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STRONGLY g # -CLOSED SET IN TOPOLOGICAL SPACES

[1]Dr.P.Thamil Selvi, [2]G.M.Kohila Gowri,[3]V.RuthDayana
[1&2&3] Assistant Professor, Department of Mathematics,
[1&2&3]Parvathy's Arts and Science College, Dindigul, Tamil Nadu, India

Abstract

The aim of this paper is to introduce and study stronger form of generalized $g^{\#}$-closed sets in a topological space. Also we investigate topo- logical properties of strongly $g^{\#}$-closed sets. Throughout this paper (Y, r_1) represent non empty topological spaces on which no separation axioms are assumed, unlesses otherwise mentioned. For a subset A of (Y, r_1), cl(A) and int(A) represent the closure of A withrespect to r_1 and the interior of A with respect to r_1 respectively.

Keywords: $g^{\#}$-closed,gsp-closed, gs-closed, r-g-closed, λ g closed, strongly $g^{\#}$-closed sets

1. Introduction

Levine (1960) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality etc are all g-closed hereditary. Andrijevic (1986), Arya and Nour(1990), Bhattacharya and Lahiri(1987), Dontchev (1995,1996), Ganambal(1997), Levine(1960,1963), Maki(1993,1994,1996),Mashhour et.al(1982), Njastad(1965), Palaniappan(1993),Velicko(1968) and Veerakumar(2000) introduced and investigated semi-preopen sets, generalized semiopen sets, semi- generalized open sets, generalized semi-preopen sets, $\lambda-$generalized closed sets, η -generalized closed sets, pre regular closed sets, generalized open sets, semi open sets,β-closed sets regular generalized closed sets, H-closed sets and
$g^{\#}$-closed sets which are some of the weak and stronger form of open sets and complements of these sets are called the same type g-closed sets respectively.

Veerakumar (2000) introduced and investigated between closed sets and $g^{\#}$-closed sets. The aim of this paper is to introduce and study stronger form of generalized $g^{\#}$-closed sets in a topological space. Also we investigate topo- logical properties of strongly $g^{\#}$-closed sets. Throughout this paper (Y, r_1) represent non empty topological spaces on which no separation axioms are assumed, unlesses otherwisementioned. For a subset A of (Y, r_1), cl(A) and int(A) represent the closure of A with respect to r_1 and the interior of A with respect to r_1 respectively.

2. Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine. Definition 2.1. [13]: A subset A of a topological space (Y, r_1) is called a pre- open set if $A \subseteq int(cl(A))$
and pre-closed set if $cl(int(A)) \subseteq A$.

Definition 2.2. [8] A subset A of a topological space (Y, r_1) is called a semi- open set if $A \subseteq cl(int(A))$
and semi closed set if $int(cl(A)) \subseteq A$.

Definition 2.3. [14] A subset A of a topological space (Y, r_1) is called an β- open set if $A \subseteq int(cl(int(A)))$ and an α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition 2.4. [1] A subset A of a topological space (Y, r_1) is called a semi pre- open set (γ-open set) if $A \subseteq cl(int(cl(A)))$ and semi pre-closed set if $int\ (cl(int(A))) \subseteq A$.

Definition 2.5. [16] A subset A of a topological space (Y, r_1) is called a λ- closed set if $A = cl_{\square}(A)$
where $cl_{\square}(A) = \{x \in X : int(cl(U)) \cap A \neq \varphi, U \in r_1$ and $y \in U\}$.

Definition 2.6. [16] A subset A of a topological space (Y, r_1) is called a η- closed set if $A = cl_y(A)$ where $cl_y(A) = \{x \in X : (cl(U)) \cap A \neq \varphi, U \in r_1 \ and\ y \in U\}$.

Definition 2.7. [9] A subset A of a topological space (Y, r_1) is called a g-closed if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.8. [3] A subset A of a topological space (Y, r_1) is called a semi- generalized closed set(briefly sg-closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$, U is semi open in (Y, r_1).

Definition 2.9. [2] A subset A of a topological space (Y, r_1) is called ageneralized semi-closed set (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$, U is open in (Y, r_1).

Definition 2.10. [11] A subset A of a topological space (Y, r_1) is called a generalized β -closed (brieflyg β -closed) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is β-open in (Y, r_1).

Definition 2.11. [10] A subset A of a topological space $(Y, r_1)$is called an β generalized closed set (briefly β g-closed) if $\beta\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.12. [14] A subset A of a topological space (Y, r_1) is called a generalized semi pre-closed set (briefly gsp-closed) if $spcl(A) \subseteq U$whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.13. [154] A subset A of a topological space (Y, r_1) is called a regular generalized closed set (briefly r-g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (Y, r_1).

Definition 2.14. [12] A subset A of a topological space (Y, r_1) is called a generalized pre closed set(briefly gp-closed) if $(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.15. [7] A subset A of a topological space (Y, r_1) is called a gen-eralized pre regular closed set (briefly gpr-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in (Y, r_1).

Definition 2.16. [6] A subset A of a topological space (Y, r_1) is called a $\eta-$ generalized closed set(briefly ηg-closed) if $cl_y \subseteq U$ whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.17. [5] A subset A of a topological space (Y, r_1) is called a λ generalized closed set (briefly λ g closed) if $cl_{\square}\ (A) \subseteq U$ whenever $A \subseteq U$ and U is open in (Y, r_1).

Definition 2.18. [17] A subset A of a topological space (Y, r_1) is called a g#-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (Y, r_1).

3. Strongly g#-closed sets

In this section we have introduce the concept of strongly g#-closed sets in topological space and we investigate the group of structure of the set of all strongly g#-closed sets.

Definition 3.1. Let (Y, r_1) be a topological space and A be its subset, then A is strongly g#-closed set if $(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

Theorem 3.1. Every closed set is strongly g#-closed set.

Proof. The proof is immediate from the definition of closed set.

Example 3.1. The converse of the above theorem need not be true from the following example.

Let $Y = \{a, b, c, d\}$. $r_1 = \{\varphi, \{a\}, \{a, c\}, Y\}$. Let $A = \{a, b\}$ is astrongly g# closed set but not a closed set of (Y, r_1).

Theorem 3.2. If a subset A of a topological space Y is g#-closed then it is strongly g#-closed in Y butnot conversely.

Proof. Suppose A is g#-closed in Y.

Let G be an open set containing A in Y. Then G contains cl(A). Now $G \supseteq (A) \supseteq c(int(A))$. ThusA is strongly g#-closed in Y.

Example 3.2. The converse of the above theorem need not be true as seen from the following example.

Let $Y = \{a, b, c, d\}$ with topology $r_1 = \{Y, \varphi, \{a\}, \{a, b\}\}$. In this topological space the subset $\{b\}$ isstrongly g#-closed but not g#-closed set.

Theorem 3.3. If A is a subset of a topological space Y is open and stronglyg#-closed then it is closed.

Proof. Suppose a subset A of Y is both open and strongly g#-closed. Now $A \supseteq (in(A)) \supseteq cl(A)$.Therefore $A \supseteq (A)$. Since $(A) \supseteq A$.

We have $A \subseteq (A)$. Thus Ais closed in Y.

Corollary 3.1. If A is both open and strongly g#-closed in Y then it is both regular open and regular closed in Y.

Proof. As A is open $A = (A) = int(cl(A))$, since A is closed. Thus A is regular open . AgainA is open in Y, $(in(A)) = cl(A)$. As A is closed $(in(A)) = A$. Thus A is regular closed.

Corollary 3.2. If A is both open and strongly g#-closed then it is rg-closed.

Theorem 3.4. If a subset A of a topological space Y is both strongly g#-closed and semi open then it is g#-closed.

Proof. Suppose A is both strongly g#-closed and semi open in Y, Let G be an open set containing A.As A is strongly g#-closed , $G \supseteq cl(int(A))$. Now $G \supseteq (A)$. since A is semi open . Thus A is g#- closed in Y.

Corollary 3.3. If A subset A of a topological space Y is both strongly g#-closed and open then it is g#- closed set.

Proof. As every open set is semiopen by the above theorem the proof follows.

Theorem 3.5. A set A is strongly g#-closed iff $(in(A)) - A$ contains no non empty closed set. Proof. **Necessary :** Suppose that F is non empty closed subset of $cl(int(A))$. Now $F \subseteq (in(A)) - A$ implies $F \subseteq (in(A)) \cap A^C$, since $cl(int(A)) - A = cl(int(A)) \cap A^c$. Thus $F \subseteq cl(int(A))$. Now $F \subseteq A^c$ implies $A \subseteq F^c$. Here F^c is g-open and A is strongly g#-closed, we have $cl(int(A)) \subseteq F^c$. Thus $F \subseteq ((in(A)))^c$. Hence $F \subseteq ((in(A))) \cap (cl(int(A)))^c = \varphi$.Therefore F = φ ⇒ cl(int(A)) − A contains no non empty closed sets.

Sufficient: Let $A \subseteq G$, G is g-open. suppose that $cl(int(A))$ is not contained in G then $(cl(int(A)))^c$ is a non empty closed set of $(in(A)) - A$ which is a contradiction. Therefore $(int(A)) \subseteq G$ and hence A is strongly g# -closed.

Corollary 3.4. A strongly g#-closed set A is regular closed iff $(in(A)) - A$ is closed and $cl(int(A)) \supseteq A$.

Proof. Assume A that A is regular closed. Since $(in(A)) = A, cl(int(A)) - A = \varphi$ is regular closed and hence closed.

conversely assume that $cl(int(A)) - A$ is closed.By the above theorem $cl(int(A)) - A$ contains no nonempty Sclosed set.Therefore $cl(int(A) - A = \Phi$.Thus A is regular closed.

Theorem 3.6. Suppose that $B \subseteq A \subseteq Y$, B is strongly g#-closed set relative to A and that both open and strongly g# closed subset of Y then B is strongly g# closed set relative to Y.

Proof. Let $B \subseteq G$ and G be an open set in Y. But given that $B \subseteq A \subseteq Y$, therefore $B \subseteq A$ and $B \subseteq G$. This implies B⊆ A ∩G. Since B is strongly $g^{\#}$- closed relative to A, $(in(B)) \subseteq A \cap G. (ie) A \cap cl(int(B)) \subseteq A \cap G$. This implies $A \cap ((in(B))) \subseteq G$. Thus $(A \cap (cl(int(B)))) \cup (cl(int(B)))^c \subseteq G \cup (cl(int(B)))^c$ implies $A \cup (cl(int(B)))^c \subseteq G \cup (cl(int(B)))^c$. since A is strongly $g^{\#}$ closed in Y, we have $(cl(int(A))) \subseteq G \cup (cl(int(B)))^c$. Also $B \subseteq A \Rightarrow (in(B)) \subseteq cl(int(A))$. Thus $(in(B)) \subseteq cl(int(A)) \subseteq G \cup (cl(int(B)))^c$. Therefore B is strongly $g^{\#}$ closed set relative to Y.

Corollary 3.5: Let A be strongly g#closed and suppose that F is closed then $A \cap F$ is strongly g# closedset.

Proof. To show that $A \cap F$ is strongly $g^{\#}$-closed, we have to show $(in(A \cap F)) \subseteq G$ whenever $A \cap F \subseteq G$ and G is g-open. $A \cap F$ is closed in A and so strongly g#closed in B. By the above theorem $\cap$ is strongly g#closed in Y. Since $\cap F \subseteq A \subseteq Y$.

Theorem 3.7. Theorem 3.15: If A is strongly g#closed and $A \subseteq B \subseteq (in(A))$ then B is strongly g#closed .

Proof. Given that B ⊆ cl(int(A)) then cl(int(B)) ⊆ cl(int(A)), cl(int(B)) - B ⊆ cl(int(A)) - A . Since A ⊆ B. As A is strongly g# closed by the above theorem $(in(A)) - A$ contains no non empty closed set, $cl(int(B)) - B$ contains no empty closed set. Again by theorem 3.13, B is strongly g#-closed set.

Theorem 3.8. Theorem 3.16: Let A ⊆ Y ⊆ X and suppose that A is strongly g#closed in X then A is strongly g# closed relative to Y.

Proof. Given that A⊆ Y ⊆X and A is strongly $g^\#$closed in X. To show that A is strongly $g^\#$ - closed relative to Y, let A ⊆Y∩ G, where G is g-open in X. Since A is strongly $g^\#$-closed in X, A ⊆G impliescl(int(A)) ⊆ G. (ie) Y ∩ cl(int(A)) ⊆ Y ∩ G, where Y ∩ cl(int(A)) is closure of interior of A in Y. Thus A is strongly $g^\#$closed relative to Y.

Theorem 3.9. If a subset A of a topological space Y is gsp-closed then it is strongly $g^\#$-closed but not conversely.

Proof. Suppose that A is gsp- closed set in Y, let G be open set containing A. Then G ⊇ spcl(A), A ∪G⊇A∪(int(cl(int(A)))) which implies G ⊇ int(cl(int(A))) as G is open. (ie)G⊇cl(int(A)) - A is strongly $g^\#$closed set in Y.

Example 3.3: The converse of the above theorem need not be true from the following example.

Let Y= {a,b,c,d} with topology $r_1 = \{\Phi, X, \{a\}, \{b,c\}\}$ and B={b}. B is not strongly $g^\#$closed . since {b} is a g-open set of (Y, r_1) such that B ⊆ {b} but cl(B) = cl({b}) = {b, c} ⊆ {b}. However B is a gsp-closed set of (Y, r_1).

Theorem 3.10: Every λ closed set is a strongly $g^\#$closed set.

Proof. The Proof of the theorem is immediate from the definition.

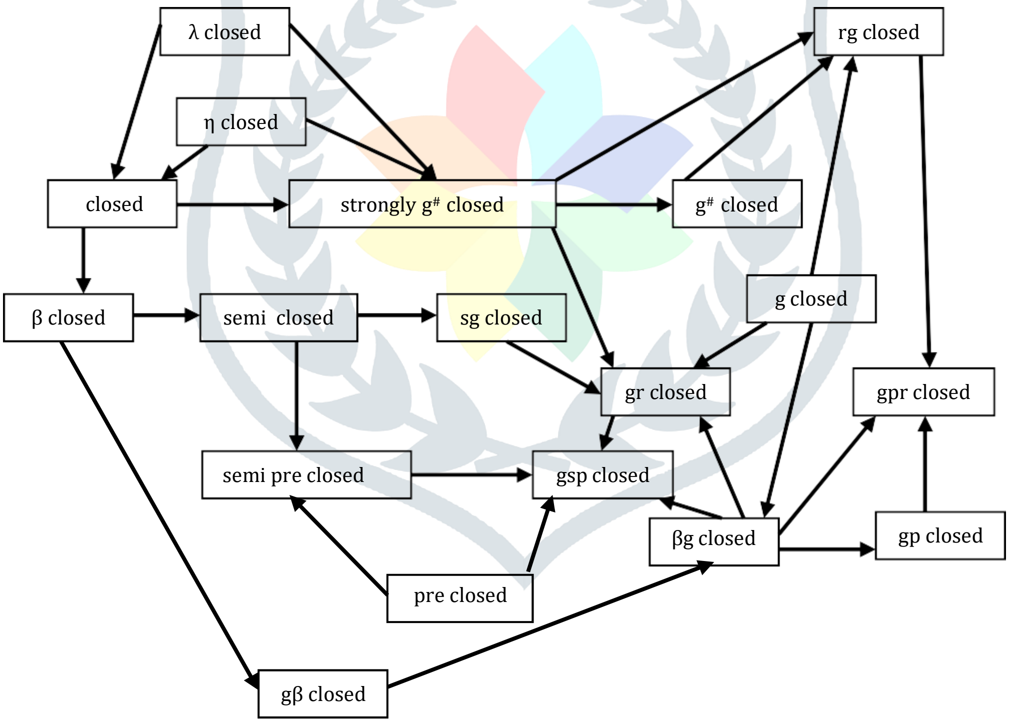
Example 3.4. The converse of the above theorem need not be true from the following example.

Let $Y = \{a, b, c, d\}, r_1 = \{\varphi, Y, \{a, b\}\} = \{a, c\}$. D is not a λ -closed set and also not even closed set.Hence D is strongly $g^\#$ closed set.

Theorem 3.11. Every η -closed set is a strongly $g^\#$closed set. Proof. The Proof of the theorem is immediate from the definition.

Example 3.5. The converse of the above theorem need not be true from the following example.

Let Y = {a, b, c,d}, r_1 = {Φ, Y, {a}, {a, b}{a, c}} and E ={c}. Clearly E is closed and hence strongly $g^\#$−closed. E is not λ − closed set of (Y, r_1).



Theorem 3.12. Every strongly $g^\#$-closed set in an β g-closed set and hence gs-closed, gsp-closed, gp- closed, gpr closed set and rg closed set but not conversely.

Proof. Let A be a strongly $g^\#$-closed set of (Y, r_1). By above theorem, A is g-closed. By implications (2.4) in Maki et.al(1993) A is β g-closed. From the investigations of Dontchev(1996) and Ganambal (1997), we know that every g-closed set is gs-closed, gsp-closed, gp-closed, gpr-closed and rg-closed. By above theorem every strongly $g^\#$closed set is gs-closed, gsp closed and rg- closed.

Example 3.6. The converse of the above theorem need not be true from the following example.

Let Y = {a, b, c,d} , r_1 = {Φ, X, {a, b}},D ={b} . D is not a β g closed, gs closed, gp closed, gpr closed and regular-closed but not strongly $g^\#$closed.

Remark 3.1. The following are the implications of strongly $g^\#$-closed set.

4. References

1. Andrijevic.D, Semi-preopen sets, Mat. Vesink, **38** (1986), 24 - 32.
2. S.P.Arya and T.Nour, Characterizations of S-normal spaces, Indian J.Pure.Appl.Math., **38** 21(8)(1990),717-719.
3. Bhattacharyya.P and Lahiri.B.K, Semi-generalised closed sets in topology, Indian J. Math., 29(1987), 375 - 382.
4. J.Dontchev, On generalizing semi-preopen sets,Mem.Fac.Sci.Kochi.Ser.A, Math., **29** 16(1995),35-38.
5. J.Dontchev and M.Ganster, On δ-generalized closed sets and $T_{3/4}$ - spaces, Mem.Fac.Sci.Kochi.Univ.Ser.A, Math., **17** 17(1996),15-31.
6. J.Dontchev and H.Maki, on θ -generalized closed sets,Internat.J.Math.Math.Sci,22(2) **17** (1998), 239-249.
7. Y.Ganambal, on generalized preregular closed sets in topological spaces,Indian J.Pure.Appl.Math., **38** 28(3)(1997), 351-360.
8. N.Levine, Semi-open sets and semi-continuity in topological spaces,Amer.Math.Monthly, **17** 70(1963), 36-41.
9. N.Levine, Generalized closed sets in topology,Rend.Circ.Math.Palermo, 17 19(2)(1970), 89-96.
10. H.Maki, R.Devi and K.Balachandran, Generalized α -closed sets in topol- ogy,Bull.Fukuoka Univ.E. Part III, **17** 42(1993), 13-21.
11. H.Maki, R.Devi and K.Balachandran, Associated topologies of general- ized α -closed sets and α -generalized closed sets, Mem.Fac.Sci.Kochi Univ.Ser.A, Math., **17** 15(1994), 51-63.
12. H.Maki, J.Umehara and T.Noiri, Every topological spaces in pre- $T_{1/2}$, Mem.Fac.Sci.Kochi. Univ.Ser A, Math., 17 17(1996),33-42.
13. A.S.Mashhour, M.E.Abd EI-Monsef and S.N.EI-Deeb, on Pre-continuous and weak pre- continuous mapping,Proc.Math. and Physics.Soc.Egypt, **17** 53(1982), 47-53.
14. O.Njastad, on some classes of nearly open sets,Pacific J.Math.,**17** 15(1965), 961-970.
15. N.Palaniappan and K.C. Rao, Regular generalized closed sets,Kyngpook.Math.J., **17** 33(2)(1993), 211-219.
16. N.V.Velicko, H-closed topological spaces, Amer.Soc.Transe., **17** 78(1968), 103-1