3D Space Mapping Using Ultrasonic Sensor

Viraj Sawant, Ruthvik Vachhani, Newton Nadar, Tanisha Kadam

Mechatronics Department
Terna Engineering College, Navi Mumbai 400706, India

Abstract : This study has been undertaken which proposes a way to use the Ultrasonic sensor to map the space in 3D. In this project, an Ultrasonic sensor is used to map the area using Multivariate Gaussian Distribution. The Ultrasonic sensors give the reading for the distance of object. This reading is then used to calculate the probability of position of obstacle in the field of vision of that particular sensor. The Multivariate Probability Distribution function has two variables 'r' and 'θ' (theta) which are varied at particular instants to find the probability at each cell.

***IndexTerms* – Gaussian Probability Distribution Function, Ultrasonic sensors, Baye's Law, 3D mapping**

1. INTRODUCTION

The fundamental property of an ultrasonic sensor is to measure the distance of the object in its field of vision. The operator certainly does not know the position, shape, size of the obstacle. The only information that can be achieved is the distance of the obstacle. Using the method given the paper one can map the surrounding using multiple ultrasonic sensors and approximate the nature of the surrounding. This method of mapping can be used to map surrounding in automotive for parking assistance, to map sea beds, and underwater landscape.

2. SYSTEM OVERVIEW

The project aims to map the surrounding so that the detection of space is possible. The mapping is done using ultrasonic sensors. Multiple sensors will be used to get the distance of an object. Using the distance, and the probabilistic function **F(r, θ)** we find the probability of occupancy of each cube in the field of vision of each sensor[1][2] . On the basis of this probability a color is allotted to the cube which will result in a depth map of the surrounding[6].

All ultrasonic sensors have different beam widths and sensing range, even every ultrasonic sensor shows different behavior in different environments. We have considered a sensor having a beam of 30cm in radius. Also the environment for our research is considered to be static.

In this research we have considered cylindrical coordinate system for calculating probability of cubes as the beam of a ultrasonic sensor is a cone and mapped as per cartesian coordinate of the corresponding cubes[4].

3. PROBABILISTIC MODEL FOR ULTRASONIC SENSOR

Different sensor readings will be from an ultrasonic sensor array. As per the obstacle there will be different sensor readings. Each sensor reading will be used to calculate probability of the cubes under the field of vision of the corresponding sensor. These probabilities are used to allot a particular color to the cell. This will result in a depth map of the surrounding area.

Derivation for Probabilistic Distribution Function (P.D.F)

The generic form for multivariate P.D.F is given by;

$$\mathrm{F}(x_1, x_2) = \frac{1}{(2\pi)^{\frac{d}{2}}\,|\Sigma|^{\frac{1}{2}}} e^{\{\frac{-1}{2}[(X-\mu)^T \Sigma^{-1}[X-\mu]]\}}$$

In our case we are using two variables which are independent of each other i.e. 'r' & 'θ'

Therefore, d=2

Mean matrix;

$$\mu = [\mu_r \; \mu_\theta \,] \;\ldots\ldots\ldots\; (1)$$

Covariance matrix;

$$\Sigma = [\sigma_r^2 \; 0 \; 0 \; \sigma_\theta^2 \,] \;\ldots\ldots\ldots\; (2)$$

element (1,2) & (2,1) is zero because covariance between r & θ is zero as both are independent variables.

Now;

$$\mathrm{X} = (r \; \theta \,) \;\ldots\ldots\ldots\ldots\; (3)$$

from (1) & (3)

$$\mathrm{X} - \mu = [r - \mu_r \; \theta - \mu_\theta \,] \;\ldots\ldots\ldots\; (4)$$

$$[X - \mu]^T = [r - \mu_r \;\; \theta - \mu_0] \;\ldots\ldots\ldots(5)$$

from (2);

$$|\Sigma| = (\sigma_r^2 . \sigma_\theta^2) - 0 = \sigma_r^2 . \sigma_\theta^2$$

$$\Sigma^{-1} = \frac{1}{|\Sigma|} \operatorname{adj}(\Sigma)$$

$$\operatorname{adj}(\Sigma) = [a_{11} \; a_{12} \; a_{21} \; a_{22} \,] = [\sigma_r^2 \; 0 \; 0 \; \sigma_\theta^2 \,]$$

$$a_{ij} = (-1)^{i+j} \, |a_{ij}|$$

$$a_{11} = (-1)^{1+1} \, |a_{11}| = (-1)^2 . \sigma_r^2 = \sigma_r^2$$

$$a_{12} = (-1)^{1+2}\,|a_{12}| = (-1)^3.0 = 0$$
$$a_{21} = (-1)^{1+2}\,|a_{21}| = (-1)^3.0 = 0$$
$$a_{22} = (-1)^{2+2}\,|a_{22}| = (-1)^2\sigma_\theta^2 = \sigma_\theta^2$$

$$\Sigma^{-1} = \frac{1}{\sigma_r^2.\sigma_\theta^2}\,[\sigma_r^2\ 0\ 0\ \sigma_\theta^2\,] = \left[\frac{1}{\sigma_r^2}\ 0\ 0\ \frac{1}{\sigma_\theta^2}\right]$$

$$\Sigma^{-1} = \left[\frac{1}{\sigma_r^2}\ 0\ 0\ \frac{1}{\sigma_\theta^2}\right] \text{.......... (6)}$$

Now, from (4), (5) & (6);

$$(X-\mu)^T\ \Sigma^{-1}\ (X\text{-}\mu)\ =\ [r-\mu_r\ \ \theta-\mu_\theta].\left[\frac{1}{\sigma_r^2}\ 0\ 0\ \frac{1}{\sigma_\theta^2}\right].\,[r-\mu_r\ \ \theta-\mu_\theta\,]$$

$$= \left[\frac{r-\mu_r}{\sigma_r^2}+0\quad 0+\frac{\theta-\mu_\theta}{\sigma_\theta^2}\right][r-\mu_r\ \ \theta-\mu_\theta\,]$$

$$(X-\mu)^T\ \Sigma^{-1}\ (X\text{-}\mu)\ = \frac{(r-\mu_r\,)^2}{\sigma_r^2} + \frac{(\theta-\mu_\theta)^2}{\sigma_\theta^2} \text{.......... (7)}$$

So;

$$F(r,\theta) = \frac{1}{(2\pi)^{\frac{2}{2}}\left|\sigma_r^2.\sigma_\theta^2\right|^{\frac{1}{2}}}\,e^{\{\frac{-1}{2}[\frac{(r-\mu_r\,)^2}{\sigma_r^2}+\frac{(\theta-\mu_\theta)^2}{\sigma_\theta^2}]\}}$$

Hence;

$$F(r,\theta) = \frac{1}{(2\pi).(\sigma_r.\sigma_\theta)}\,e^{\{\frac{-1}{2}[\frac{(r-\mu_r\,)^2}{\sigma_r^2}+\frac{(\theta-\mu_\theta)^2}{\sigma_\theta^2}]\}} \text{..........(8)}$$

4. METHODOLOGY

4.1 Probability of single cube

Consider the sensor cone to be a cone. The cone is divided into number of small cubes of same dimensions. The multivariate P.D.F consist of ‘r’ and ‘θ’[4], the r is the length of the vector joining the center of a particular cube to sensor position (i.e. r1 in Fig.4.1) and θ (theta) is the angle made by the vector joining the particular cube center and the center axis (i.e. θ1 in Fig.4.1)[3].

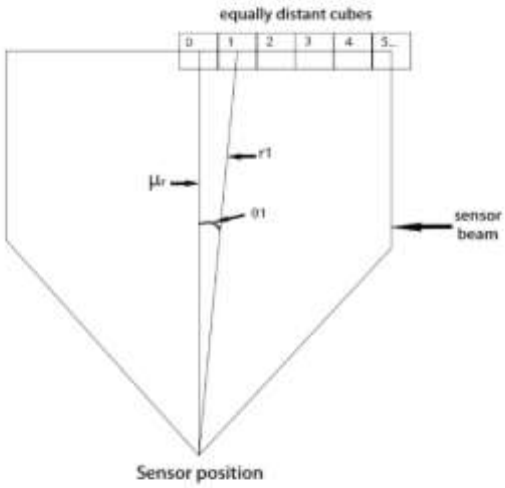

Fig.4.1 Sensor Wave

Using trigonometry one can find ‘r’ and ‘θ’ for a particular cube. ‘μr’ and ‘μθ’ are the mean values for r and θ. μr will be the reading given by the ultrasonic sensor, it will be fixed for all r and θ values. μθ is the angle where the probability of object is maximum i.e. the center axis and θ will be zero at the center axis.

Variance i.e. ‘σr^2’ and ‘$\sigma\theta^2$’ can be calculated by the formula given below;

$$\sigma_r^2 = \frac{\sum_{i=0}^{n} \ (r_i-\mu_r)^2}{n} \text{ and } \sigma_\theta^2 = \frac{\sum_{i=0}^{n} \ (\theta_i-\mu_\theta)^2}{n}$$

Here,
σ_r^2 & σ_θ^2 are the variance for r and θ
μ_r & μ_θ are the mean values for r and θ, ($\mu_\theta = 0$)
n is the number of cubes
The standard deviation in the square root of the variance i.e. σr and σθ.
By substituting these values is the Multivariate Probability Distribution Function (8) one will get the probability of a particular cube.

4.2 Merging values from two or more sensors

When one uses two or more sensors for plotting. The sensor beam should coincide with each other. The value of the probability of the individual sensor is not sufficient for accurate mapping. So, we use Baye's filter, Baye's filter considers the Probability of the current reading, Probability of reading till previous reading & the probability set at the start of the scanning and finding their odds to calculate odds of the cell at particular reading & timestamp and convert odds to Probability.
Baye's Law states that;

$$P(y) = \frac{P(x).P(x)}{P(y)}$$

To find Odds;

$$Odds = \frac{Probability\ of\ something\ happening}{Probability\ of\ something\ not\ happening}$$

$$Odds = \frac{P(x)}{1-P(x)} = \frac{P(x)}{P(\underline{x})}$$

Estimation of Map;

$$\frac{P(m_i|z_{1:t})}{1-P(m_i|z_{1:t})} = \frac{P(m_i|z_t)}{1-P(m_i|z_t)} \times \frac{P(m_i|z_{1:t-1})}{1-P(m_i|z_{1:t-1})} \times \frac{1-P(m_i)}{P(m_i)}$$

Here;
LHS is the odds
In the RHS 1st term is the current reading odds, 2nd term is the odds of reading latest point, 3rd term is Prior odds (starting value)
Let,

$$P(z_{1:t}) = P(x)$$

Therefore,

$$odds(x) = \frac{P(x)}{1+P(x)}$$

$$\therefore P(x) = \frac{1}{1+\frac{1}{odds(x)}} = \frac{odds(x)}{odds(x)+1}$$

5. SIMULATION AND ANALYSIS

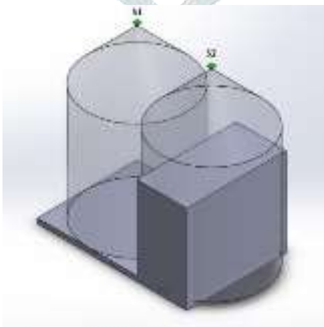

Fig.5.1 Simulation scenario

Assuming the scenario in Fig5.1 where sensor S1 gives the reading as 100cm and sensor S2 gives the reading as 50cm. After recording these readings and applying to the algorithm and simulating the map on excel we get. We also assume that the field of vision of the sensor is divided into a number of cubes of dimensions 3cm x 3cm x 3cm.

Sensor reading	100	μθ	0		μr1	100	
resolution	3						
			θ				
cube distance	cube no.		degrees	(θ-μθ)^2	r_n	(r-μr)^2	f(r,θ)
0	0		0	0	100	0	0.007514305
3	1		1.718358002	2.952754222	100.0449899	0.002024089	0.007402005
6	2		3.433630362	11.78981747	100.1798383	0.032341811	0.007056823
9	3		5.142764558	26.4480273	100.4041832	0.163364043	0.006461263
12	4		6.842773413	46.82354798	100.7174265	0.514700777	0.005609677
15	5		8.53076561	72.77396189	101.1187421	1.251583843	0.00453754
18	6		10.20397372	104.1210797	101.6070864	2.582726591	0.003344247
21	7		11.85977912	140.6543608	102.1812116	4.757683958	0.002186793
24	8		13.49573328	182.1348168	102.8396811	8.063788508	0.001230343
27	9		15.10957512	228.2992604	103.5808863	12.82274646	0.000575478
30	10		16.69924423	278.864758	104.4030651	19.38698218	0.000215503
			sum	1094.862385	sum	49.57794226	
			divide	99.53294405	divide	4.50708566	
			σθ	9.976619871	σr	2.122989793	

Table 5.1 Function values for sensor S1

Table 5.1 shows the r and θ values for different cubes over a half beam width. The probability values when plotted form a Half Gaussian curve for half beam width as shown in Fig.5.1.

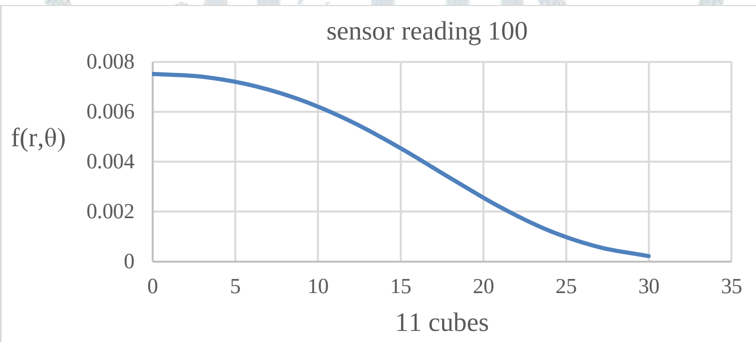

Fig. 5.1 Half Gaussian Probability Distribution Function

In Figure 5.1, the X axis shows the cube distance from the sensor position and the Y axis shows the function value of the corresponding cubes. As we move away from the sensor position the probability of a cell being empty decreases and eventually will tend to zero. Similar plot was plotted for a sensor giving a value of 50cm (Fig5.2).

sensor reading	50	μθ	0		μr2	50	
			θ				
cube distance	cube no.		degrees	(θ-μθ)^2	r2	(r-μr)^2	f(r,θ)
0	0		0	0	50	0	0.002069765
3	1		3.433630362	11.78981747	50.08991915	0.008085453	0.002035613
6	2		6.842773413	46.82354798	50.35871325	0.128675194	0.001931738
9	3		10.20397372	104.1210797	50.80354318	0.645681648	0.001755991
12	4		13.49573328	182.1348168	51.41984053	2.015947127	0.001511188
15	5		16.69924423	278.864758	52.20153254	4.846745545	0.001212061
18	6		19.79887635	391.9955049	53.14132102	9.867897765	0.000888996
21	7		22.78240573	519.0380109	54.23098745	17.90125482	0.00058357
24	8		25.64100582	657.4611797	55.4616985	29.83015046	0.00033494
27	9		28.36904629	804.8027876	56.82429058	46.57094193	0.000164024
30	10		30.96375653	958.7542186	58.30951895	69.04810515	6.68491E-05
			sum	3955.785722	sum	180.8634851	
			divide	359.6168838	divide	16.44213501	
			σθ	18.96356727	σr	4.054890258	

Table 5.1 Function values for sensor S2

Table 5.1 shows the r and θ values for different cubes over a half beam width. The probability values when plotted form a half Gaussian curve for half beam width as shown in Fig.5.2.

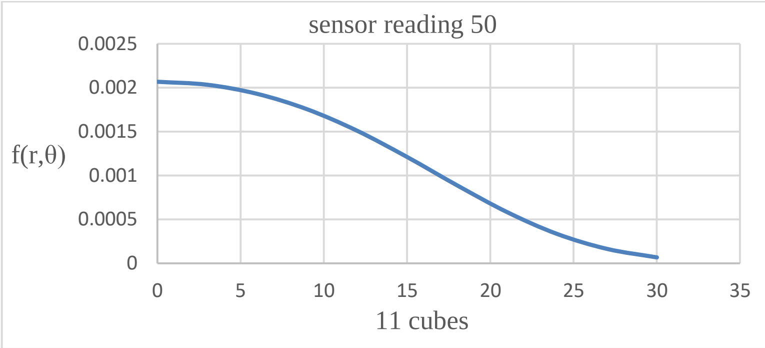

Fig. 5.2 Half Gaussian Probability Distribution Function

As ultrasonic sensors are symmetric around the axis of the wave, after finding probabilities in one plane we can rotate those probability around the axis of the wave and a 3D probability map can be plotted. An Excel simulation is shown Table 5.3 of sensor which gives the reading of 100cm.

equi-dimensional cube numbers

equi-dimensional cube numbers	$100	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10
	10	0.0002	0.0002	0.0002	0.00022	0.0002	0.00022	0.0002	0.00022	0.0002	0.00022	0.00022	0.00022	0.0002	0.00022	0.00022	0.00022	0.00022	0.0002	0.0002	0.0002	0.0002
	9	0.0002	0.0006	0.0006	0.00058	0.0006	0.00058	0.0006	0.00058	0.0006	0.00058	0.00058	0.00058	0.0006	0.00058	0.00058	0.00058	0.00058	0.0006	0.0006	0.0006	0.0002
	8	0.0002	0.0006	0.0012	0.00123	0.0012	0.00123	0.0012	0.00123	0.0012	0.00123	0.00123	0.00123	0.0012	0.00123	0.00123	0.00123	0.00123	0.0012	0.0012	0.0006	0.0002
	7	0.0002	0.0006	0.0012	0.00219	0.0022	0.00219	0.0022	0.00219	0.0022	0.00219	0.00219	0.00219	0.0022	0.00219	0.00219	0.00219	0.00219	0.0022	0.0012	0.0006	0.0002
	6	0.0002	0.0006	0.0012	0.00219	0.0033	0.00334	0.0033	0.00334	0.0033	0.00334	0.00334	0.00334	0.0033	0.00334	0.00334	0.00334	0.00334	0.0022	0.0012	0.0006	0.0002
	5	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0045	0.00454	0.0045	0.00454	0.00454	0.00454	0.0045	0.00454	0.00454	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	4	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00561	0.0056	0.00561	0.00561	0.00561	0.0056	0.00561	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	3	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0065	0.00646	0.00646	0.00646	0.0065	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	2	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0071	0.00706	0.00706	0.00706	0.0071	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	1	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0071	0.0074	0.0074	0.0074	0.0071	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	0	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0071	0.0074	0.00751	0.0074	0.0071	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	1	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0071	0.0074	0.0074	0.0074	0.0071	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	2	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0071	0.00706	0.00706	0.00706	0.0071	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	3	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00646	0.0065	0.00646	0.00646	0.00646	0.0065	0.00646	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	4	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0056	0.00561	0.0056	0.00561	0.00561	0.00561	0.0056	0.00561	0.00561	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	5	0.0002	0.0006	0.0012	0.00219	0.0033	0.00454	0.0045	0.00454	0.0045	0.00454	0.00454	0.00454	0.0045	0.00454	0.00454	0.00454	0.00334	0.0022	0.0012	0.0006	0.0002
	6	0.0002	0.0006	0.0012	0.00219	0.0033	0.00334	0.0033	0.00334	0.0033	0.00334	0.00334	0.00334	0.0033	0.00334	0.00334	0.00334	0.00334	0.0022	0.0012	0.0006	0.0002
	7	0.0002	0.0006	0.0012	0.00219	0.0022	0.00219	0.0022	0.00219	0.0022	0.00219	0.00219	0.00219	0.0022	0.00219	0.00219	0.00219	0.00219	0.0022	0.0012	0.0006	0.0002
	8	0.0002	0.0006	0.0012	0.00123	0.0012	0.00123	0.0012	0.00123	0.0012	0.00123	0.00123	0.00123	0.0012	0.00123	0.00123	0.00123	0.00123	0.0012	0.0012	0.0006	0.0002
	9	0.0002	0.0006	0.0006	0.00058	0.0006	0.00058	0.0006	0.00058	0.0006	0.00058	0.00058	0.00058	0.0006	0.00058	0.00058	0.00058	0.00058	0.0006	0.0006	0.0006	0.0002
	10	0.0002	0.0002	0.0002	0.00022	0.0002	0.00022	0.0002	0.00022	0.0002	0.00022	0.00022	0.00022	0.0002	0.00022	0.00022	0.00022	0.00022	0.0002	0.0002	0.0002	0.0002

Table 5.3 Mesh of probability distribution function

Table 5.3 shows the mesh of the probability distribution function values. The center cell is the cell right in front of the sensor and has the greatest function value (probability) than other cubes. The number in the first row and First column are the cube numbers in the X and Y axes, zero means the center position and the number goes on increasing in both the direction as we move away from the center. The P.D.F values are in the form of mesh, equal values are in the same color rings. Plotting a graph on the bases of this mesh we get a 3D probability Distribution Graph as shown in Fig.5.3;

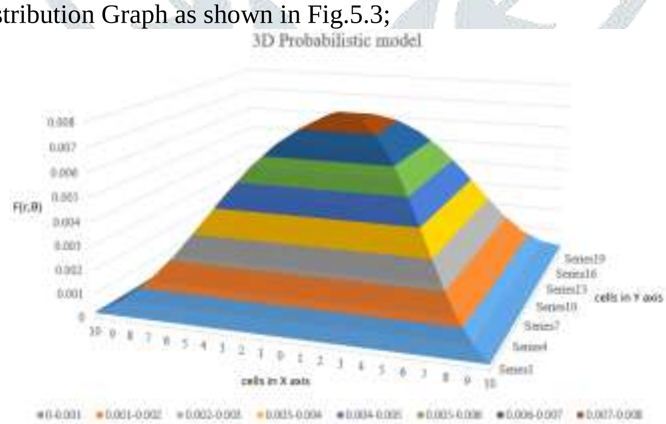

Fig.5.3 3D probabilistic plot for sensor S1

In figure 5.3 X and Y axes show the cube positions as in Table 5.3 (row 1 & column 1) and Z axis show the function values of corresponding cubes. Figure5.3 shows Probability Distribution of XY plane at a distance Z (sensor reading) from the sensor. Similarly, we can plot a 3D probabilistic plot for sensor 2 (S2) refer figure 5.1. which gives a reading of 50 cm.

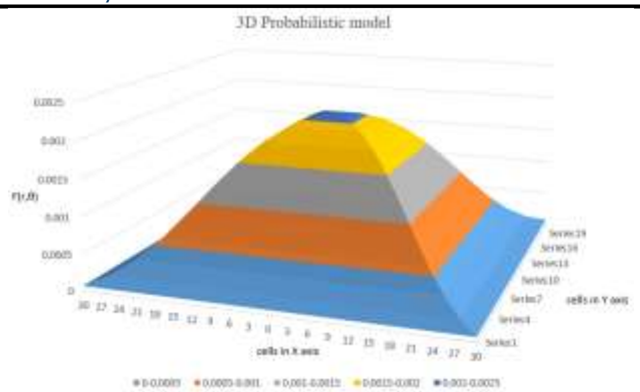

Fig.5.4 3D probabilistic plot for sensor S2

When the two plots are equally scaled on the same plot. We get the following plot. (Figure 5.5)

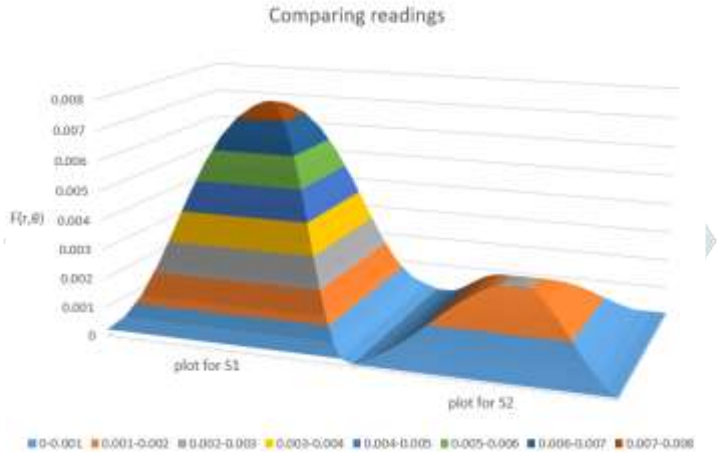

Fig.5.5 Comparing sensor plots

From Fig. 5.5 we can conclude that the function value for sensor S1 giving a reading of 100cm is larger than the function value for the sensor S2 which gives a reading of 50cm. Hence here it is proved that the probability od having a empty space is greater where sensor reading is greater.

6. Conclusion

From this research we conclude that, the derived, Multivariate Gaussian Probability Distribution function gives a function value to a particular cube in space on the basis of the cubes position with respect to the position of the ultrasonic sensor. Further the cube from the sensor, greater the function value of the corresponding cube.

7. Acknowledgment

This method of 3D mapping using ultrasonic sensor helps to scan the volume using Gaussian Probabilistic Distribution Function. This method can be used in space scanning in robots, in parking assistance for vehicles. As ultrasonics is used, one can also scan surface in other mediums like under water to scan sea beds and river beds.

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