Changing and Unchanging of Nonsplit Eccentric Domination number in graphs.

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ABSTRACT

A subset D of the vertex set V (G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is said to be an eccentric dominating set if for every $v \in V - D$, there exists at least one eccentric point of v in D. An eccentric dominating set D of G is a nonsplit eccentric dominating set if the induced sub graph $< V - D >$ is connected. The minimum of the cardinalities of the nonsplit eccentric dominating sets of G is called the nonsplit eccentric domination number $\gamma_{nsed}(G)$. In this paper, we have studied the changing and unchanging of Nonsplit eccentric domination number in graphs.

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Key Words and Phrases: Domination number, distance, eccentricity, radius, diameter, self centered graph, neighborhood, eccentric domination ,nonsplit eccentric domination.

1 Introduction

Let G be a finite, simple, undirected graph on n vertices with vertex set V (G) and edge set E(G). For graph theoretic terminology refer Harary [5], Buckley and Harary [1]. The concept of distance in graphs plays a dominant role in the study of structural properties of a graph in various angles using related concept of eccentricity of vertices in graphs. The study of structural properties of graphs using distance and eccentricity started with the study of center of tree and propagated in different directions in the study of structural properties of graph such as unique eccentric point graphs. K-eccentric point graphs, self centered graphs. Let G be a connected graph and u be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus $e(v) = \max\{d(u, v) : u \in V \}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. The vertex v is a central vertex of G if e(v) = r(G). The center C(G) is the set of all central vertices of G. The central subgraph $< C(G) >$ of a graph G is the subgraph induced by the center. The vertex v is a peripheral vertex if e(v) = diam(G). The periphery P(G) is the set of all peripheral vertices of G. For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V (G) : d(u, v) = e(v)\}$. The open neighborhood N(u) of a vertex u is the set of all vertices adjacent to v in V . $N[u] = N(u) \cup \{u\}$ is called the closed neighborhood of u. For a vertex $u \in V (G)$, $N_i(u) = \{u \in V (G) : d(u, v) = i\}$ is defined to be the i th neighborhood of u in G. The concept of domination in graphs was introduced by Ore [8] and Cockayne et al. studied various bounds and results to domination in [4]. A set $D \subseteq V$ is said to be a dominating set of G, if every vertex in $V - D$ is adjacent to some vertex in D.

In 2010, Janakiraman, Bhanumathi and Muthammai defined eccentric domination in graphs [6] and studied eccentric domination in trees [2] and various bounds of eccentric domination in graphs. V.R. Kulli and Janakiram introduced the concept of split and nonsplit domination number of a graph in 1997 [10] and in 2000 [11] Motivated by these, we have defined split and nonsplit eccentric domination number of graphs.The changing and unchanging terminology was first suggested by Harary [5]. It is useful to partition the vertex set or the edge set of a graph G into three sets according to how the addition of vertex (or edge) or removal of addition (edge) affects the domination number.

A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists atleast one eccentric vertex of v in D. If D is an eccentric dominating set, then every superset $D'' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set. The eccentric domination number $\gamma ed(G)$ of a graph G is the minimum cardinality of an eccentric dominating set. An eccentric dominating set D of G is a nonsplit eccentric dominating set if the induced subgraph $< V - D >$ is connected

The nonsplit eccentric domination number $\gamma_{nsed}(G)$ of a graph G equals the minimum cardinality of a nonsplit eccentric dominating set. That is $\gamma_{nsed}(G) = \min |D|$, where the minimum is taken over D in D, where D is the set of all minimal nonsplit eccentric dominating sets of G.

In this paper, we have studied the changing and unchanging of the Nonsplit eccentric domination number in graphs.

2 Prior Results

Theorem 2.1

1. $\gamma_{ed}(K_n) = 1, n \geq 3$.

2. $\gamma_{ed}(K_{1,n}) = 2, n \geq 2$.

3. $\gamma_{ed}(W_n) = 3, n \geq 7$

. 4. $\gamma_{ed}(K_{m,n}) = 2, n \geq 2$.

$$5.\ \gamma_{ed}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & if\ n = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ = 3k\ or\ 3k+2 \end{cases}$$

$$6.\ \gamma_{ed}(C_n) = \begin{cases} \frac{n}{3}, & if\ n = 3m\ and\ is\ odd \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n = 3m+1\ and\ is\ odd \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ = 3m+2\ and\ is\ odd \end{cases}$$

Theorem 2.2

(i) $\gamma_{nsed}(K_n) = 1$, for $n \geq 3$.

(ii) $\gamma_{nsed}(K_{m,n}) = 2$, for $m, n \geq 2$.

(iii) $\gamma_{nsed}(C_n) = n - 2$, for $n \geq 3$.

(iv) $\gamma_{nsed}(W_n) = 3$, for $n \geq 4$.

(v) $\gamma_{nsed}(T) = \gamma_{ns}(T)$ for any tree T.

(vi) $\gamma_{nsed}(C_n \circ K_1) = n$ for $n \geq 3$.

(vii) $\gamma_{nsed}(K_{1,n}) = n$, for $n \geq 2$

(viii) $\gamma_{nsed}(P_n) = n - 2$, for $n \geq 3$.

Theorem2. 3.

. Let G be a connected graph obtained from a complete graph K_n by attaching pendant edges at at least one of the vertices of the complete graph and not all the vertices of the complete graph then $\gamma_{nsed}(G) = s + 1$ for $n \geq 3$, here S is the set of all pendant vertices of G and $|S| = s$.

Theorem 2.4

If G is a spider, then $\gamma_{nsed}(G) = e + 1$, where e is the number of pendant vertices of G.

Theorem2. 5

If G is a caterpillar such that each non pendant vertex is of degree three then $\gamma_{nsed}(G) = e$, where e is the number of pendant vertices of G.

Theorem 2.6

If G is a wounded spider, then $\gamma_{nsed}(G) = s + e$ where s is the number of pendant vertices of non wounded legs, and e is the number of pendant vertices of wounded legs.

Theorem2. 7

If G is a unicyclic graph with p vertices and if G has an induced cycle of length $(p - 1)$ then $\gamma_{nsed}(G) = p - 2$.

Theorem2. 8

If G is a graph obtained from a path P_{p-1} by attaching one pendant vertex with any vertex of degree 2 of a path P_{p-1} then $\gamma_{nsed}(G) = p-2$.

Theorem 2.

Let n be an even integer. Let G be obtained from the complete graph K_n by deleting edges of a linear factor. Then $\gamma_{ed}(G) = \frac{n}{2}$

3 Changing and Unchanging of Nonsplit Eccentric Domination Number in Graphs

In this section, a study of changing and unchanging of nonsplit eccentric domination number in connected graphs is initiated.

3.1 Changing and unchanging of γ_{nsed} due to vertex removal

If a vertex is removed from G, then the vertex lies in one of the following three sets. There are vertices whose removal will maintain the nonsplit eccentric domination number.

Define,

$V E^0_{ns} = \{v \in V(G)/\gamma_{nsed}(G - v) = \gamma_{nsed}(G)\}$

$V E^-_{ns} = \{v \in V(G)/\gamma_{nsed}(G - v) < \gamma_{nsed}(G)\}$

$V E^+_{ns} = \{v \in V(G)/\gamma_{nsed}(G - v) > \gamma_{nsed}(G)\}$

Theorem 3.1.1.

1 If G is a complete graph with atleast three vertices then $V = V E^0_{ns}$.

2. Let G be a star $K_{1,n}$, $n \geq 3$,

(i) If v is a pendant vertex then $v \in V E^-_{ns}$

(ii) If v is a central vertex then $v \in V E^0_{ns}$.

3. If G is a complete bipartite graph then $V = V E^0_{ns}$ for $m, n \geq 3$.

4. Let G be a path on atleast four vertices.

(i) If v is a pendant vertex,then $v \in V E^-_{ns}$,

(ii) If v is a support vertex, then $v \in V E^-_{ns}$,

(iii) If v is not a pendant vertex and not a support vertex, then $v \in V E^-_{ns}$.

5. If G is a cycle C_n, $n \geq 3$, then $v \in V E^-_{ns}$.

6. If G is a wheel and v is not a central vertex of G, then $v \in V E^0_{ns}$ forn ≥ 3.

If G is a wheel and v is a central vertex of G then $v \in V E^+_{ns}$ for $n \geq 10$.

7. If G is a corona $C_n \circ K_1$ ($n \geq 3$), and if v is a pendant vertex of G, then $v \in V E^0_{ns}$.

Proof.

(1) $G \cong K_n$, $n \geq 3$ by Theorem 2 , $\gamma_{nsed}(Kn) = 1$. Let $v \in V(G)$. Then $G - v \cong K_{n-1}$ and $\gamma_{nsed}(G - v) = 1 = \gamma_{nsed}(G)$. Therefore $v \in VE^0{}_{ns}$ and hence $V = VE^0{}_{ns}$.

(2) Let $G \cong K_{1,n}$, for $n > 3$. By Theorem 2, $\gamma_{nsed}(K_{1,n}) = n$. Let $v \in V(G)$ be the central vertex then $G-v = nK_1$, totally disconnected. Therefore $\gamma_{nsed}(G - v) = \gamma_{nsed}(G)$. Thus $v \in VE^0{}_{ns}$. Let $v \in V(G)$ be a pendant vertex. Then $G - v \cong K_{1,n-1}$ and $\gamma_{nsed}(G - v) = n - 1 < \gamma_{nsed}(G) = n$. Therefore $v \in VE^-{}_{ns}$..

(3) Let G be a complete bipartite graph Km,n, where $m, n \geq 3$. By Theorem 2, $\gamma_{nsed}(Km,n) = 2$. Let $v \in V(G)$. Then $\gamma_{nsed}(G-v) = 2 = \gamma_{nsed}(G)$. Therefore $v \in VE^0{}_{ns}$ and hence $V = VE^0{}_{ns}$

(4) Let $G \cong P_n$, $n \geq 4$. By Theorem 2, $\gamma_{nsed}(Pn) = n - 2$.

(i) If v is a pendant vertex of G, then $G-v \cong P_{n-1}$, $\gamma_{nsed}(G-v) = n-3 < \gamma_{nsed}(G)$. Therefore $v \in VE^-{}_{ns}$.

(ii) If v is support vertex then $\gamma_{nsed}(G - v) = 1 + (n - 2) - 2 = n - 3$. Therefore $\gamma_{nsed}(G - v) < \gamma_{nsed}(G)$. Hence $v \in VE^-{}_{ns}$.

(iii) If v is not a pendant and not a support vertex then P_{n1} and P_{n2} be the component of $G-v$ with $n_1+n_2 = n-1$ and $n_1 \geq n_2 \geq 2$ then $\gamma_{nsed}(G - v) = (n_1 - 2) + n_2 = n_1 + n_2 - 2 = n - 3 < \gamma_{nsed}(G)$. Therefore $v \in VE^-{}_{ns}$.

(5) Let $G \cong C_n$, $n \geq 3$. By Theorem 2, $\gamma_{nsed}(C_n) = n-2$. Let $v \in V(G)$. Then $G - v \cong P_{n-1}$ and $\gamma_{nsed}(G - v) = n - 3 < \gamma_{nsed}(G)$. Therefore $v \in VE^-{}_{ns}$ and hence $V(G) = VE^-{}_{ns}$..

(6)Let G be a wheel on $(n + 1)$ vertices, where $W_n = C_n + K1$, $n \geq 10$. Then by Theorem 2, $\gamma_{nsed}(W_n) = 3$. Let v be a vertex of W_n.

(a) Let $v \in V(C_n)$. Then $G - v \cong K_1 + P_{n-1}$ and $\gamma_{nsed}(G - v) = 3 = \gamma_{nsed}(G)$ for $n \geq 3$. Thus $v \in VE^0{}_{ns}$.

(b) Let v be the central vertex of Wn. Then $G - v \cong C_n$ and $\gamma_{nsed}(G - v) \geq \gamma_{nsed}(G)$. Therefore $v \in VE^+{}_{ns}$.

(7) Let G be the corona $C_n \circ K_1$ and v be a pendant vertex of G. By Theorem 2, $\gamma_{nsed}(C_n \circ K_1) = n$. Now, $G - v$ is a graph obtained by attaching exactly one pendant edge at each of $(n - 1)$ vertices of C_n. Then a minimum nonsplit eccentric dominating set of $G-v$ contains all the $(n - 1)$ pendant vertices and a vertex of degree 2 of C_n, and hence $\gamma_{nsed}(G - v) = n = \gamma_{nsed}(G)$. Therefore $v \in VE^0{}_{ns}$

Theorem 3.1.1.

Let T be a tree such that each vertex of degree atleast 2 is a support. Then $\gamma nsed(T) = e$, where e is the number of pendant vertices in T. Also if v is a pendant vertex then $v \in VE^-{}_{ns} \cup VE^0{}_{ns}$.

Proof. T is a tree with each vertex of degree atleast 2 is a support, where e is the number of pendant vertices in T. The set of all pendant vertices forms a minimum non split eccentric dominating set. Thus $\gamma_{nsed}(T) = e$. Let v be a pendant vertex of T. When we remove v from T, $\gamma_{nsed}(T - v) \leq \gamma_{nsed}(T)$. Therefore $v \in VE^-{}_{ns} \cup VE^0{}_{ns}$.

Theorem 3.1.2.

Let G be a spider. Then $V = V E^0_{ns} \cup V E^-_{ns}$

Proof.

Let G be a spider, then by Theorem 2.4 $\gamma_{nsed}(G) = s + 1$, where s is the number of pendant vertices of G. When we remove a pendant vertex v from G, $G - v$ is a wounded spider with one wounded leg, then $\gamma_{nsed}(G - v) = s < \gamma_{nsed}(G)$, using Theorem 2.6. Therefore $v \in V E^-_{ns}$. When we remove a vertex from G which is not a pendant vertex, then $G-v$ is disconnected. $G - v = G_1 \cup K_1$ where G_1 is a spider with $(s - 1)$ pendant vertices and $\gamma_{nsed}(G_1) = (s - 1) + 1$. Hence $\gamma_{nsed}(G - v) = ((s - 1) + 1) + 1 = s + 1 = \gamma_{nsed}(G)$. Therefore $v \in V E^0_{ns}$. Thus $V = V E^0_{ns} \cup V E^-_{ns}$

Theorem 3.1.3.

Let G be a caterpillar such that each non pendant vertices is of degree 3. Then $V = V E^0_{ns} \cup V E^+_{ns} \cup V E^-_{ns}$.

Proof.

Let G be a caterpillar such that each non pendant vertices is of degree 3. Then $\gamma_{nsed}(G) = e = \frac{p}{2}+1$, where e is the number of pendant vertices in G.

Case (i): When we remove a pendant vertex v which is peripheral, $\gamma_{nsed}(G - v) = e - 1 < \gamma_{nsed}(G)$.

Case (ii): When we remove a pendant vertex v which is not peripheral, $\gamma_{nsed}(G - v) = e - 1 < \gamma_{nsed}(G)$.

Case (iii): When we remove a vertex v of degree 3, which is not a pendant vertex, $G - v$ is disconnected.

Subcase (a): $\deg(v) = 3$ and v is adjacent to two pendant vertices. $G-v$ has 3 components with two isolated vertices. $\gamma_{nsed}(G - v) = \frac{p-3+1}{2}+ 2 = \frac{p+2}{2} = \frac{p}{2} + 1 = \gamma_{nsed}(G)$. Therefore $v \in V E^0_{ns}$

Subcase (b): $\deg(v) = 3$ and v is adjacent to only one pendant vertex. $G-v$ has 3 components G_1, G_2 and G_3 with one isolated vertex. Let the pendant vertices of G_1 be e_1 and the pendant vertices of G_2 be e_2 Then $e = e_1 +e_2 +1$. Let $e_1 \leq e_2$. Then G_1 has $(e_1+e_1-_1)$ vertices and G_2 has $e_2+(e_2-1)$ vertices. Hence $\gamma_{nsed}(G - v) = 1 + (e_1 + e_2 - 1) + e_2 = 2e_1 + e_2 = e_1 + (e_1 + e_2) = e_1 + (e - 1) \geq 2 + e - 1 = e + 1 > e = \gamma_{nsed}(G)$. Therefore $v \in V E^+_{ns}$.

Hence in general $V = V E^0_{ns} \cup V E^+_{ns} \cup V E^-_{ns}$.

Theorem 3.1.4.

Let G be a connected graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph but not all the vertices of a complete graph *then* $V = V E^0_{ns} \cup V E^-_{ns}$

Proof.

Let G be a graph obtained from a complete graph K_n by attaching pendant edges at at least one of the vertices of the complete graph but not attached to all the vertices of a complete graph. By Theorem 2.3 $\gamma_{nsed}(G) = s + 1$ where s is the set of all pendant vertices of G and $|S| = s$.

Case (i): If deg(v) = 1 and v is the only one pendant vertex of G. Then $G - v \cong K_n$ and $\gamma_{nsed}(G - v) = 1 < \gamma_{nsed}(G)$. Hence $v \in V E^{-}_{ns}$

Case (ii): If v is a support vertex of G. Then $G - v$ is a disconnected graph.

Subcase (a): $G-v$ has K_{n-1} as a component. Then $\gamma_{nsed}(G-v) = \gamma(G-v) = 2$. Also $\gamma_{nsed}(G) = 2$. Therefore $\gamma_{nsed}(G - v) = \gamma_{nsed}(G)$. Hence $v \in V E^{0}_{ns}$

Subcase (b): $G - v$ has a pendant vertex, $s \geq 2$. In this case $G - v = K_1 \cup G_1$, where G_1 has $(s - 1)$ pendant vertices and $\gamma_{nsed}(G) = s + 1$. Therefore $\gamma_{nsed}(G - v) = (s - 1) + 1 = s < \gamma_{nsed}(G)$. Hence $v \in V E^{-}_{ns.}$

Case (iii): Let $deg_G(v) = 1$ and let there exist t $(t \geq 2)$ vertices of degree 1 in G. Then $G - v$ is a graph with $(p - 1)$ vertices obtained from a complete graph by attaching $(s - 1)$ pendant edges at atleast one of the vertices of the complete graph. Then $\gamma_{nsed}(G-v) = (s-1) + 1 < \gamma_{nsed}(G)$. Hence $v \in V E^{-}_{ns}$

Case (iv): Let v be a vertex of the complete graph and be not a support vertex of G. Then $degG(v) = n - 1$ when n $(n < p)$ is the number of vertices of the complete graph K_n. Since $G - v$ has $(p - 1)$ vertices and $\gamma_{nsed}(G - v) = s + 1 = \gamma_{nsed}(G)$. Hence $v \in V E^{0}_{ns}$

From the above cases $V = V E^{0}_{ns} \cup V E^{-}_{ns}$

3.2 Changing and unchanging of γ_{nsed} due to edge removal

Here, the edge set of a graph G is classified in such a way that removal of an edge affect or does not affect the non split eccentric domination number of G.

Define,

$EE^{0}_{ns} = \{e \in E(G)/\gamma_{nsed}(G - e) = \gamma_{nsed}(G)\}$

$EE^{-}_{ns} = \{e \in E(G)/\gamma_{nsed}(G - e) < \gamma_{nsed}(G)\}$

$EE^{+}_{ns} = \{e \in E(G)/\gamma_{nsed}(G - e) > \gamma_{nsed}(G)\}$

Clearly, $E = EE^{0}_{ns} \cup EE^{+}_{ns} \cup EE^{-}_{n}$

The following are the results of some special classes of graphs.

Theorem 3.2.1

(i) If G is a complete graph then $E = EE^{+}_{ns}$

(ii) If G is a star K_1,n, $n \geq 2$, then $E = EE^{-}_{ns}$

(iii) Let G be a complete bipartite graph Km,n, $n \geq m \geq 2$. Then $E = EE^{0}_{ns}$.

(iv) If G is a path and e is a pendant edge then $e \in EE^{0}_{ns}$

(v) If G is a wheel Wn, then $E = EE^{0}_{ns}$

(vi) If G is a cycle then $E = EE^{0}_{ns}$

Proof.

(i)Let $G \cong K_n$. Let $e \in E(G)$. Then $G - e$ is $K_n - e$. Therefore $\gamma_{nsed}(G - e) = 2 > \gamma_{nsed}(G) = 1$. Hence $e \in EE^{+}_{ns}$ and $E = EE^{+}_{ns.}$

(ii) Let $G \cong K_{1},_{n}$, $n > 2$. Let $e \in E(G)$. Then $G - e$ is a disconnected graph. Therefore $\gamma_{nsed}(G - e) = n = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns}$. Therefore $E = EE^{0}_{ns}$

(iii) Let G be a complete bipartite graph $K_{m,n}$. Let $e \in E(G)$. Then $G - e \cong K_{m,n} - e$. Therefore $\gamma_{nsed}(G - e) = 2 = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns}$ and $E = EE^{0}_{ns}$

(iv) Let G be path P_n, $n \geq 4$. Let e be a pendant edge of P_n, $n \geq 4$. Then $G - e \cong P_{n-1} \cup K_1$ and also disconnected. By Theorem 2.2 $\gamma_{nsed}(G-e) = ((n-1)-2)+1 = n-2$. Hence $e \in EE^{0}_{ns}$ and $E = EE^{0}_{ns}$

(v) Let G be a wheel W_n, $n \geq 7$ and $W_n = C_n + K_1$. By Theorem 2.2 $\gamma_{nsed}(G) = 3$.

Case (i): Let $e = xy \in E(C_n)$. Then $G-e \cong K_1 + P_n$. $D = \{xzy\}$ is a nonsplit eccentric dominating set of $G - e$, where z is the central vertex of G. Therefore $\gamma_{nsed}(G - e) = 3 = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns}$

Case (ii): Let $e = xy \in E(G)$ be an edge joining the vertex of K_1 and a vertex of C_n. Then $G - e$ is a graph with radius 2 and diameter 3. Also $G - e$ is a graph with $\delta(G) = 2$ and $\Delta(G) = n - 1$ and $\gamma_{nsed}(G - e) = 3 = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns.}$

From cases (i) and (ii) $E = EE^{0}_{ns}$.

(vi) Let G be a cycle C_n, $n \geq 3$. Let $e \in E(C_n)$. Then $G - e \cong P_n$. By Theorem 2.2 $\gamma_{nsed}(Pn) = n - 2$ and $\gamma_{nsed}(C_n) = n - 2$. Therefore $\gamma_{nsed}(G - e) = n - 2 = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns}$ and $E = EE^{0}_{ns}$

Theorem 3.2.2

If G is a unicyclic graph with p vertices and G has an induced cycle of length $p - 1$ then

$E = EE^{0}_{ns} \cup EE^{+}_{ns}$

Proof.

Let $v_1, v_2, \ldots, v_{p-1}$ be the vertices of G and let x be the pendant vertex of G. $\gamma_{nsed}(G) = p - 2$. Then by Theorem 2.7. Let us remove an edge $e = uv \in V(G)$.

Case (i): If $d(u) = 3$ and $d(v) = 2$. Then $G - e$ is a path Pp. By Theorem 2.2 $\gamma_{nsed}(Pp) = p - 2$. Therefore $\gamma_{nsed}(G - e) = p - 2 = \gamma_{nsed}(G)$. Hence $e \in EE^{0}_{ns}$.

Case (ii): If d(u) = 2 and d(v) = 2. Then G − e is a graph obtained from a path Pp−1 by attaching one pendant vertex with any vertex of degree 2 of a path Pp−1. By Theorem 2.8 $\gamma_{nsed}(G - e) = p - 2 = \gamma_{nsed}(G)$. Hence $e \in EE^0_{ns}$.

Case (iii): If d(u) = 1 and d(v) = 3. Then G − e is a disconnected graph. Therefore $\gamma_{nsed}(G-e) = p-2+1 = p-1 > \gamma_{nsed}(G) = p-2$. Hence $e \in EE^+_{ns}$. From the above cases $E = EE^0_{ns} \cup EE^+_{ns}$

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