RELATIONSHIP BETWEEN, DYNAMIC & STATIC YOUNG'S MODULUS, POISSON'S RATIO AND OTHER ENGINEERING PROPERTIES WITH TRI-AXIAL ON GRANITE AND BASALT.

[1] Mr. J. S. Salunkhe, [2] Miss. Damini Pawar
[1] Managing Director, [2] Geotech Engineer,
S & R Geotechniques Pvt. Ltd, Navi Mumbai, India.

Abstract : In many application, rock is subjected to an intense loading characterized by high level of pressure and very high rate of loading ranging from few tons to several hundred of MPa. A good grasp of behavior under confined compression is essential to understand and model their performance to improve the efficiency. Present study is carried out on rock cores with different geological setup to determine Dynamic modulus of elasticity with resonance frequency of Sonic Velocity, Static modulus of elasticity and Tri-axial test (Cohesion, Angle of friction, Modulus of elasticity and Poisson's ratio) with Engineering properties (Water absorption, Density, Specific gravity, porosity & Brazilian). Different rocks with variable mineral composition have wide selection of moduli values. Interpretation of results with regression analysis indicates significant correlations between moduli values which can be further used for designing and planning purpose. A brief summary and a few aspects of prospective research are presented.

***IndexTerms* - Ultrasonic pulse velocity, Modulus of elasticity, Poisson's ratio, Tri-axial.**

I. INTRODUCTION

Uniaxial and triaxial tests are common lab testing methods to assess the behavior under varying stress conditions (Fjar et al. 2008). These tests can provide information about static mechanical parameters (e.g., Poisson's ratio, Young's modulus, yield and failure stress) for a single sample under defined experimental conditions (Jaeger et al. 2007). The tests help to predict the volumetric changes due to compaction (Zoback 2011). The main objectives are the prediction of failure under varying confining pressures, using different approaches and concepts. Different concepts can be used: (1) a linear correlation between the horizontal and vertical stresses or (2) specific failure envelope to describe mathematically and empirically the behavior of different rock types.). Triaxial tests offers the opportunity to estimate the rock strength of a potential rock masses to reduce risks for reservoir stability (Lama and Vutukuri 1978). Simulations are used to analyze mechanical and hydraulic processes and their linkage between each other in detail and to quantify their influence on both the results and vice versa (Rutqvist et al. 2002). The differences between dynamic and static elastic moduli is highly associated with mineralogical differences in porosity. A deviation of dynamic elastic modulus from static elastic modulus can be attributed to the presence of fractures, cracks, cavities and planes of weakness and foliation. (Al-shayea et al. 2004). The goal of our investigation is to study rock cores with different geological setup and determine Dynamic modulus of elasticity with resonance frequency of Ultrasonic Velocity, Static modulus of elasticity and Tri-axial test (Cohesion, Angle of friction, Modulus of elasticity and Poisson's ratio) with Engineering properties (Water absorption, Density, Specific gravity, porosity & Brazilian). Numerical investigations, based on the mechanical properties of two comparative rock samples, Granite and Basalt, have been performed to investigate their mechanical behavior. The advantage of the coupled numerical investigation is the possibility to use and combine independent experimental setups and data, while combined measurements of deformation are lengthy and expensive

II. SAMPLING OF DATA AND METHODS.

Two types of rock, Granite and Basalt with different mineralogical composition have been used in Ultrasonic pulse velocity & triaxial experiments. Granite rock core is obtained from Banswara (Rajasthan) and Basalt from Palghar (Maharashtra). 18 samples of both are been used in this study. Relationship between P & S waves velocities, Engineering properties and Tri axial (Cohesion, Angle of friction & Modulus of elasticity & Poisson's ratio). All the test is carried out in accordance with the IS codes & ASTM

.

Figure 1. Basalt

Figure 2. Granite

III.RESEARCH METHODOLOGY

3.1 Test procedures.

This section elaborates the test procedures which are conducted on the rock core samples as per IS code and ASTM. The details of methodology are given as follows.

3.1.1 Dynamic elastic properties. (Non Destructive testing)

The ultrasonic test was applied using the transmission method, which consists of coupling two piezoelectric sensors on opposite faces of the sample. The core specimens were taken at diameter of 54mm and a length diameter thickness ratio of 2.5 from all the rocks. The end surfaces of the core specimens are covered with stiffer grease to provide a good coupling between the transducer face and the specimen surface to maximize accuracy of the transmit time measurement. The Vp and Vs velocity through the specimen were calculated from travel time from generator to a receiver at the opposite end. For each granite and basalt rock, five specimens were prepared and their Vp and Vs were determined according to ASTM D 2845. The calculus of the E and μ are calculated by means of following equations, respectively:

- Young's Modulus of elasticity.

$$E_{dyn} = \frac{\rho Vp^2(3Vp^2\text{-}4Vs^2)}{(Vp^2 - Vs^2)} \qquad (3.1)$$

- Poisson's ratio.

$$\mu_{dyn} = \frac{Vp^2 - 2Vs^2}{2(Vp^2 - Vs^2)} \qquad (3.2)$$

- Dynamic modulus of rigidity .

$$G_{dyn} = \frac{E_{dyn}}{2(1+\mu_{dyn})} \qquad (3.3)$$

- Dynamic bulk modulus.

$$K_{dyn} = \frac{E_{dyn}}{3(1-2\mu_{dyn})} \qquad (3.4)$$

Where,
ρ = density g/cm^2.
Vs = Shear velocity, Km/s
Vp= Longitudinal infinite medium velocity, Km/s.

3.1.2 Triaxial Shear test.

The rock specimens for conducting triaxial shear test are prepared as per the specification. The test may be conducted in dry or saturated state as per requirement. The specimen is enclosed in a flexible impervious sleeve and sealed at the top with the cap and at the bottom with the pedestal. The confining pressure is applied and maintained through the oil, filled in the cell. Axial load is applied to the specimen at a constant rate of deformation or loading in loading machine. If the confining pressure acts on the top of the specimen, due to the design of the cell and loading system, its load has to be added to the axial load measured during shear. Strain gauges are attached to measure the lateral strains as well as deformation indicated by LVDT.The stress-strain curve is usually plotted with lateral and longitudinal stresses; Poisson's ratio is estimated. At least three specimen tests under different confining pressures are conducted to provide value of shear strength parameters. The parameters, ć and φ, are based on the assumption that the failure envelope is linear within the range of confining pressure adopted. Test are conducted under confining pressures ranging from low to high pressure.

3.1.3 Brazilian test.

The tensile strength is greatly influenced by loading along the plane of weakness or across it in anisotropic specimen. Usually Brazilian test is conducted on NX size specimen with length equal to the diameter as per IS 10082(1981). The tensile strength increases with the increasing rate of loading above 0.1 T/min. It is subjected to line load in an CTM with the Brazilian mould. Load is applied on the specimen gradually. The fracture is supposed to initiate at the center and progress towards the loaded ends; if the fracture starts from periphery, the test is discarded.

3.2 Analyzing the results for Granite and Basalt.

3.2.1 *Vs to Vp correlation*.

As the first step, a correlation between compressional and shear wave velocities were obtained showing the linear relationship between Vp and Vs. It shows a strong relationship between two velocities which enables estimation of one velocity having another one. The following equation defines this relationship.
Basalt- Vs = 0.4198Vp - 24.35
Granite Vs = 0.5333Vp - 379.05

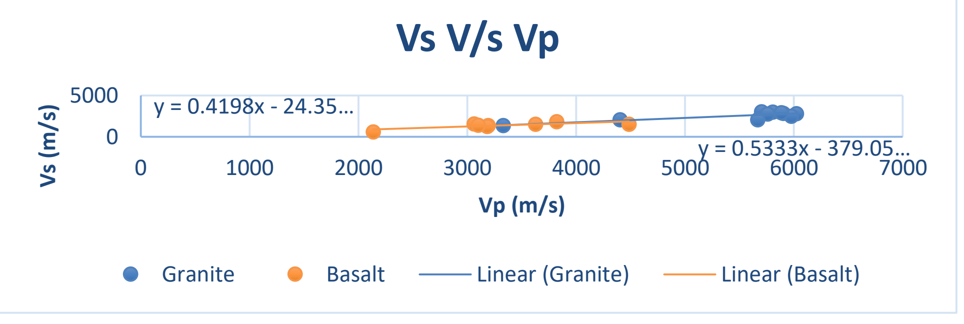

Figure 3 Vs Versus Vp.

3.2.2 *Static Young's Modulus Versus Dynamic Young Modulus (Es and Ed)*

The most important relation, which is the aim of the research is the correlation between static and dynamic Young's modulus (Es and Ed). The importance of this relation, especially in oil & gas industry, because of the fact that sonic dada is usually available in oil and gas wells which makes it possible to calculate dynamic modulus; however, obtaining cores in order to measure static modulus in lab is cumbersome and expensive process. Finding a good correlation between Es and Ed enables estimation of Es without requirements to perform expensive and time consuming compressive strength test. The Equation below gives a fair relationship between static and dynamic Young's modulus for Granite and Basalt.

Basalt - Ed = 1.1195Es - 0.3949.

Granite- Ed = 1.0297Es + 0.152.

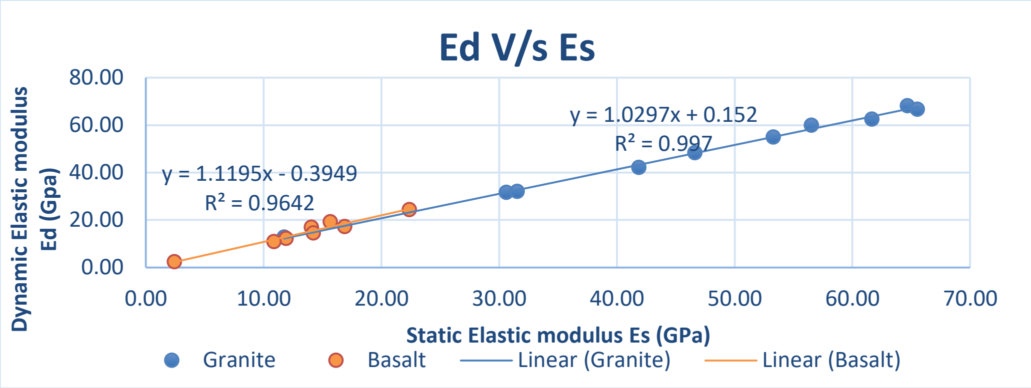

Figure 4 Young modulus (Dynamic versus Static)

3.2.3 *Correlation between Gd and Gs.*

Another important relationship is between Dynamic shear modulus (Gd) and Static shear modulus (Gs). Having found a good equation between Gs and Gd, we can evaluate Gs and Gd without doing expensive and time consuming static tests. The results of analyzing the sample, data is obtained from laboratory and the graph is plotted which gives an appropriate power relationship between these two parameters with $R^2 = 0.96$ for Basalt and $R^2 = 0.97$ for Granite. The relation can be defined as

Basalt - Gdyn = 1.0899Gstat - 0.1835.

Granite – Gdyn = 1.0188Gstat - 0.6007.

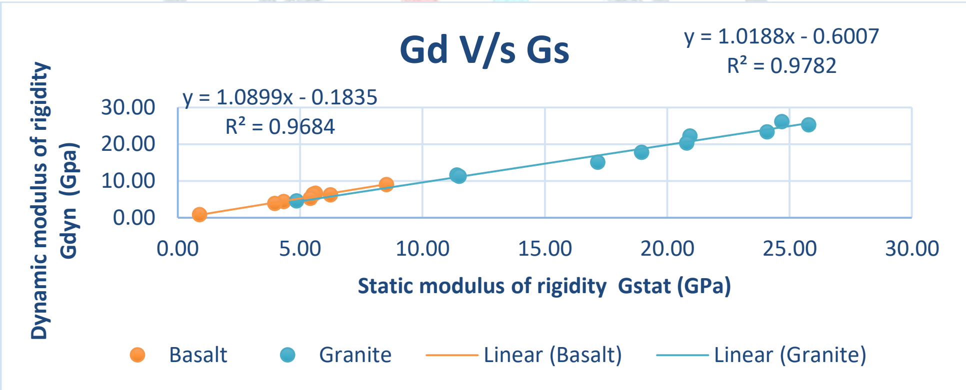

Figure 5 Shear Modulus (Dynamic versus Static)

3.2.4 *Correlation between Bulk Modulus Kdyn and Kstat.*

The relation between Dynamic bulk modulus (Kdyn) and Static bulk modulus (Kstat) gives a strong equation of Basalt Kdyn = 1.5637x + 0. 9536. Figure 6 reveals a fairly appropriate relationship between Kdyn and Kstat with $R^2 = 0.74$ for Basalt and $R^2 = 0.44$ for Granite.

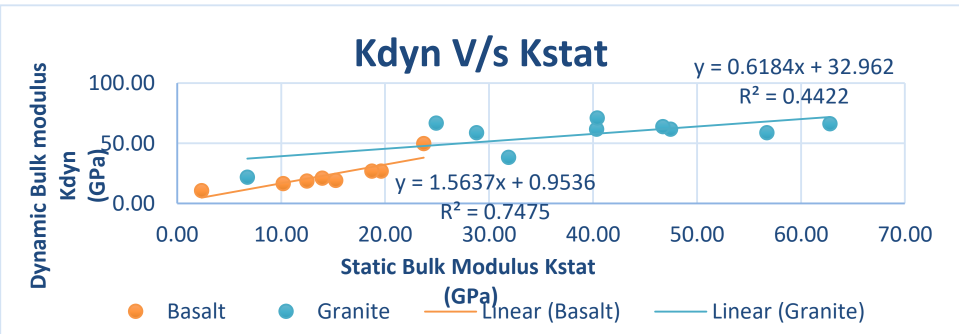

Figure 6 Bulk modulus (Kdyn Versus Kstat)

3.2.5 *UCS versus elastic waves velocity.*

Above mentioned parameter have some influence on rock strength. Therefore, UCS is expected to show relationship with elastic wave velocity. To scrutinize this results graphs were plotted between UCS and Vp and Vs. These cross plots showed rather weak correlation due to dispersion of data. Following are the equations

Basalt - UCS = 0.0264Vp -35.692.

Basalt - UCS = 0.0365Vs + 2.037.

Granite - UCS = 0.0289 Vp- 99.114.

Granite – UCS = 0.0375 Vs -36.512.

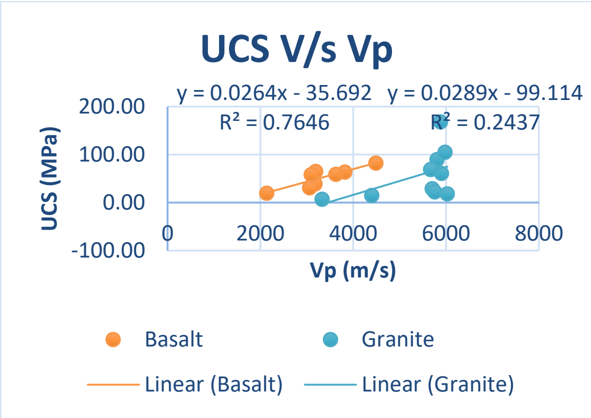

Figure 7 Uniaxial compressive strength versus Vp

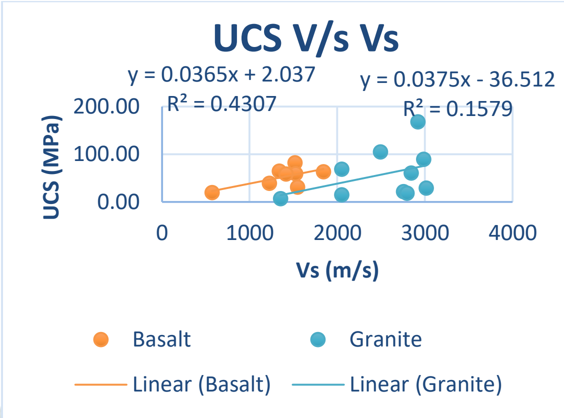

Figure 8 Uniaxial compressive strength versus Vs.

3.2.6 *Cohesion, angle of friction Versus Dynamic Elastic modulus.*

The correlation of cohesion and angle of friction estimated the R^2= 1. The equation mentioned below can give the values of C and φ more precisely with respect to Dynamic modulus of elasticity.

C = -0.3314 Ed + 66.107.

ᶲ = 0.4368 Ed - 4.56

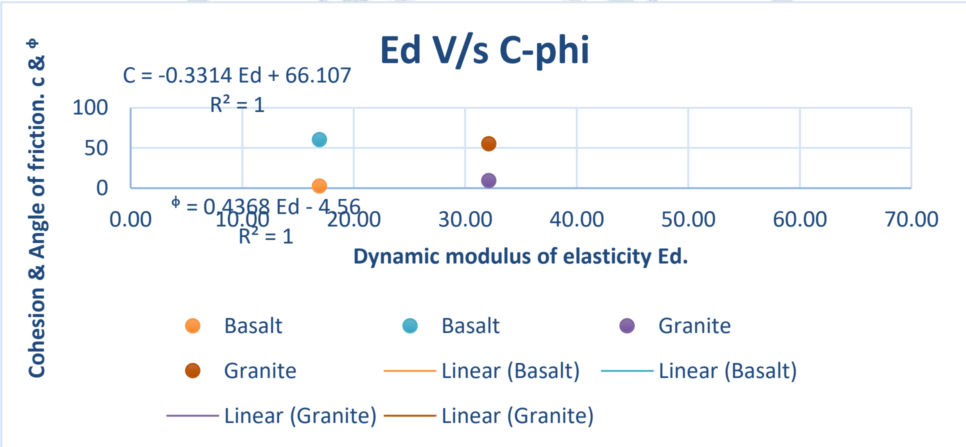

Figure 9 C, ᶲ Versus Dynamic Modulus of elasticity Ed.

3.2.7 *Tensile strength Versus elastic waves velocity.*

Brazilian test was conducted on samples in order to determine the tensile strength. The result was used to establish correlation between Tensile strength (T) and elastic wave velocity. Following are the equations of Vp and Vs versus Tensile strength.

Basalt - T = 2E-08Vp + 0.0076.

Basalt - T = -2E-07Vs + 0.0079.

Granite - T = -4E-07Vp + 0.013.

Granite – T = -2E-06Vs + 0.0147

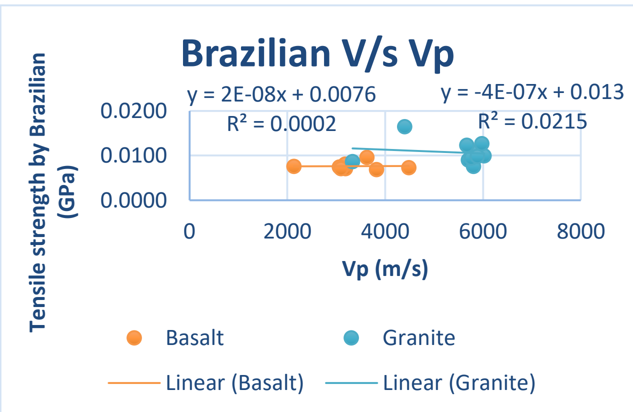

Figure 10 Tensile strength Versus Vp.

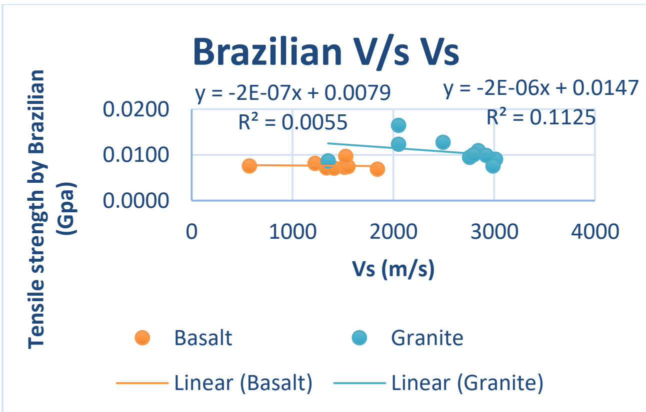

Figure 11 Tensile strength Versus Vs.

3.2.8 ***Water absorption versus Elastic wave velocity.***

Water absorption is an important rock index depending on mineralogy and porosity of rock. This are the characteristics that sonic velocity also depends on them. These cross plots showed rather weak correlation due to dispersion of data. In overall, the following equation can be used to estimate water absorption from Vp with reasonable accuracy.

Basalt- Iw = 4E-05Vp + 0.3697.

Granite- Iw = 3E-05Vp + 0.195.

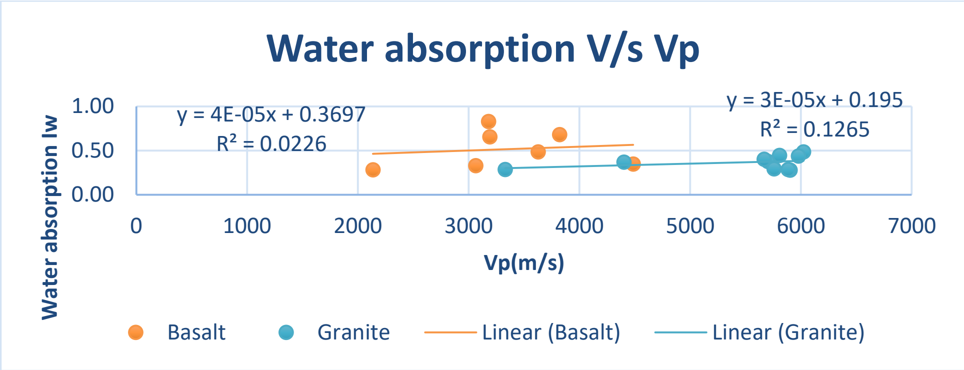

Figure 12 Water absorption versus Vp.

3.2.9 ***Porosity versus elastic wave theory.***

Contrasting the general idea that porosity can be estimated from sonic data with a high accuracy, the results of tests showed an intermediate correlation between parameters. The reason for that was due to discrepancy in rock types used in correlation.

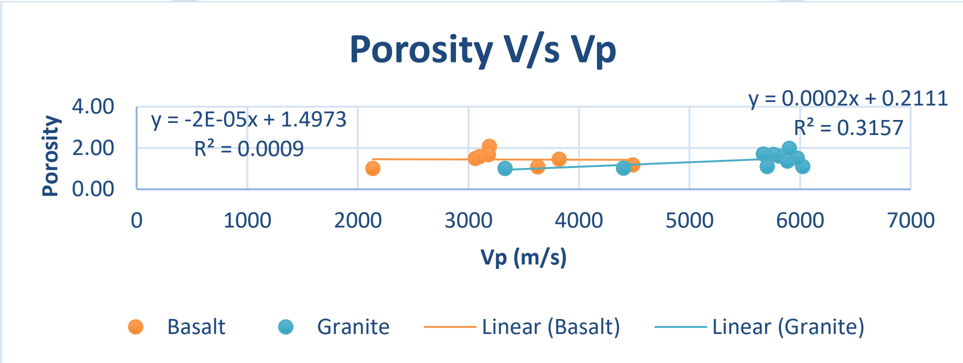

Figure 13 Porosity versus Vp.

3.2.10 ***Specific gravity versus elastic wave theory.***

Specific gravity is also an important index considered in rock index. The result of the study, gives the following equation for Vp respectively. These cross plots showed rather weak correlation due to dispersion of data.

Basalt – G = 0.0002Vp + 2.0169.

Granite – G = 0.0001Vp + 1.7945.

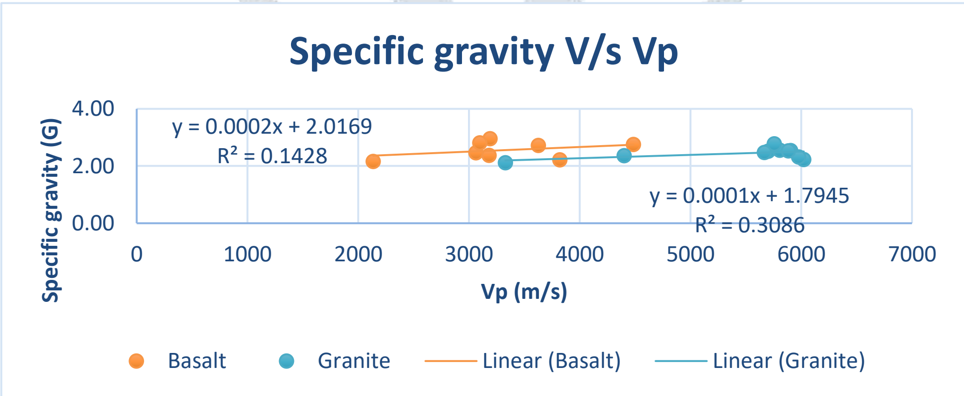

Figure 14 Specific gravity Versus Vp.

3.2.11 ***Density versus elastic wave theory.***

According to the plots, compressional and shear wave velocities increases with increasing density, due to the fact that sonic velocity is higher in solids than fluids. As a results, having Vp and Vs of certain rock, it is possible to estimate density with acceptable accuracy.

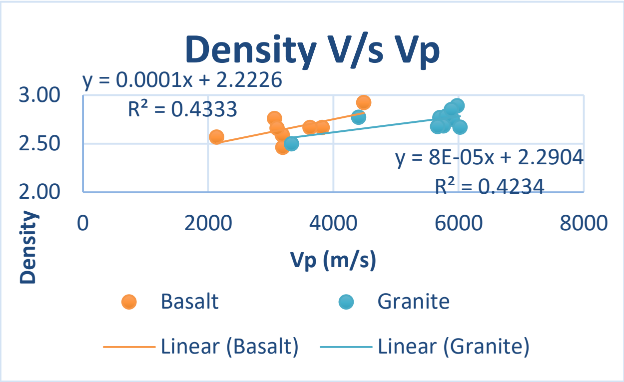

Figure 15 Density Versus Vp

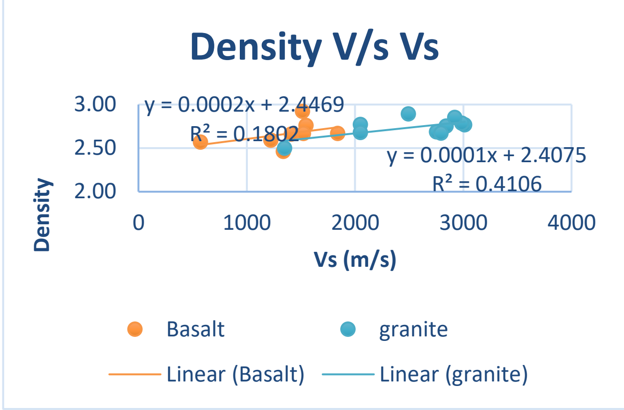

Figure 16 Density Versus Vs

3.2.12 *Poisson's ratio (Static V/s Dynamic).*

Poisson's ratio depends upon Modulus of Elasticity. It has some influence on rock. The R^2 value is 0.4902 for Basalt and R^2 value for Granite is 0.0043 due to dispersion of data. The dynamic Poisson's ratio is almost higher than the Static Poisson's ratio, but there is no obvious relationship between the static and dynamic Poisson's ratio. The static Poisson's ratio distributes widely, while dynamic Poisson's ratio is concentrated.

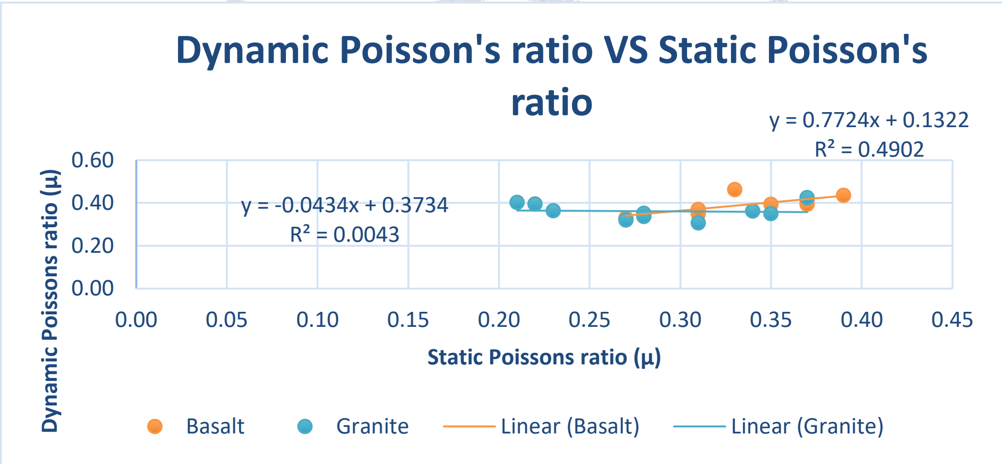

Figure 17 Dynamic Poisson's ratio Versus Static Poisson's ratio.

IV.RESULTS AND DISCUSSION

4.1 Results of Descriptive Statics of Study.

Table 4.1: Descriptive Statics

Sr no	Rock Type	Core No	Confined/ Unconfined	Vp (m/s)	Vs (m/s)	Edynamic. (GPa)	µdyn	Estatic (GPa)	µstatic	Gdyn (GPa)	Kdyn (Gpa)	Gstatic (GPa)	Kstat (GPa)	Failure load. (kN)	UCS (MPa)	Cohesion. (Kg/cm^2)	Angle of friction.(φ)
1	Basalt	34B	Unconfined	3191	1337	12.26	0.39	11.89	0.37	4.40	19.19	4.34	15.24	154.10	64.52		
2	Basalt	93	Unconfined	3821	1841	24.41	0.35	22.37	0.31	9.05	26.91	8.54	19.62	150.10	63.41		
3	Basalt	117A	Unconfined	3181	1223	10.95	0.41	10.87	0.37	3.87	21.04	3.97	13.94	91.40	38.93		
4	Basalt	38	Unconfined	2137	572	2.46	0.46	2.40	0.33	0.84	10.63	0.90	2.35	45.30	19.28		
5	Basalt	14	Unconfined	4486	1515	19.25	0.44	15.64	0.39	6.70	49.84	5.63	23.70	193.0	82.13		
6	Basalt	108B	Confined	3063	1547	16.92	0.33	14.05	0.27	6.37	16.47	5.53	10.18	70.00	31.02		
7	Basalt	82	Confined	3627	1525	17.29	0.39	16.86	0.35	6.21	26.84	6.24	18.73	132.10	58.98	2.83	60.50
8	Basalt	83	Confined	3099	1413	14.53	0.37	14.21	0.31	5.31	18.45	5.42	12.46	129.60	58.30		
9	Granite	34	Unconfined	4402	2049	31.68	0.36	30.59	0.34	11.63	38.18	11.42	31.87	34.80	15.01		
10	Granite	22	Unconfined	3330	1351	12.78	0.40	11.74	0.21	4.56	21.62	4.85	6.75	14.70	6.47		
11	Granite	56	Unconfined	5702	3012	68.20	0.31	64.67	0.31	26.10	58.74	24.68	56.73	65.90	28.67		
12	Granite	44	Unconfined	5903	2841	59.91	0.35	56.51	0.35	22.20	66.24	20.93	62.79	139.80	60.49		
13	Granite	53	Unconfined	5806	2986	66.67	0.32	65.50	0.27	25.25	61.80	25.79	47.47	205.50	89.40		
14	Granite	103	Unconfined	5758	2753	54.92	0.35	53.26	0.28	20.31	61.78	20.81	40.35	50.70	21.81		
15	Granite	129	Unconfined	6024	2794	48.40	0.36	46.62	0.23	17.76	58.86	18.95	28.78	43.10	17.98		
16	Granite	46	Confined	5668	2050	32.10	0.42	31.51	0.37	11.27	71.10	11.50	40.40	156.30	68.64		
17	Granite	85	Confined	5976	2492	42.12	0.39	41.87	0.22	15.10	66.70	17.16	24.92	242.00	105.00	9.46	55.47
18	Granite	67A	Confined	5885	2920	62.47	0.34	61.65	0.28	23.37	63.76	24.08	46.71	377.80	168.06		

Table 4.2: Range of Modulus.

Variable	Young's Modulus (E)	Bulk Modulus (K)	Modulus of rigidity (G)	Poisson's ratio. (µ)
Static	2.40 to 64.67	2.35 to 62.79	0.90 to 24.68	0.21to 0.39
Dynamic	2.46 to 68.20	10.63 to 71.10	0.84 to 26.10	0.31to 0.46

4.2. Discussion.

In the process of dynamic test, the core sample is exsiccation condition. If the confinement pressure increases, the sample micro crack will closure, pore fluid will compress, the equivalent of pore fluid stiffness will increase, and it will provide additional Young's modulus. The static Young's modulus is the ratio of axial stress and axial strain. When confining pressure increases, the core sample will be longer, the axial stress will increase, if temperature is varied some of the minerals in core sample will expand with heat, which can offset some of the deformation, and axial strain will decrease. Once you get Young's modulus you can calculate Bulk modulus and modulus of rigidity and the differences between Static modulus and Dynamic modulus can be estimated.

CONCLUSIONS

It can be concluded that for both the rocks Granite and Basalt from Laboratory tests and results, that in some cases we can replace the expensive and complex tests with cheap and quick nondestructive test.

- Density, water absorption percentage and porosity can be determined by ultrasonic pulse velocity test with higher precision.
- The dynamic Young's modulus is greater than the static Young's modulus.
- The dynamic Poisson's ratio is almost higher than the Static Poisson's ratio, but there is no obvious relationship between the static and dynamic Poisson's ratio.
- The static Poisson's ratio distributes widely, while dynamic Poisson's ratio is concentrated.
- Static Elasticity modulus can be estimated by having dynamic elasticity modulus with a rather high precision.
- Static shear modulus can be estimated by having dynamic shear modulus with a moderate precision.
- The general equation encompassing all types of rock do not give reliable results and show large relative errors. Such errors are unacceptable and cannot be used for planning and design purpose. It is suggested that the rock should be taken into account separately in statistical analysis and seeking the relationship between elastic wave theories.

REFERENCES

[1] Al-Shayea NA (2014) Effects of testing methods and conditions on the elastic properties of limestone rock. Eng Geology 74:139-156. https://doi.org/10.1016/j.enggeo.2004.03.007.

[2] ASTM D 2845 Standard test method for Laboratory Determination of Pulse Velocities and Ultrasonic Elastic Constant of Rock.

[3] Fjar E, Holt RM, RaaenAM.Petroleum related rock mechanics,2nd ed.2.Aufl Development in petroleum science, vol .53. Elsevier professional.s.l.2008.

[4] Jaeger JC, Cook NGW, Zimmerman RW. Fundamentals of rock mechanics 4th ed. Malden Mass: Blackwell Publ;2007.

[5] Lama RD, Vutukuri VS. Handbook on mechanical properties of rocks: testing techniques and results, 1.print.vol 2 no.1.Clausthal: Trans Tech Publication. 1978.

[6] Rutqvist J, Wu Y-S, Tsang C-F, Bodvarsson G. A modeling approach for analysis of coupled multiphase fluid flow, heat transfer, and deformation in fractured porous rock. Int J Rock Mech Min Sci.2002; 39:429-42. https://doi.org/10.1016/S1365-1609(02)00022-9

[7] Zoback MD. Reservoir geomechanics 5 print. Cambridge: Cambridge Univ. Press;2011.