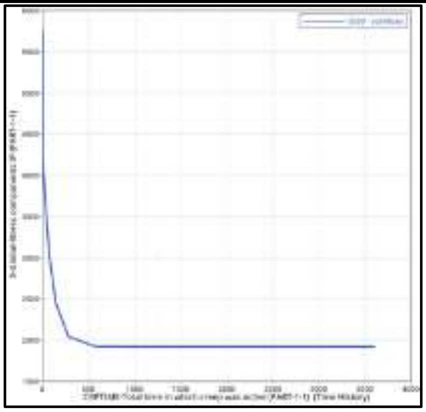

Figure 11.1: stress relaxation simulation behavior for Bending load

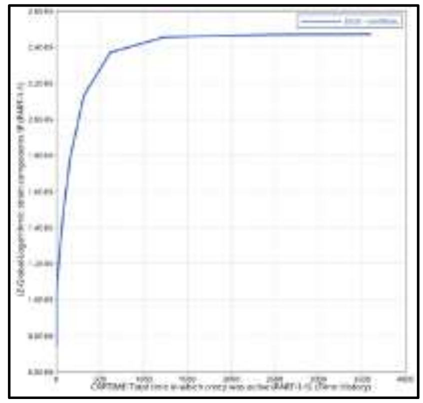

Figure 11.2: stress relaxation simulation behavior for Bending load

The viscoelastic behaviors in Figures 10.1, 10.2, 11.1, and 11.2 are for the period of 3600s for both axial load and bending load. The linear behavior in the previous graphs is seen only due to the short duration of 10s and is a plot for 1000 points. This same applies to the analytical solution as well.

V. CONCLUSION

The present work proposes a methodology to analyze the failure in any component due to viscoelasticity and validation with analytical calculation. This methodology is described using a standard shape i.e., a cantilever beam, subject to bending and axial loads. The cantilever beam is chosen to owe ease of performing analytical calculations on it.

A Copper-Nickel material of generic grade was selected for this study and instantaneous and viscoelastic behaviors were modelled using appropriate elastic modulus and Prony constants. The correlation for the axial and bending load for the Copper-Nickle material selected, analytical calculation using the generalized maxwell model has no significant variation from the simulated results and has a similar curve trend like it was expected to be.

A review of the analytical calculations and simulation results, suggests a good correlation of output both in terms of magnitude and trend. A 3% variation of the magnitude observed in simulation is negligible and is attributed to approximations in FEA. This study, therefore, establishes a reliable procedure for evaluating the viscoelastic behavior of a component in FEA. Moreover, an average 30% drop in output before and after viscoelastic behavior although, dependent on the material used suggests the importance of considering the viscoelastic property of materials, in applications involving Time-dependent thermal events.

REFERENCES

[1] Tzikang, Chen. (2000). Determining a Prony Series for a Viscoelastic Material from Time Varying Strain Data.

[2] Tschoegl, Nicholas w., 1989. The phenomenological theory of linear viscoelastic behavior an introduction. ISBN-13:978-3-642-73604-9,page 69-180

[3] lv, Huijie & Liu, Hanqi & Tan, Yiqiu & Meng, Anxin & Cyriaque, Assogba & Xiao, Shenqing. (2021). An extended search method for identifying optimal parameters of the generalized Maxwell model. Construction and Building Materials. 266. 10.1016/j.conbuildmat.2020.120796.

[4] Babaei, Behzad & Davarian, Ali & Pryse, Kenneth & Elson, Elliot & Genin, Guy. (2015). Efficient and optimized identification of generalized Maxwell viscoelastic relaxation spectra. Journal of the Mechanical Behavior of Biomedical Materials. 55. 32-41. 10.1016/j.jmbbm.2015.10.008.

[5] Juan, Kong & Ju-yun, Yuan. (2010). Application of linear viscoelastic differential constitutive equation in ABAQUS. 10.1109/ICCDA.2010.5541456.

[6] King H. Yang,Chapter 5 - Material Laws and Properties,Basic Finite Element Method as Applied to Injury Biomechanics,Academic Press,2018,Pages 231-256,https://doi.org/10.1016/B978-0-12-809831-8.00005-2.

[7] Pacheco, Juliana & Bavastri, Carlos & Pereira, Jucelio. (2015). Viscoelastic Relaxation Modulus Characterization Using Prony Series. Latin American Journal of Solids and Structures. 12. 420-445. 10.1590/1679-78251412.

[8] Drabousky, David. "Prony series representation and interconversion of viscoelastic material functions of equine cortical bone." Electronic Thesis or Dissertation. Case Western Reserve University, 2009. OhioLINK Electronic Theses and Dissertations Center. 26 May 2021.

[9] HD, Karlsson BI, Sorensen EP.(2012) ABAQUS 6.12 analysis user's manual , Version 6.12, (Elastic Mechanical properties >Viscoelasticity > time domain viscoelasticity)22.7.1