Review Paper of Jacobian

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Abstract : Let a_1 ...., a_m be such real numbers that can be expressed as a finite product of prime powers with rational exponents. Using arithmetic partial derivatives, we define the arithmetic Jacobian matrix J_a of the vector $a=(a_1,...., a_m)$ anologusly to the jocobian matrix J_f of a vector function f(9), we introduce the concept of multiplicative independence of $\{a_1,..., a_m\}$and show that J_a plays in it a similar role as J_f does in functional independence. We also present a kind of arithmetic implicit function theorem and show that J_a applies to it somewhat analogously as J_f applies to the ordinary implicit function theorem.

Index terms : multiplicative independence, arithmetic derivative, continuously differentiable function.

I) Introduction :

Let IR, Q, X, N and P stand for the set of real numbers, rational numbers, integers, nonnegative integers and primes, respectively.

If a∈IR, there may be a sequence of rational numbers $(Vp(a))_{P\in P}$ with only finitely many nonzero terms satisfying.

$$a = (sgn\ a) \quad \pi_{p \in P} P^{Vp(a)}, \text{ ....(1)}$$

Where sign is the sıgn function. We let R^1 and R^1_+ denote the set of all such real numbers and the subset consisting of its positive elements, respectively. Formula (1) is also valid for a=0, as we define Vp (0) = 0 for all p∈P. If Vp(a) ≠0, we say that p divides a and denote p | a otherwise, we denote p ∤ a.

<u>Proposition</u> :-

We are here further discussing about the sequence $(V_p(a))_{pEP}$ is unique. (Study from the textbook of J.G. Jacobi's Sammtliche Werke, Erster Band. i.e. (2)).

If a ∈IR. If then the sequence $(Vp(a))_{P\in P}$ is unique.

→ proof:

This is well known if a ∈ Q

We define the arithmetic derivative of a ∈ IR^1 by

$$a^1 = a \sum_{p\in P} \frac{Vp(a)}{P} = \sum_{p\in P} a^1 p \quad ,$$

where

$$a^1 p = \frac{Vp(a)}{P} \text{ a} \qquad \text{.................(2)}$$

is the arithmetic partial derivative of a with respect to p. These references mainly concern the arithmetic derivative in IN, Z, or Q, but most of the results can be extended to IR^1 in an obvious way.

Let $f=(f_1, ..., f_m) : E \to IR^m$ be a continuously differentiable function, where $E\in IR^n$ is a connected open set. Its Jacobion matrix at $t = (t_1, ..., t_n) \in E$ is defined by

$$J_f(t) = \begin{bmatrix} (f_1)^1_{t1}(t) & (f_1)^1_{t2}(t) \ldots\ldots (f_1)^1_{tn}(t) \\ (f_2)^1_{t1}(t) & (f_2)^1_{t2}(t) \ldots\ldots (f_2)^1_{tn}(t) \\ (f_m)^1_{t1}(t) & (f_m)^1_{t2}(t) \ldots\ldots (f_m)^1_{tn}(t) \end{bmatrix},$$

where

$(f_1)\frac{1}{t_j} = \frac{\partial f_i}{\partial f_j}$ If m=n, then det $J_f(x)$ is the Jacobian determinant (or, more briefly, the Jacobian) of f.

Let a_1, , $a_m \in IR_+^1$ (actually, we could study IR^1 instead of IR_+^1, which, however, would not benefit us in any significant way), and denote

$$P = \{P_1, \dots\dots, P_n\} = \{P \in IP \ |\exists\ a_i : P|\ a_i\} \dots\dots\dots (3)$$

and

$$a_{ij} = Vp_j(a_i), i = 1, \dots, m, i = 1, \dots, n \ \dots\dots\dots (4)$$

then

$$a_i = \pi_{P\in P}\ P^{Vp(ai)} = P_1^{ai1} P_2^{ai2} \dots\dots P_n^{ain}, i=1, \dots, m \dots\dots(5)$$

Further, let

$$a = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_m \end{bmatrix}, \quad a_i = \begin{bmatrix} a_{i1} \\ a_{i2} \\ . \\ . \\ a_{in} \end{bmatrix} \quad i = 1, \dots, m \dots\dots\dots (6)$$

and

$$A_a = \begin{bmatrix} a_{11} & a_{12} \dots\dots a_{1n} \\ a_{21} & a_{22} \dots\dots a_{2n} \\ . & . & . \\ . & . & . \\ a_m & a_{m2} \dots\dots a_{mn} \end{bmatrix}, = \begin{bmatrix} a_1^T \\ a_2^T \\ . \\ . \\ a_m^T \end{bmatrix} \dots\dots\dots (7)$$

we define the arithmetic Jacobian matrix of a by

$$J_a = \begin{bmatrix} (a_1)_{p_1}^1 & (a_1)_{p_2}^1 \dots\dots (a_1)_{p_n}^1 \\ (a_2)_{p_1}^1 & (a_2)_{P_2}^1 \dots\dots (a_2)_{p_n}^1 \\ . & . & . \\ . & . & . \\ (a_m)_{p_1}^1 & (a_m)_{p_2}^1 \dots\dots (a_m)_{p_n}^1 \end{bmatrix}$$

and, if m=n, the arithmetic Jacobian determinant (or, more briefly, the arithmetic Jacobian) of a by

$$\det J_0 = \begin{vmatrix} (a_1)_{p_1}^1 & (a_1)_{p_2}^1 \dots\dots (a_1)_{p_m}^1 \\ (a_2)_{p_1}^1 & (a_2)_{P_2}^1 \dots\dots (a_2)_{p_m}^1 \\ . & . & . \\ . & . & . \\ (a_m)_{p_1}^1 & (a_m)_{p_2}^1 \dots\dots (a_m)_{p_m}^1 \end{vmatrix}$$

Let f be as above. The functions f_1,...., fm are functionally independent (i.e. there is no function ϕ:$IR^m \to IR$ such that $\nabla\phi\big(f(t)\big) \neq 0$ and $\phi((f_1(t), \dots \dots, f_m(t)) = 0$ for all t ∈E) if and only if m≤n and rank J_f (t) =m for all t ∈E (8).

Applications :-

Here I studied that the wast use as Jacobian in Engineering field (5). An Jacobian study is useful in engineering students for solving examples Jacobian is used to solve system of differential equations. The use of Jacobian matrices allows the local (approximate) linerisation of non-linear system around a gives equilibrium point. An Jacobian is useful for calculation of Eigen values. The Jacobian Study is useful for solving examples change of variable formulas. An Jacobian Study is useful for studying multiple integrals. It is useful for studying linear algebra, linear transformation L with determinant. Also Jacobian study is wastly useful in statistics.

Conclusion :-

By using the Hadamard matrix product concept, this paper introduces to generalized matrix formation forms of numerical analogue of non- linear differential operators. We also present and prove simple underlying relationship between general nonlinear analogue polynomials & their corresponding Jacoboian matrices, which forms the basis of this paper. (7)

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