Medical Image Compression and Reconstruction Using Compressive Sensing

Mr. Utsav Bhatt[1], Kishor Bamniya[2]
PG Student, Asst. Professor
Kalol Institute and Research Center, Kalol, North Gujarat, India[1]

Abstract: Compressive sensing is new era and emerging platform for data acquisition and signal processing. Magical statement of Compressive sensing tells that one can recover certain signal or images from far fewer samples than traditionally required. Image Compression, the art and science of reducing the amount of data required tore present image is one of the most useful and commercially successful in the field of digital image processing. Image compression plays important role in many other areas, including tele video conferencing, remote sensing, document and medical imaging, and facsimile transmission. On encoding side it require two property of a signal that are sparsity and incoherence. First, any signal is converted into particular transform i.e. wavelet or DCT, with help of sensing matrix it extracts required coefficients which has less dimensionality than image dimensions and hence we can get resultant matrix. which is also called measurements which are non-adaptive. In certain areas like magnetic resonance imaging (MRI), it is urgent to reduce the time of the patients' exposure in the electromagnetic radiation. CS recovery algorithms basically divided into two types like L1 minimization techniques and greedy CS recovery algorithms. Where L1 minimization technique is based on linear optimization solving and this technique provide good security but more computation time is required for image reconstruction. When greedy CS recovery algorithms are based on iteration calculation of approximation of image coefficients until a convergence criterion is met and these algorithm is faster but they are no provide stability. So that we have proposed Hybrid technique for reconstruction of image and by using it we can reduce the elapsed time.

I. Introduction

The area of compressed sensing was initiated in 2006 by two ground breaking papers, namely [1] by Donoho and [2] by Cand´es, Romberg, and Tao. Nowadays, after only 6 years, an abundance of theoretical aspects of compressed sensing are already explored in more than 1000 articles. Moreover, this methodology is to date extensively utilized by applied mathematicians, computer scientists, and engineers for a variety of applications in astronomy, biology, medicine, radar, and seismology, to name a few.
The key idea of compressed sensing is to recover a sparse signal from very few non adaptive, linear measurements by convex optimization. Taking a different viewpoint, it concerns the exact recovery of a high-dimensional sparse vector after a dimension reduction step. From a yet again different standpoint, we can regard the problem as computing a sparse coefficient vector for a signal with respect to an over complete system. The theoretical foundation of compressed sensing has links to and also explores methodologies from various other fields such as, for example, applied harmonic analysis, frame theory, geometric functional analysis, numerical linear algebra, optimization theory, and random matrix theory.
It is interesting to notice that this development – the problem of sparse recovery – can in fact be traced back to earlier papers from the 90s such as [3] and later the prominent papers by Donoho and Huo [4] and Donoho and Elad [5]. When the previously mentioned two fundamental papers introducing compressed sensing were published, the term 'compressed sensing' was initially utilized for random sensing matrices, since those allow for a minimal number of non-adaptive, linear measurements. Nowadays, the terminology 'compressed sensing' is more and more often used interchangeably with 'sparse recovery' in general, which is a viewpoint we will also take in this survey paper.

II. Related Work

A. The Compressed Sensing problem

To state the problem mathematically precisely, let now $x = (x_i)_{i=1}^{n} \in R^n$ be our signal of interest. As prior information, we either assume that x itself is sparse, i.e., it has very few non-zero coefficients in the sense that

$$\|x\|_0 := \#\{i: x_i \neq 0\}$$

is small, or that there exists an orthonormal basis or a frame1 Φ such that $x = \Phi c$ with c being sparse. For this, we let Φ be the matrix with the elements of the orthonormal basis or the frame as column vectors. In fact, a frame provides more flexibility than an orthonormal basis due to its redundancy and hence leads to improved sparsifying properties, wherefore in this setting customarily frames are more often employed than orthonormal bases. Sometimes the notion of sparsity is weakened, For that we will refer to as approximately sparse. Further, let A be an $m \times n$ matrix, which is typically called sensing matrix or measurement matrix. Throughout we will always assume that $m < n$ and that A does not possess any zero columns, even if not explicitly mentioned.
Then the Compressed Sensing Problem can be formulated as follows: Recover x from knowledge of

$$y = Ax$$

or recover c from knowledge of

$$y = A\Phi c$$

In both cases, we face an underdetermined linear system of equations with sparsity as prior information about the vector to be recovered. This leads us to the following questions:

•What are suitable signal and sparsity models?
•What are suitable sensing matrices?
•How can the signal be algorithmically recovered?
•When and with which accuracy can the signal be recovered?

B. Sparsity: A Reasonable Assumption

As a first consideration, one might question whether sparsity is indeed a reasonable assumption. Due to the complexity of real data certainly only a heuristic answer is possible. If a natural image is taken, it is well known that wavelets typically provide sparse approximations. This is illustrated in Figure 1, which shows a wavelet decomposition [6] of an exemplary image. It can clearly be seen that most coefficients are small in absolute value, indicated by a darker color.

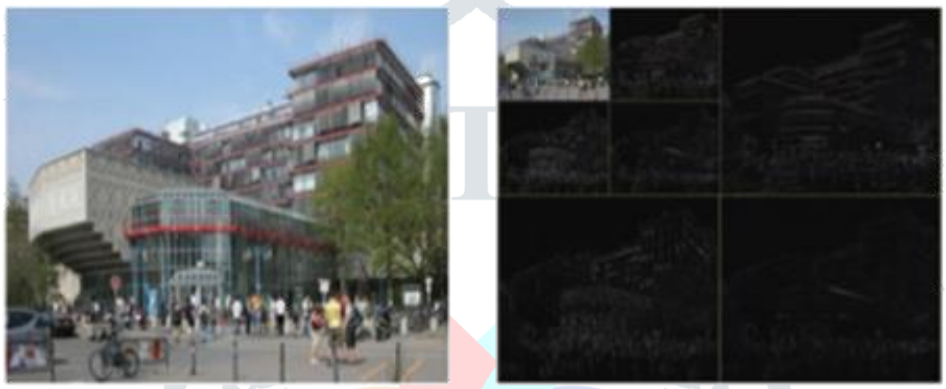


Figure 1: (a) Mathematics building of TU Berlin (Photo by TU-Pressestelle); (b) Wavelet decomposition

Depending on the signal, a variety of representation systems which can be used to provide sparse approximations is available and is constantly expanded. In fact, it was recently shown that wavelet systems do not provide optimally sparse approximations of most natural images, but the novel system of shear lets does.[7,8] Hence, assuming some foreknowledge of the signal to be sensed or compressed, typically suitable, well analyzed representation systems are already at hand. If this is not the case, more data sensitive methods such as dictionary learning algorithms, in which a suitable representation system is computed for a given set of test signals, are available.

Depending on the application at hand, often x is already sparse itself. Think, for instance, of digital communication, when a cell phone network with n antennas and m users needs to be modelled. Or in genomics, when in a test study m genes shall be analyzed with n patients taking part in the study. In the first scenario, very few of the users have an ongoing call at a specific time; in the second scenario, very few of the genes are actually active. Thus, x being sparse itself is also a very natural assumption.

In the compressed sensing literature, most results indeed assume that x itself is sparse, and the problem $y = Ax$ is considered. Very few articles study the problem of incorporating a sparsifying orthonormal basis or frame, and we mention [9,10]. In this paper, we will also assume throughout that x is already a sparse vector. It should be emphasized that 'exact' sparsity is often too restricting or unnatural, and weakened sparsity notions need to be taken into account. On the other hand, sometimes such as with the tree structure of wavelet coefficients some structural information of the non-zero coefficients is known, which leads to diverse structured sparsity models.

C. Sensing Matrices: How Much Freedom is Allowed

As already mentioned, sensing matrices are required to satisfy certain incoherence conditions such as, for instance, a small so-called mutual coherence. If we are allowed to choose the sensing matrix freely, the best choice are random matrices such as Gaussian iid matrices, uniform random ortho-projectors, or Bernoulli matrices, see for instance [2]. It is still an open question whether deterministic matrices can be carefully constructed to have similar properties with respect to compressed sensing problems. At the moment, different approaches towards this problem are being taken such as structured random matrices.[11,12] Moreover, most applications do not allow a free choice of the sensing matrix and enforce a particularly structured matrix. Exemplary situations are the application of data separation, in which the sensing matrix has to consist of two or more orthonormal bases or frames , or for high resolution radar, for which the sensing matrix has to bear a particular time-frequency structure.

D. Recovery Algorithms: Optimization Theory and More

1. Minimization of L1 norm

The first use of L1-norm for signal reconstruction goes back to a PhD thesis in 1965 – by Logan. L1-norm was used in geophysics way back in 1973 (ref: Claerbout and Muir, "Robust modeling with erratic data", Geophysics, 1973).L1-norm is the most popular sparsity-promoting prior in contemporary CS, but not the only one. The L_1 norm is a "softer" version of the L_0 norm. Other Lp-norms where $0 < p < 1$ are possible and impose a stronger form of sparsity, but they lead to non-convex problems. Hence L_1 is preferred.L_1 is good but L_2 is bad,it can be understand from the below figure.
The $\ell 1$ norm in the objective is a crucial feature of the whole approach. Minimizing the $\ell 1$ norm of an objective often results in a sparse solution. On the other hand, minimizing the $\ell 2$ norm and commonly used for regularization because of its simplicity, does not result in a sparse solution and hence is not suitable for use as objective function in Eq. 3.Intuitively, the $\ell 2$ norm penalizes large coefficients heavily, therefore solutions tend to have many smaller coefficients – hence not be sparse. In the $\ell 1$ norm, many small coefficients tend to carry a larger penalty than a few large coefficients, therefore small coefficients are suppressed and solutions are often sparse.[13]

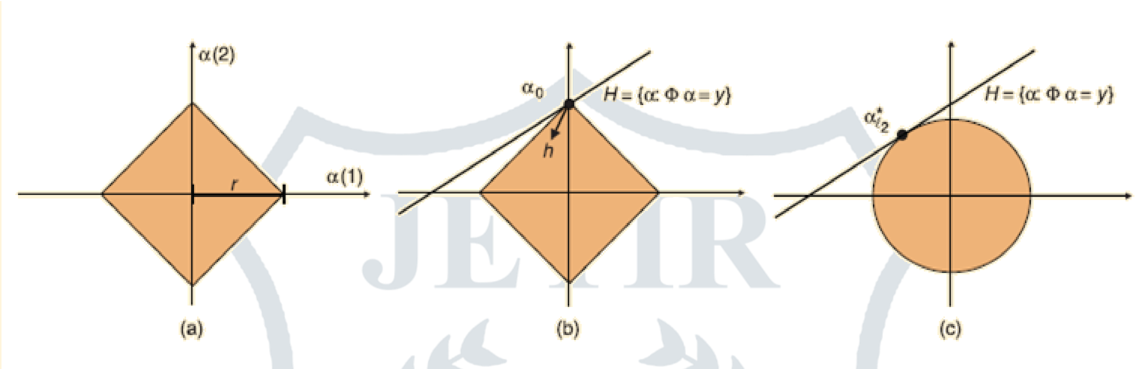


Figure 2: Geometry of L1 recovery.
Note that the line $y = Ax$ represents a HARD constraint, i.e. any vector α must satisfy this constraint.We want to find the vector α with minimum Lp-norm which satisfies this constraint.So we grow circles of increasing radius (denoted as r and defined in the sense of the Lp-norm) starting from the origin, until the circle touches the line $y = Ax$. The very first time the circle touches the line, gives us the solution (the second time, the circle will have a greater radius and hence the solution vector α will have higher Lp-norm). For the L1-norm, the solution vector α will be sparse (see middle figure). For the L2-norm, the solution vector will not be sparse (rightmost figure).

2. Greedy Technique

Greedy Techniques can solve this problem by selecting the most significant variable in$x$ for decreasing the least square error $\|y - Ax\|_2^2$ once a time. The greedy search starts from $x = 0$. In each iteration, the most important and unselected variable (different importance criterion can be designed) is added to the support set, then $\|y - Ax\|_2^2$ is minimized on the support set composed of the selected variables. The iterations stop when the number of the selected variables reach k. Famous representatives are Orthogonal matching pursuit (OMP), compressive sampling matching pursuit (CoSaMP), and original lasso.
Orthogonal matching pursuit (OMP) is an improvement on matching pursuit. The principle is the same, at every iteration an element is picked from the dictionary that best approximates the residual. However rather than simply taking the scalar product of the residual and the new dictionary element to get the coefficient weight, we fit the original function to all the already selected dictionary elements via least squares or projecting the function orthogonally onto all selected dictionary atoms, hence the term orthogonal matching pursuit. (Pati, Rezaiifar, and Krishna prasad 1993; Mallat, Davis, and Zhang 1994; Davis, Mallat, and Avellaneda 1997) Orthogonal matching pursuit has been successfully used for signal recovery, however many complaints appeared concerning its performance in compressive sensing, such as (DeVore and Temlyakov 1996). More recent work though in (Tropp and Gilbert 2007) suggests that OMP can indeed perform well in the compressive sensing arena. The recent paper (Tropp and Gilbert 2007) contains two main theorems concerning the application of orthogonal matching pursuitto compressed sensing. Consider the task of finding the k sparse signal $x \in R^n$ given the measurements $y = Ax$ and the measurement matrix A $\in R^{m \times n}$.
An extension to orthogonal matching pursuit algorithms is the CoSaMP (COmpressive SAmpling Matching Pursuit) algorithm published in (Needell and Tropp 2008). The basis of the algorithm is OMP but CoSaMP, can be shown to have tighter bounds on its convergence and performance. The CoSaMP consists of five main steps, each of which will be covered by a lemma analysing its performance.[14]

- Identification: Finds the largest 2s components of the signal proxy covered in Lemma.
- Support Merge: Merges the support of the signal proxy with the support of the solution from the previous iteration.
- Estimation: Estimates a solution via least squares with the constraint that the solution lies on a particular support.
- Pruning: Takes the solution estimate and compresses it to the required support.
- Sample Update: Updates the "sample", namely the residual in F-space.

We have proposed gradient projection algorithms for solving a quadratic programming reformulation of a class of convex non smooth unconstrained optimization problems arising in compressed sensing and other inverse problems in signal processing and statistics. In experimental comparisons to state-of-the-art algorithms, the proposed methods are significantly faster (in some cases by orders of magnitude), especially in large-scale settings. Instances of poor performance have been observed when the regularization parameter is small, but in such cases the gradient projection methods can be embedded in a simple continuation heuristic to recover their efficient practical performance.

III. The proposed Hybrid technique for reconstruction

1. Description

Given an initial low resolution image $\tilde{x}$, we would like to solve for the wavelet transform of the sharp, high-resolution image $\widehat{X_s}$. The idea is that once we solve for $\widehat{X_s}$, we can take its inverse wavelet transform $\Psi\widehat{X_s}$ to recover our desired high-resolution image $\widehat{X_s}$. To do this, we use the Hybrid algorithm. Hybrid algorithm is preferable over non-linear methods like linear programming or Basis pursuit for our experiments because it is faster and can handle large vectors and matrices, which is necessary when working with images because the size of the matrices involved are $n \times n$ where n is on the order of 256^2.
In that it approximates the transform coefficients on every iteration and then sorts them in non-increasing order. However, the main difference is that instead of only selecting the largest coefficient, Hybrid algorithm selects the continuous sub-group of coefficients with the largest energy, with the restriction that the largest coefficient in the group cannot be more than twice as big as the smallest member. These coefficients are then added to a list of non-zero coefficients and a least-squares problem is then solved to find the best approximation for these non-zero coefficients. The approximation error is then computed based on the measured results and the algorithm iterates again. In this work, we make the slight modification in that we limit the number of coefficients added in each iteration. We found experimentally that we get better results than the L1 minimization and Greedy technique.

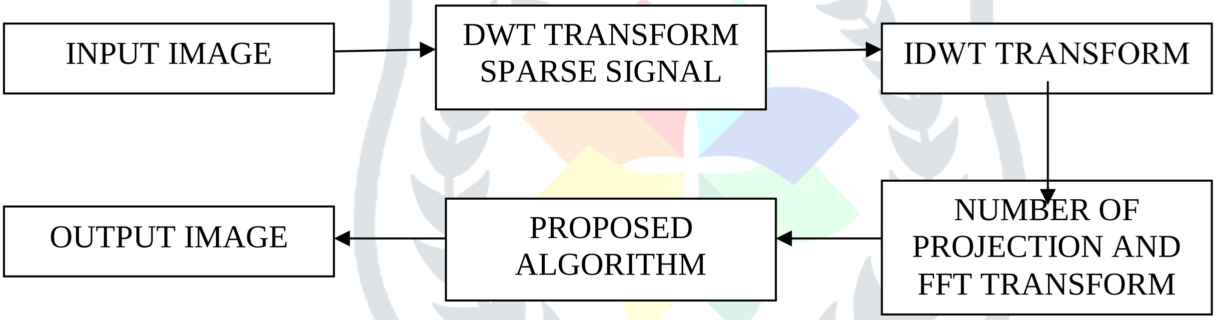


Figure 3: Block diagram of proposed method

2. Flow chart

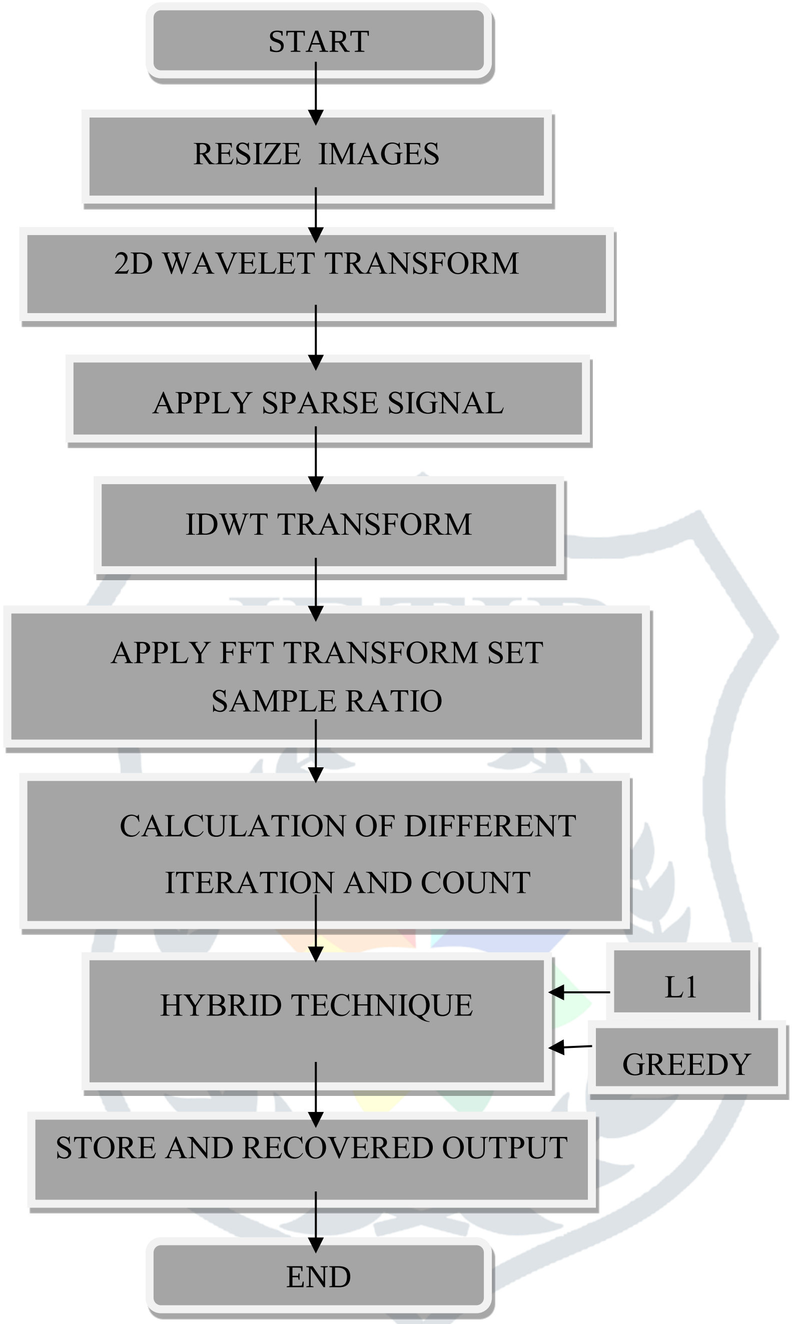


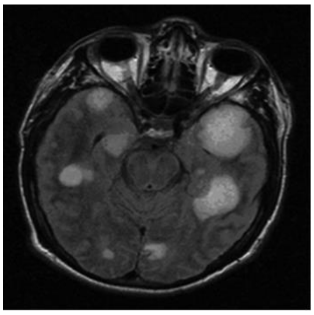
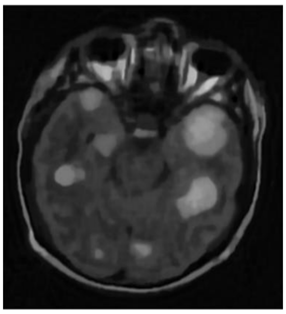
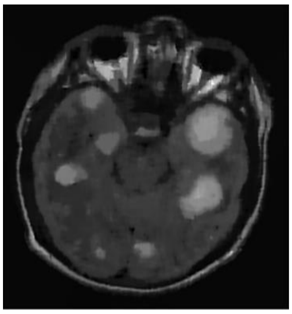
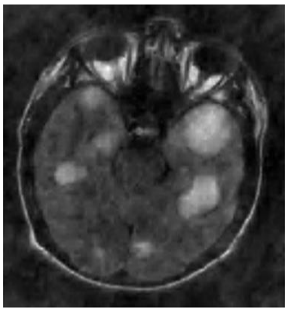
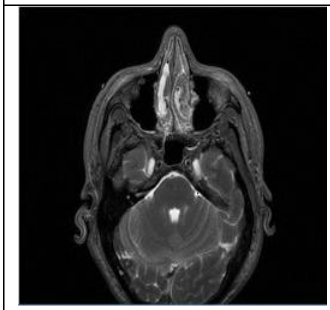
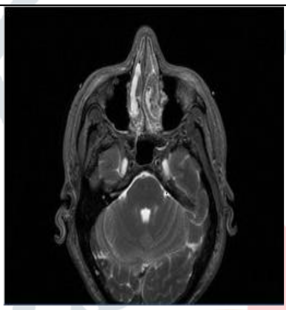
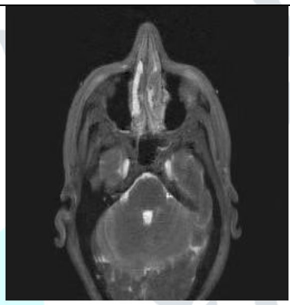
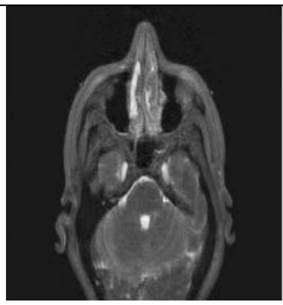
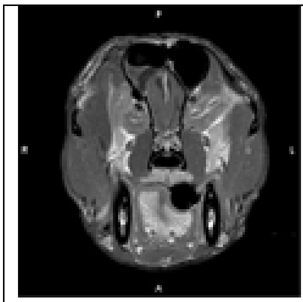
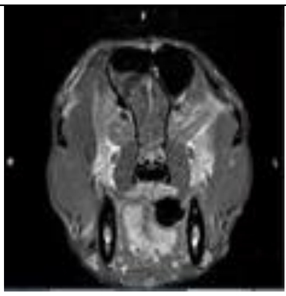
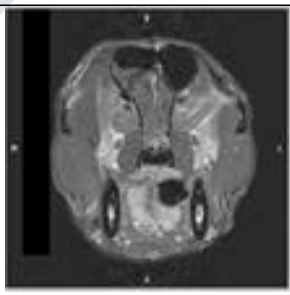
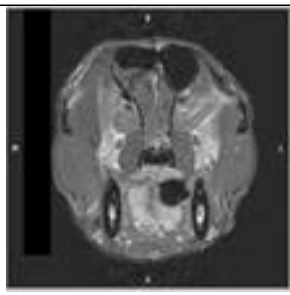
Figure 4: Proposed Hybrid algorithm

IV. Experimental results

We first evaluate our proposed Hybrid technique for reconstruction with combining of two different technique L1 and Greedy. Besides presenting the PSNR values, we also compare our results with the results of L1 and Greedy technique. Also check the computational time of different images with different techniques. After the combination of these two techniques, This is to verify that our proposed framework is taking less computational time as compared to those two techniques. It is also seen that the computational time is depends on the processor of the system.

Below is the table of different results and compare them with one another. It is obvious that if you are combining two techniques than you definitely get improvement in your resultant hybrid output.

TABLE 1 CHECK THE DIFFERENT VALUE OF PSNR AND ELAPSED TIME FOR THE IMAGES

Original Image	**Reconstruct Image**		
	Using L1 Technique	Using Greedy Technique	Using Hybrid Technique
 PSNR value Elapsed time	 29.5486 237.912055	 30.1232 156.432970	 31.5143 127.453430
 PSNR value Elapsed time	 28.9758 235.935840	 33.9358 158.453256	 41.7985 130.895632
 PSNR value Elapsed time	 29.1768 236.753492	 34.1305 159.745125	 40.4679 131.896523
 PSNR value Elapsed time	 29.2474 239.491673	 32.8990 157.427683	 42.4091 129.482619

V. Conclusion

This paper proposed a novel in-reconstruction of image with less computational time. Compressive sensing theory is basically used in application where image reconstruction task is performed in compressed domain. CS theory is used in one pixel camera, image acquisition, radar signal processing, medical imaging, pattern recognition. Many CS recovery algorithms have been developed in last decade. This paper is focused on quantitative study and analysis of different reconstruction techniques like L1 technique, Greedy technique and proposed Hybrid technique for different image transforms and shows that how they are used for image reconstruction in medical image. Comparison of these three techniques taken place based on PSNR and computational time and concluded that the proposed Hybrid technique provides a less computational time and better image quality of reconstructed image compare to other two techniques.

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