



Flavor mixing patterns of neutrinos-An overview

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Abstract: The observation of neutrino oscillations at SuperKamiokande, SNO etc. has undoubtedly established neutrino physics as one of the most interesting regimes of high energy physics. This phenomenon, hinting at the non-zero neutrino masses and mixings, provided a smoking gun signal for exploring the physics beyond the standard model (BSM). Owing to the significant advances on the theoretical and experimental fronts, the last couple of decades have turned out to be quite fruitful for our understanding of leptonic mixing parameters. In this context, the recent 'precise' measurement of lepton mixing angle θ_{13} has not only signaled the existence of CP violation in the leptonic sector, but has also deepened the mystery of flavor mixing further as the quark and lepton mixing patterns now seem to be quite different from each other. To this end, several ideas have been proposed, by introducing various flavor symmetries, in order to explain the observed leptonic mixing pattern. In the present article, we aim to present an overview of advances made in this direction by discussing the trimaximal, bimaximal, democratic, hexagonal flavor mixing patterns. Some of the upcoming experiments intending to unravel these mysteries of flavor mixing patterns would also be discussed briefly.

Keywords: BSM, neutrino oscillations, flavor mixing patterns

I. INTRODUCTION

Flavor physics have played a central role in the development of Standard Model (SM). An important aspect of flavor physics is the understanding of fermion masses and mixings which is one of the outstanding problems of present day high energy physics. Since the inception of neutrino by Pauli [1] in 1930, neutrino physics has been full of excitement and rich in its implications for New Physics (NP). Apart from playing crucial role in the development of V-A theory of weak interactions and in the discovery of neutral currents, neutrino physics continues to play an important role in revealing several mysteries of nature.

The strong evidence of discovery of neutrino oscillation phenomena comes from the series of experiments performed during the last four decades with very different neutrino beams and detection techniques. These include solar neutrino experiments [2] e.g; SAGE, Super-Kamiokande (SK) and Sudbery Neutrino Observatory (SNO); atmospheric neutrino experiments [3] eg; MACRO, Soudan-2; the Long Baseline reactor neutrino experiment KamLAND [4]; and the Long Baseline accelerator neutrino experiment K2K [5]. All these experiments show that neutrinos have mass and that similar to quarks, the neutrino flavor eigenstates are different from neutrino mass eigenstates i.e. neutrinos mix. This implies that neutrinos produced in a well defined flavor eigenstate can be detected, after propagating a distance as a different flavor eigenstate. For example, electron type neutrino produced in a core of sun, propagates as a superposition of three mass eigenstates which pick up different phases as they travel. The recombined wave will have a μ and τ component, in addition to the e component it started with.

With the discovery of flavor oscillations, the role of flavor physics has shifted from the discovery of the building blocks of the SM to the measurement of its parameters. The majority of the SM parameters is related to the flavor sector and can thus be determined in flavor violating decays. Fermion mass, flavor mixing and CP violation constitute three central concepts of flavor physics. SM itself does not make any quantitative predictions for the values of fermion masses, flavor mixing angles and CP-violating phases and hence any deeper understanding of such flavor issues must go beyond the scope of the SM. With an increasing experimental and theoretical accuracy, their determination has by now reached an impressive precision.

The observed patterns of quark as well as lepton mixing and the fact they are very different from each other remain puzzling till today. As an example of the similarities, the general structure of the mixing matrices and the mass matrices of neutrinos and quarks could be very much related to each other. However, the mixing pattern in the neutrinos as well as the

hierarchy of neutrino masses could be very different from those observed in charged leptons and quarks. The smallest neutrino mass is order of a fraction of an eV and top quark mass being of the order of 10^{11} eV. Similarly, fermion mixing angles again span several orders of magnitude, these being quite small in the quark sector whereas in the leptonic sector these are somewhat large. Measurements [6] from RENO, T2K, MINOS reported a mixing angle θ_{13} is not small, given a big impetus to the sharpening of the implications of the neutrino oscillations and also signaled the existence of CP violation in the leptonic sector. Also, a look at the pattern of quark and neutrinos indicate that there may be some kind of universality, at the lowest order of masses and mixings that brings forth quark–lepton complementarity at unified scale. Thus, the experimental data give us a food for thought for the flavor symmetry as these belong to one of a few promising approaches toward deeper understanding of the observed mass spectra and flavor mixing patterns of leptons and quarks.

The paper is organized as follows. Section II illustrates the idea of flavor mixing and summarizes the experimental data from the recent global fit analysis of flavor parameters. Section III presents a detailed discussion on different flavor patterns in case of neutrinos. Future upcoming experiments aiming to relate flavor of neutrinos are discussed in Section IV. Section V summarizes and concludes our discussion.

III KNOWLEDGE ABOUT THE FLAVOR PARAMETERS AND THEIR CURRENT STATUS

The possibility of flavor oscillations was examined by B. Pontecarvo on one hand and by Maki, Nakawaga and Sakata [7] on the other hand. The flavor oscillation of massive neutrinos travelling in space, change from one flavor to another, is a striking quantum phenomenon sensitive to the tiny neutrino mass squared differences. To illustrate the idea of neutrino mixings and oscillations, we briefly discuss the idea of neutrino oscillations. In the two flavor scenario, the flavor eigenstates ν_a and ν_b are related to the mass eigenstates ν_1 and ν_2 through the leptonic mixing matrix U given as

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{1}$$

Using the above transformation, the transition probability of ν_a changing to ν_b can be written as

$$P_{ab} = \sin^2 2\theta \sin^2 \left(1.267 \frac{\Delta m^2 L}{E} \right) \tag{2}$$

In the above equation L is the distance from the source in meters, E is the neutrino energy in MeV and Δm^2 is neutrino mass squared difference in eV^2 . In the case of three flavors, the relation between the neutrino mass eigenstates and the flavor eigenstates is expressed as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3}$$

where ν_e, ν_μ, ν_τ are the flavor eigenstates and ν_1, ν_2, ν_3 are the mass eigenstates and the 3×3 mixing matrix is referred as Pontecorvo-Maki-Nakagata-Saki (PMNS) leptonic mixing matrix. It is usually parameterized by three mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and 1 CP violating phase (δ). If neutrinos are Majorana particles, there results in additional two phases (ρ, σ) related to two relative phases of three neutrino mass eigenstates. One may parameterized U in terms of three flavor mixing angles and one (or three) CP violating phase(s) in a standard way advocated by the Particle Data Group [8] :

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_\nu} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} P_\nu$$

$$= \begin{bmatrix} c_{13}c_{12} & & s_{12}c_{13} & s_{13}e^{-i\delta_\nu} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_\nu} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_\nu} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_\nu} & -s_{12}c_{23} - s_{12}s_{13}c_{23}e^{i\delta_\nu} & c_{13}c_{23} \end{bmatrix} P_\nu \tag{4}$$

in which $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, δ_ν is the irreducible CP violating phase usually referred as Dirac phase and $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is the diagonal phase matrix containing two independent phase parameters ρ and σ . Thus the neutrino oscillations are sensitive to six fundamental flavor parameters: two independent neutrino mass square differences (Δm_{12}^2 and Δm_{31}^2), three flavor mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and one CP violating phase (δ_ν).

The current experimentally allowed ranges [9] of neutrino mass square differences and flavor mixing angles at 3σ C.L. can be summarized as:

Table 1: Current status of neutrino mixing oscillation data at 3σ C.L.

Parameters	Normal Hierarchy (NH)	Inverted Hierarchy (IH)
$\Delta m_{sol}^2 / 10^{-5} eV^2$	6.79 - 8.01	6.79 - 8.01
$\Delta m_{atm}^2 / 10^{-3} eV^2$	2.432 - 2.618	2.416 - 2.603
$\sin^2\theta_{12}$	0.275 - 0.350	0.275 - 0.350
$\sin^2\theta_{23}$	0.427 - 0.609	0.430 - 0.612
$\sin^2\theta_{13}$	0.02046 - 0.02440	0.02066 - 0.02461
δ_{CP}	$0.783\pi - 2.056\pi$	$1.139\pi - 1.967\pi$

where NH refer to normal hierarchy of neutrino masses as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$ and IH as $m_{\nu_1} \ll m_{\nu_2} < m_{\nu_3}$.

To explain the nature of neutrinos whether they are Dirac or Majorana and CP violation, β -decays and neutrinoless double β decays provide us useful information. The effective neutrino mass can be extracted from the distorted spectrum of end point of beta decay and given as

$$m_\beta = \sqrt{m_1^2 |V_{e1}|^2 + m_2^2 |V_{e2}|^2 + m_3^2 |V_{e3}|^2} \quad (5)$$

The effective neutrino mass in the lepton number violating neutrinoless beta decay in the standard three-neutrino scheme can be expressed as

$$\langle m_{ee} \rangle = m_1 U_{e1}^2 \exp(i\phi_{e1}) + m_2 U_{e2}^2 + m_3 U_{e3}^2 \exp(i\phi_{e3}) \quad (6)$$

where the phase parameters may vary from $[0, 2\pi]$.

Also, the observation of cosmic microwave background would constrain the sum of three neutrino masses given by

$$\Sigma_\nu = m_1 + m_2 + m_3 \quad (7)$$

Thus, a combination of the future measurements of $\langle m_{ee} \rangle$, $\langle m_e \rangle$ and Σ_ν will be greatly helpful to pin down the absolute neutrino mass scale and test the self consistency of the standard three-flavor scheme.

III. DESCRIPTION OF VARIOUS FLAVOR MIXING PATTERNS OF NEUTRINOS

The above mentioned PMNS matrix given by equation (4) has two terms i.e. $U = U_0 + \Delta U$ in which the leading term is U_0 containing at least two large but special flavor mixing angles and receives small corrections described by ΔU which may depend on CP violating parameters and mass ratios of charged leptons and neutrinos. This section focuses on the number of interesting patterns of U_0 , which links the flavor eigenstates of three neutrinos to their mass eigenstates and have an impact on our understanding of lepton flavor mixing and CP violation. Several attempts have been made to find the lowest-order mixing matrices in different scenarios, in view of fact that mixing angle θ_{23} is maximal as well as in the absence of lower limit on θ_{13} , the popular ones are tribimaximal, bimaximal etc.. It is expected that spontaneous symmetry breaking of possible flavor symmetries hidden in such constant mixing patterns could help us in finding viable clues to the dynamics responsible for neutrino mass matrices. Further, these patterns also prove to be as useful in studying the neutrino oscillation phenomenology.

1. The “tribimaximal” flavor mixing pattern [10] also called threefold maximal mixing which assures each neutrino flavor eigenstate to receive equal and maximally allowed contributions in magnitude from the three neutrino mass eigenstates. Thus, refers to highly symmetric, maximally CP violating fermion mixing configuration characterized by a unitary matrix having all its elements equal in modulus $|U_{ai}| = 1/\sqrt{3}$, $a, i = 1, 2, 3$, written as

$$U_\omega = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \\ \frac{\bar{\omega}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{bmatrix} \quad (8)$$

where $\omega = \exp(i2\pi/3)$ is the complex cube root of unity. It predicts $\theta_{12} = \theta_{23} = 45^\circ$, $\theta_{13} = \arctan(1/\sqrt{2}) \sim 35.3^\circ$ and $\delta = 90$ in the standard parametrization of U . The matrix U_ω is unique in accommodating maximal CP violation in the lepton sector as it defines six congruent regular triangles in the complex plane and each one of them has the maximal area equal to $1/(12\sqrt{3})$. Originally proposed as a candidate lepton mixing matrix and actively studied as such (and even as a candidate quark mixing matrix), tribimaximal mixing is now definitively ruled-out as a phenomenologically viable lepton mixing scheme by neutrino oscillation experiments, especially the Chooz reactor experiment, in favour of the no longer acceptable tribimaximal mixing scheme.

2. The “democratic” flavor mixing pattern U_{DC} [11], originating from the transpose of the orthogonal matrix which has been used to diagonalize the flavor “democracy” texture of a fermion mass matrix. The pattern will become stable after small corrections to ΔU , introduced by slightly breaking flavor democracy of the corresponding fermion mass matrix.

$$U_{DC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \quad (9)$$

It predicts $\theta_{12} = 45^\circ$, $\theta_{23} = \arctan(\sqrt{2}) \sim 54.7^\circ$, $\theta_{13} = 0$ and $\delta = 90^\circ$ in the standard parameterization of U . A natural derivation of the democratic mixing pattern as the leading term of a viable PMNS matrix U for massive Dirac neutrinos needs to introduce the flavor symmetry $S_4 \times Z_2$ in a warped extra-dimension model with complicated custodial symmetry.

3. The “bimaximal” flavor mixing pattern U_{bm} , proposed by Barger et al. and Vissani [12] is obtained from a product of two rotation matrices in the (2,3) and (1,2) planes with the same rotation angles $\theta_{23} = \theta_{12} = 45^\circ$ and expressed as

$$U_{bm} = U_{23}^m U_{12}^m \quad (10)$$

Explicitly, we have

$$U_{BM} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{12}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (11)$$

This predicts $\theta_{12} = \theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ in the standard parametrization of U. It may be noted that although the PMNS mixing matrix cannot be identified exactly with the above mentioned mixing matrix owing to strong deviation of the 1-2 mixing from maximal, yet it can play the role of a dominant structure.

The case (2) and (3) are, however fully consistent with the CHOOZ and all other data as long as LSND data for sterile neutrinos is not included.

4. The “tribimaximal” or Harisson-Perkins-Scott flavor mixing pattern [13], popularly known as TBM can be obtained from the trimaximal flavor mixing pattern multiplied by a bimaximal (1,3) rotation matrix on the right side. This pattern can be viewed as a twisted form of Eq. (9) interchanging between the first and second columns of the same entries:

$$U_{\text{TBM}} = \begin{bmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (12)$$

It predicts $\theta_{13} = 0^\circ$, $\theta_{23} = 45^\circ$, $\theta_{12} = \arctan(1/\sqrt{2}) \sim 35.3^\circ$ and $\delta = 90^\circ$ in the standard parametrization of U. One can see that the values of θ_{12} and θ_{23} predicted by above equation are very close to their best-fit values listed in Table (1). Further, to this end we have considered a parametrization of the PMNS matrix in the modified tribimaximal scenario expressed as

$$U_{\text{TBM}} = \begin{bmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & U_{e3} \\ -\frac{1}{\sqrt{6}} - \frac{\sqrt{3}}{2}U_{e3} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{2}U_{e3} \\ \frac{1}{\sqrt{6}} + \frac{\sqrt{3}}{2}U_{e3} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} - \frac{1}{2}U_{e3} \end{bmatrix} \quad (13)$$

This parameterization is important as it does not involve the three mixing angles; instead it enables one to directly deal with the complex parameter U_{e3} of the mixing matrix. This flavor mixing pattern has attracted the most attention and help in building neutrino mass models based on the discrete flavor group symmetry.

5. The “hexagonal” flavor mixing pattern [14], which contains a special rotation angle $\theta_{12} = 30^\circ$ equal to half of the external angle of the hexagon:

$$U_0 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (14)$$

It predicts $\theta_{12} = 30^\circ$, $\theta_{13} = 0^\circ$, $\theta_{23} = 45^\circ$ in the standard parametrization of U. Its phenomenological consequences on neutrino oscillations are quite similar to those of tribimaximal flavor mixing pattern.

6. In the literature some more constant flavor mixing patterns such as the “golden ratio” pattern [15] with $\theta_{12} = \arctan(2/\sqrt{1+5}) \sim 31.7^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ and “tetra-maximal mixing pattern” [16] with $\theta_{12} = \arctan(2-\sqrt{2}) \sim 30.4^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 8.4^\circ$ have also been proposed. These patterns are quite more complicated than discussed above.

More efforts are underway to explore more successful applications of the flavor symmetries in understanding a number of well known flavor puzzles. Thus some enlightening ideas are expected to be very helpful in the study of other flavor symmetries and their consequences on fermion mass generation, flavor mixing and CP violation.

IV UPCOMING EXPERIMENTS

Neutrinos are known to oscillate between flavors, as they move through space. But in 1995, physicists working on the Liquid Scintillator Neutrino Detector (LSND) [17], at Los Alamos National Laboratory reported that there may be an extra neutrino flavor that would not interact like the others flavors and called it as “a sterile neutrino”. In the years since the LSND anomaly, physicists have been designing experiments geared towards chasing down this hidden flavor. In 2002, the MiniBooNE experiment [18] began collecting data related to this at Fermi National Accelerator Laboratory. Results have shown an excess of MiniBooNE events that is consistent with the LSND signal, but it isn’t clear how this fits into a model of sterile neutrinos. In addition, an experiment MicroBooNE will eventually be joined at Fermilab by ICARUS and SBND, forming a suite of three detectors known as the Short-Baseline Neutrino Program. Beyond these accelerator-based experiments which also include the Japan-based JSNS2, a number of radioactive-source and reactor-based experiments [19], including PROSPECT, STEREO, DANSS, CHANDLER and SOLID are also working and hope to catch the hidden flavor sometimes in the near future.

The two major long-baseline experiments that experiments [20] tackled the question of mass ordering are the T2K experiment hosted by KEK accelerator laboratory, which monitors a beam of neutrinos traveling more than 180 miles across Japan, and NOvA hosted by Fermilab, which studies a beam that originates about 500 miles from the detector in the United States. Fermilab just completed an upgrade of its accelerators, and the detector for the T2K experiment will gain sensitivity with an upgrade. Reactor-based experiments, such as the Daya Bay Reactor Neutrino Experiment in China, are also involved in the investigation. Results from NOvA and T2K that could figure out the ordering of neutrino flavor. The next generation precision observations of the CMB anisotropies [21] e.g., Pixie, CORE projects and large scale structures e.g., the DES, Euclid, LSST projects in cosmology are expected to convincingly tell whether the inverted neutrino mass ordering is really true after all uncertainties from the relevant cosmological parameters.

Other experiments are working to measure the combined mass of the three types of neutrinos. KATRIN [22], a neutrino experiment in Germany with a 200-ton spectrometer at its core, has just started taking data. The experiment will measure the energy of the electrons spit out during the decay of the radioactive isotope tritium and look for very slight distortions that will clue researchers in to the neutrino's absolute mass. In the near future, KATRIN experiment would pin down the value of $m_\beta < 0.2$ eV. If the neutrinos are considered to Majorana neutrinos, the neutrinoless double beta decay ($0\nu 2\beta$) could take place for some even even nuclei such as ^{76}Ge and ^{136}Xe . Project 8, another experiment going after the absolute mass of the neutrino, will also use tritium, instead measuring the energy of individual electrons by measuring the frequency of their spiraling motion in a magnetic field.

Experiments such as CUORE, Majorana Demonstrator, GERDA and NEXT [23] published their results for neutrinoless double decay. A large number of experiments are under process which are expected to yield results in coming few years such as JUNO [24], an underground observatory in China that will be able to pin down the neutrino mass ordering at the 4 σ confidence level. The Fermilab-hosted Deep Underground Neutrino Experiment, DUNE [25], will send neutrinos racing more than 1300 km away from the neutrino-antineutrino sources, may hopefully reach at 5 σ sensitivity to the true neutrino mass ordering for any possible values of δ_ν .

Thus, these future precision flavor experiments will help complete the flavor phenomenology and even shed light on the underlying flavor theory which is anticipated to be more fundamental and profound than the SM.

VI SUMMARY AND CONCLUSIONS

Understanding the origin of flavor has been a major goal of particle physics research for last few decades. The fermion masses and mixing not only provide a fertile ground to hunt for physics beyond the SM but also pose a big challenge to understand these from more fundamental considerations. The fact that the masses of three active neutrinos are extremely small strongly indicates that the new physics responsible for neutrino mass generation and lepton flavor mixing seems to be essentially decoupled from the other part of SM. We have discussed the various flavor mixing patterns and reported that the results obtained by using TBM pattern is so close to the experimental oscillation data. A large number of experiments are under process which is expected to yield precise results in coming few years and give us a clear picture of flavor mixing parameters.

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