Sequential Linear Quadratic Programming: A Review

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ABSTRACT: Optimal Power Flow (OPF) is used in accordance with energy balancing constraints and power system operating inequalities to maximize the specified target function. For power planning and operation, OPF has been extensively used. In general, OPF is neither a linear nor a convex issue. With semi-defined programming, this research presents a new method to addressing optimal power flow problems. SDP (Semi-definite Programming) is a programming technique. The proposed method is putting the OPF problems into an SDP model and creating an algorithm. In other words, in a non-linear programming model, OPF has been properly transformed into a convex SDP model. The OPF issue can be solved using SDP and primal dual inner point algorithms with super linear convergence. The proposed method was tested on four different kinds of OPF goal functions. Because of its resilience to OPF problems, comprehensive numerical simulations of 4 to 300 bus-sized test systems have shown that this method is promiscuous. The suggested method failed to address computer resource limitations that were too large, such as Random Access Memory (RAM). The importance of an overview of the state of the art, an assessment of recent achievements, and suggestions for future study cannot be overstated. As a result, developing better, sparse, and parallel computations algorithms utilizing OPF and SDP is an intriguing future research topic.

KEYWORDS: Buses, Interior Point Method, Optimal Power Flow, Optimization, Semidefinite Programming.

1. INTRODUCTION

As a strong instrument for optimising power systems issues many academics across the globe have attracted from the beginning of the 1960s[1]. Optimal Power Flow (OPF). In this field, a number of techniques based on LINEAR (LP), Q4 programming, the Newton approach, NLP and decomposition methods have been developed Recently, indoor point method (IPM) NLP-based algorithms have also been used effectively to OPF issues[2]. However, as power systems get increasingly sophisticated, OPF problems become harder to deal with. Although IPM theory is extensively established for NLP, there are still a lot of difficulties with connection creation. Firstly, because of its non-convex character, the NLP-based OPF suffers the convergence difficulty. In addition, Jacobian matrices (partial derivatives of first order) and Hessian (partial derivatives of second order) have to be generated for each particular issue in order to apply NLP IPM. Consequently, a generic and consistent software solution to NLP issues with IPM cannot be developed conveniently.

The history of optimum energy flow (OPF) research is characterised by the use of increasing every optimization technique for a problem which has been described in principle since the early 1960s[3]. The OPF is a nonlinear, static optimization problem which calculates optimal settings for power network electrical variables, load settings and system characteristics. The OPF has quickly benefited from constant numerical optimization progress and remarkable technological advancements. Progress has shown itself in an appropriate time frame to resolve larger and more complicated challenges.

The answer to the economic dispatch was historically the OPF predecessor by the Equal Incremental Cost Method (EICC). The introduction of the OPF signalled the end of the "classical," almost 30-year economic epoch of shipment. The OPB was a dramatic shutdown from shipment, however the shipping algorithms could now be observed to make the OPF crude. Both are issues of optimization with the same minimal cost goal.

However, economic dispatch only takes into account rei1 production of power. And the electric network reflects the constraint of equality of the Hinge, the equation of power balancing.

The OPF and the EICC occupy the extremes of the spectrum in terms of computing complexity. Implementation of the numerical OPF techniques for optimization and the creation of advanced shipping algorithms based on simplified OPF have drawn tG into the spectrum. With this topic in mind, several outstanding examination gapers during the previous 15 years have been created.

The most important are a summary of the state of the art, an examination of current developments and recommendations for future study. There is special attention in the two most recent publications in this list. Charpentier offers an intriguing algorithm categorization according to its solution structure, and then compares numerical results from each Group's most well-known programmes. The benefits of Carpentier's categorization were recognised by Stott and his colleagues[4]. The development of security-restricted OPF algorithms will be emphasised.

The objective of this study differs slightly from the previous because it is not a critical examination. It takes a look back on the OPF fi-eld and provides an overview of the OPF from the beginning of the 1930s to the present. It presents a classification system with a wave of optimisation approaches that is divided into some details in order to illustrate the major possibilities[5].

For more than a decade, semi-finite programming (SDP) has been one of the most active areas in numerical optimisation. Many famous algorithms have been exploited using uniform frameworks. The SDP is proved to be convex and superliner convergence theoretically can possess the primal-dual internal point methods for SDP [6]. Furthermore, the greatest advantage for the SDP based IPM is that Jacobian matrices and hessian matrices are not derived and computed for each individual issue.

Several practical issues, such as control and system theory, signal and communication processing, and combinatorial optimisation have been resolved effectively. There are no other practical problems. tackle difficulties of power shipping, often limited concerns of economic dispatch (CED). These two publications were pioneers in the use of SDP to power systems, to our knowledge. However, the work did not incorporate power flow balances and limitations on bus voltage. It is important and difficult to extend the SDP to resolve OPF challenges, known for their intricacy and practicality. Three elements are the motivations for this study:

- A non-convex NLP is the traditional OPF issue, which is more difficult than the issues referred to before. On the other hand, SDP is part of convex optimization[7] and can ensure an optimum global solution with IPM. Therefore, owing to the many advantages of the SDP technology, its convexity and a uniform algorithm software application framework, it is worth studying how to correctly reformulate the traditional OPF to an SDP model.
- The resultant SDP model should be convex and solution quality may be assured by utilising the SDP IPM when an OPF issue is re-formulated to the sDP one using the quadratic form [7]. In detail, the non-linear quadratic components in the OPF are substituted with the appropriate elements of the variable matrix X in SDP using X = xTx where x is a row vector and matrix trace operator.
- Ready approaches like IPM for LP to tackle SDP issues are available. In addition, SDP is also subject to SDP solution approaches that are in fact a particular instance of LP[7]. For SDP, IPM for LP, which can be applied in polynomial time to address convex optimization problems, can be deployed[8]. In addition, IPM improvements for LP are also available in SDP, which potentially makes IPM for SDP equally effective as for LP.

An significant field of research in the recent past has been the use of optimization techniques to power system planning and operational challenges. Optimal power flow is a general phrase describing a wide range of issues where a certain goal function is optimised while at the same time meeting restrictions determined by the electrical grid operational and physical detail. Conventional OPF formulations are intended to reduce thermal resource running costs, subject to the fulfilment of bus active and reactive energy balancing restrictions in voltages and phase angles.

To solve OPF issues, a wide range of optimization approaches have been employed. The techniques can be divided into:

i. Nonlinear Programming (NLP),
ii. Quadratic Programming (QP),
iii. Newton-Based Solution of Optimality Conditions,
iv. Linear Programming (LP),
v. Hybrid Versions of Linear Programming And Integer Programming, And
vi. Interior Point Methods.

i. Non-Linear Programming (NLP):

Nonlinear programming (NLP) addresses nonlinear objective and limitation problems. The limitations might be formulations of equal treatment and/or inequity. Inequality can be determined using the above and below limitations.

A number of approaches were employed to tackle OPF Categories 3 issues, such as the SUMT, the Lagrange based multiplier and the MINOS extended idea. This seminar takes on non-linear aims and limitations. A review of most often discovered applications indicated that around 8% of OPF formulations utilised packages for general purposes for both on-line as well as off-line operational difficulties in real time.

ii. Quadratic Programming (QP):

Quadratic programming is a particular type of nonlinear programming with a quadratic, linear function. Several QP techniques for OPF (loss, voltage economic delivery) types of issues have been utilised in this category (about 15 percent). Methods based on quasi-Newton and sensitivity have been used to solve OPF issues online.

iii. Newton-Based Solutions:

The essential optimum circumstances, generally known as the Kuhn-Tucker conditions, are established via this technique. These are often nonlinear equations which require iterative solution methods. For its quadratic convergence qualities, the Newton technique is favoured.

iv. Linear Programming (LP):

Linear programming addresses restriction issues and objective function stated with nonnegative variables in linear forms. The simplex technique is known to solve LP issues fairly effectively. Around 25% of the documents examined tackle the issues of OPF utilising approaches based on LP. The improved simplex approach is the most often utilised methodology. Target functions (voltage, loss, cost shipment and VAR) are linearized to allow LP solution.

v. Mired Integer Programming (MIP):

Mixed integral programming (MIP) is a specific kind of linear programming whose limiting equations contain only integer variables. The high demands of computer resources on integrated programming and mixed integrated programming, such as nonlinear programming, and the number of discrete variables is a significant indicator of how hard a MIP is to resolve.

vi. Interior Point Methods:

The interior point technique has amazed the research community as the system resolves linear programming more quickly and maybe better than the usual simple procedure. Extension to NLP and QP issues of the interior point technique has demonstrated outstanding characteristics and promising results.

1.1. Formulation of OPF problem:

A major non-linear optimisation issue is the OPF problem. The polar, rectangular or polar and rectangular shapes can be formed. The rectangular form of the OPF issue is used in this study to take the benefit of quadratic polynomials with no trigonometric function as power flow equations, which can then directly result in SDP modelling. The objectives to be reduced are chosen for the active or reactive power loss of transmission lines, the cost of fuel and the active system loss of total power. The OPF problem may thus be stated accordingly:

$$Min \begin{cases} P_{loss2} = -\sum_{i \in S_B} \sum_{j \in S_B} G_{ij} \left[(f_i - f_j)^2 + (e_i - e_j)^2 \right] \\ Q_{loss} = \sum_{i \in S_B} \sum_{j \in S_B} B_{ij} \left[(f_i - f_j)^2 + (e_i - e_j)^2 \right] \\ F_{cost} = \sum_{i \in S_G} (a_{fi} + a_{li} P_{Gi} + a_{qi} P_{fi}^2) \\ P_{loss2} = \sum_{i \in S_G} P_{Gi} \end{cases}$$

Which is issued to the equations as follows:

$$\begin{cases} P_{Gi} - \sum_{j \in S_B} e_i(e_j G_{ij} - f_j B_{ij}) + f_i(f_j G_{ij} + e_j B_{ij}) = P_{Di} \\ Q_{Ri} - \sum_{j \in S_B} f_i(e_j G_{ij} - f_j B_{ij}) - e_i(f_j G_{ij} + e_j B_{ij}) = Q_{Di} \end{cases} \quad i \in S_B$$

1.2. Semidefinite programming by IPM:

Half-finite programming is intended to optimise a linear function, subject to linear restrictions by picking a positive half-finite array. In other words, it is widespread to replace the variables with a symmetric matrix, whereas non-negative limitations are generally replaced with the positive half-finite limit. This generalisation still inherits from its vector counterpart some key properties: it is convex with a rich theory of duality (albeit not as strong as that of linear programming) and iteration of inner points theoretically enables fast methods to solve them.

For non-linear programming with great computational performance, the OPF issue may be resolved by IPM and its solution is optimum[9]. However, for each individual situation, NLP must be used in the Jacobian matrices and Hessian matrices. Thus, a consistent generic software solution for OPF issues with NLP is not convenient. However, for half-finite programming the problem of OPF may also be handled with IPM, and the numerical simulation in this report showed that it is possible to ensure the quality of its solution, such as the NLP[10].

The Powell and Fletcher Powell algorithms offered a not linear programming method which incorporated a penalty factor approach [2]. Real and reactive active power losses and costs of real and reactive power shipments were included in these approaches. A system based on a simplified model of the Saskatchewan Power Corporation System was used to show this technique. It covers mixed hydro-thermal cost functions with the linear cost of hydro input and non-linear costs of thermal input. Reactive optimization may only be used with a single input/output costs function for pure hydro or purity of thermal, although actual power optimisation for mixed goals may be better.

After the 1974 contributions, Barcelo et al. submitted the non-linear approach to programme optimization with a complex Hessian matrix approximation for accurate optimal delivery with safety limitations, in 1977 as a next major contribution to the solution to the economic dispatch target[11]. This approach has been intended for on-line operation but may be used for optimal output issues off-line. The new Hessian matrix approximation is fundamental to the approach and conserves sparing and significant computer core memory. The approach used sparse matrix elimination and optimization using unique patterns of switching inequality constraints. The findings are comparable to a Newton power flux solution on a 1200-bus practical system. The findings from this technique were contrasted with the straight-economic delivery penalty-function method, with substantial cash savings proposed by the author.

In 1982, an approach that utilises variable metric techniques was introduced by Housos and Irisam[12]. This approach was utilised to solve the optimal and verified energy flow. The approach was compared with famous methods; nevertheless, with large-scale Systems some convergence problems may emerge. The method worked well on tiny systems like the 14-bus system, yet for big scale issues it did not work with the same precision.

In 1982, Shoults and Sun had an optimum power flow issue based on the algorithm of decomposition (P/Q)[13]. The P-problem includes reducing the cost of producing hourly electricity by managing actual generators and taping transformers. The Q-problem included reducing actual drive losses by adjusting terminal voltages of generator, transformer tap settings and shunting condenser/reactors. Static safety constraints like bus voltage limitations, line flow rating and generator reactive power limitations also contain P- and Q issues. The study showed that the results achieved by employing the broken method would be the same if the problems were addressed. A nonlinear approach of optimization based on the gradient method utilising the SUMT technology has been created. An out-of-home penalty function is developed to make dependant functions at the ideal point (as a beginning point is not necessary). To achieve a workable solution, the first beginning point is important, and the approach breaks down big issues into smaller subsets and requires less time for calculation. This procedure is useful to operate online.

The voltage optimization approach used the Quasi-Newton process with the same convergence property as the Han-Powell method was reported in 1989. The technique chooses which restrictions are and are not active. In reality this has been restricted to a 41-bus synthetic system. A lower Lagrange multiplier estimate might lead to wrong findings. The authors did not do a comparison research and the problem of sensitivity is not addressed properly. This technique includes an infeasibility detection mechanism and the priorities list relies on heuristics that are based on the expertise of the operator. The work is compared to the GRG 1 method that has a repatriation approach for state and control variables and the 2nd order GRG algorithm. Transformer taps were not included in the design of the algorithm, and no power flow restriction was incorporated in the optimization method. The approach can achieve the best answer from an unfathomable beginning place of departure.

The sparsity and the network structure of the constraints are used to speed up the solution method and to speed up the convergence of nonlinear technology using the method of parallel tangents. This approach can achieve the ideal solution from an inviolable initial point of departure and the management of linear and nonlinear constraints. The technique has been tested on systems with 5, 39 and 118 buses. The method's features are: feasibility is achieved and optimization is carried out in two phases. The optimisation of Phase 1 is performed by the piece-wise linear approach to thermal production curves and in Phase 2, the nonlinear objective is employed in order to make further improvements with a quadratistical approximation of production costs for thermal plants. The approach is able to improve convergence from one stage to the next, and due to sparsity coding, the CPU time for this method is decreased. The authors argue that the approach is much superior than linear programming implementations; nonetheless, the system size and divergence from operational points may lead to issues due to the need for optimization.

In 1989, a continuation technique by Ponrajah and Galiana for solving nonlinear problem planning optimization. This approach was used to reduce the gasoline cost problem using a quadratic goal function and linear restrictions. It has been tested on systems with 6, 10, 30 and 118 buses The authors claim that the approach is quicker than heuristic methods and inefficiency methods. This approach can manage load and sensitivity fluctuations.

Reid and Hasdorf provided a quadratic approach for the resolution of QPF issues in 1973. The technique uses Wolfe's algorithm to solve the economic dispatch issue, which does not require penalty factors or a gradient size calculation. The technique was primarily designed for research purposes and the model employed is thus limited and uses conventional economic transmission with a limitation in voltage, reactive and actual power. It took a fairly acceptable time for the CPU; nevertheless, it grew with increasing system size. Convergence is fairly rapid and does not depend on the choice of step size or penalty elements. The approach uses a breakdown methodology based on the breakdown of Bema, Locke, Westberg (BLW). Even in an unfeasible starting point, this algorithm gives a solution; nonetheless, it was tested on tiny synthetic systems; its super linear convergence quality seems to be quick due of its power flow. For small-scale issues the technique converges quickly, however the convergence conditions do not appear realistic. More stringent testing must be performed before the production quality approach is used.

Burchett et al. revised his 1982 work to four objective functions including fuel costs, active and reactive losses and novel shunt condensers[3]. The technique has the advantages of runtime and robustness of the new method when comparing with an older Lagrangian algorithm. The approach was able to achieve an ineffective solution by developing a succession of quadratic algorithms that converge on an ideal solution to the initial non-linear issue. This strategy uses sparsity coding methods and converges quadratically.

The Aoki et al. approach was introduced in 1987 to address the problem of restricted load flow (CLF) in an efficient, practical and conclusive methodology[9]. The approach has a mechanism for adjusting variable control, and a succession of nonlinear programming issues is the problem of CFL. A problem formulation is used with quasi-quadratic programming.

2. DISCUSSION

Optimal power flow (OPF) is used to maximise the specified objective function in accordance with the restrictions on energy balance and inequality placed on power system operation. OPF was widely used for planning and operating electricity systems. In general, the OPF issue is nonlinear and non-convex. Traditional optimization techniques have been applied to the solution of the OPF issue, including non-linear programming, quadratic programming, linear programming, mixed integer and indoor pointing method.

In order to simplify the issue, however, numerous mathematical assumptions – such as convex, quadratic and differentiable goal, and linear or linear objective and constraints – were necessary. The actual global optimum cannot thus be easily attained a significant field of research in the recent past has been the use of optimization techniques to power system planning and operational challenges. Optimal power flow is a general phrase describing a wide range of issues where a certain goal function is optimised while at the same time meeting restrictions determined by the electrical grid operational and physical detail. Conventional OPF formulations aim at minimising thermal resource operational costs according to the respective restrictions.

This work has proposed a solution to the OPF problem by use of half-finite programming. Six test systems of small (4 buses) to big (300 Buses) were tested with the suggested technique, which is successful in terms of the findings. The SDP technology was not employed to tackle the OPF problem, to our best knowledge.

The OPF has been resolved by several optimization approaches. It is a highly complicated nonlinear and nonconvex issue. In NLP formats IPM was one of the top OPF algorithms [8]. The SDP is a convex programming approach which may also be carried out with IPM. Therefore, once a traditional OPF issue becomes a semidefinite programming model, it will be a convenient problem and may subsequently, as demonstrated in the numerical simulations contained in this work, be resolved with IPM SDP and profit from the SDP approach.

However, because of limitations in computer resources such as CPU and memory, the suggested approach could not address excessively large systems. The development of more powerful algorithms with the method of sparsity and parallel computing of OPF utilising SDP are therefore an intriguing issue in future for study.

3. CONCLUSION

This work has proposed a solution to the OPF problem by use of half-finite programming. Six test systems of small (4 buses) to big (300 Buses) were tested with the suggested technique, which is successful in terms of the findings. The SDP technology was not employed to tackle the OPF problem, to our best knowledge. The OPF has been resolved by several optimization approaches. It is a highly complicated nonlinear and nonconvex issue. In NLP formats IPM was one of the top OPF algorithms [8]. The SDP is a convex programming approach which may also be carried out with IPM. Therefore, once a traditional OPF issue becomes a semidefinite programming model, it will be a convenient problem and may subsequently, as demonstrated in the numerical simulations contained in this work, be resolved with IPM SDP and profit from the SDP approach. However, because of limitations in computer resources such as CPU and memory, the suggested approach could not address excessively large systems. The development of more powerful algorithms with the method of sparsity and parallel computing of OPF utilising SDP are therefore an intriguing issue in future for study.

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