Design, Mathematical modeling of a Ball Balancing Robot

Mr. Shashi bhushan Sankhyan
Department of Mechanical Engineering
PHCET Rasayani, India

***Abstract*—this research paper focuses on the recent development in the field of Robotics that is the present and future of ball balancing robots "BALLBOTS". The researcher in this paper has designed a Ball balancing robot from the scratch working on the mathematical modeling. Additionally the research which has been completed on the topic above beginning from robots and discussing their impact on our day to day lives is discussed also the researcher begins with describing various sorts of robots manufactured till the present day with their functions and drawbacks. Further, BALLBOTS are considered conferring to the developments made in this field, the various designs that have been implemented, various parts of these robots and its functioning using inverse mouse and inverted pendulum mechanism. The researcher has drafted the model on a 3D modeling software to provide the desired shape and size and also to impart the required details such as dimensions, material etc. Mathematical modeling of the robot has been done considering two vertical and one horizontal plane instead of 3D modeling furthermore the steady state equations are derived . Furthermore the researcher has discussed Performance study and the areas of the implementation and also the improvements needed in addition to which suitable conclusions based on the research will be extracted.**

IndexTerms— **Ballbots , Mathematical Modeling**

I. Introduction

Robots have become an essential part of the day today functioning of any company or MNC as the need for automation of any workplace has become the at most importance for reducing man power and so as to reduce human error and also to increase the productivity and efficiency of any process. Even NASA prefers these robots as they are light in weight, compact in nature [(1)], deployable and also fit in working in various environments which might not be suitable for human. A wider prospective is given to Robots in the last decade and they seem to have achieved immense applicability in both industrial as well as government sectors. An enormously One such admired method for greater applicability in various fields is to implement the mechanism of inverted pendulum wherein the robot can stabilize and twirl about its axis, [(21)] Imparting the robot with additional features such as better steering on uneven landscapes and a range of others in the field of medical applications etc.[(22)]

A constant comparison of every machine that has effectively reduced the work load of the humans has been compared with the humans since the beginning. This was also applicable in the field of robots, as they were made as much as user friendly and interactive in the quarters which had to intermingle with the humans. One of the characteristics was their height, but as their height increased their CG was also increased above the ground level additionally giving problems for it to slow, accelerate, move up and down a slope therefore to compensate for these problems the width of the robots was increased which further lead to the problem of restricting their movement through narrow passages and in congested rooms [(3)].

Thus to overcome these difficulties invention in the field of robots, called Ballbots [(3)] was made. [(4)(5)(6) (7) (9) (10)]. This type of the robot maintains equilibrium position whose body is mounted on a Ball further in contact with the ground, this concept is known as the "Inverse Mouse-Ball Drive" analogous to the inverted pendulum method[(5)(8)]. The complexity in the functioning of ballots is higher than any other robots working on two wheel or singe wheel as the need of balancing the body is of prime importance and the wheel moves in the direction the tilt of the body whereas in this case stability needs to be provided to both body as well as the ball and the motion in between the body and ball also needs to be taken into account. A feedback is provided by the controller which further is responsible for maintaining the balance of the Ballbot and its movement.

This research paper focuses on designing a robot mounted on a ball which stabilizes on its own without falling off and moving ahead on the ball. With their lean and loft nature these are more human adaptable and with an impeccable scheme. Moreover the ball at the bottom of the body allows it to move in all the directions with ease. A PID Controller is used which eradicates any inaccuracies in the system coordinating with the pace of the ball and tilt of the body in order the keep the body erect [(12) (14)].

II. Literature Review

A. Balancing Robots

Self-balancing robots are very well acknowledged as they do not need any support and would balance themselves on one or two wheels or a ball [(30)]. Most of the self balancing robots work on the phenomena of inverted pendulum and have been in demand for quite some time. The first of these type was the cart pendulum type which wasn't accepted widely, after which was the two wheel robot functioning on inverted pendulum which began to gain popularity after the launch of Segway RMP wheelchair [(23)(24)] known as ibot. Subsequent to which further developments in the same field emerged with tele presence [(26)], ubot [(27)] and so on [(28) (29)]. But later a major shortcoming observed in the execution of the two wheels, that the movement of the wheel was unidirectional that is in the vertical plane only furthermore not allowing it to move sideways. This led to the development of the single wheel robots which overcame this confinement and could perform various tasks in desired directions with swiftness.

B. Ballbots

Ballbots came into lime light after the drawbacks of the two wheel robots were apprehended. [(6)(35)(36)(37)] These balance themselves on a ball which can follow any course with the help of various parts which were integrated in this system to enhance the properties such as stability, acceleration, tilt, position etc. A feedback scheme is applied to the ballbot necessitating the variables used to balance the system. The key motivation which led to development of the ballbot was to make the robot more users friendly and also designing a robot for the future [(14)].

In this structure the ball is the chief imputing force which permanence's the body of the robot tilting it instinctively and twirling it. The mechanisms used are the inverse mouse and inverted pendulum which will be discussed in detail later in this research [(39)]. The principle on which the ballbot works is to maintain the Centre of gravity in line with the point of contact with surface and ball as the movement of the ball will cause the shifting of the Centre of gravity of the body mounted on it therefore to keep it vertical and in symmetry the movement of the body is supported by the movement of the ball but in the counter direction [(43)].

A Typical ballbot model produced by Iranians. [(45)]

C. Components of ballbot

1) *Body and Undercarriage:* The structure of the ballbot is formed by assembling a three part system the main frame of the ballbot, under loading compartment and the ball on which the whole system balances. The system is collaborated such together so as to place the bearing actuators such that they in turn produce a force driving on the Omni wheels which further drive the ball. The researcher in this paper has utilized three actuators therefore to maintain symmetry these are placed at a 120˚ angle. These wheels are wholly responsible for the movement of the robot in any direction as the Omni wheel produces three horizontal forces along the circular space of the ball and no oblique force.

2) Balancing Ball and its requirements:

The ball is the core of the this system therefore the design of the ball should be such that it supports the whole system that is the ball should be able to take the weight of the whole system as well move the system in any direction still imparting it with stability. The ideal preferences were either a sports ball which do not require more expenditure or a custom made ball. To plump for any of the above alternative the ball must fulfill the stipulation stated above therefore an apt size ball will be either a basketball or a soccer ball as the material provides pertinent amount of friction needed for this system but with these a few problems were encountered. Firstly these were available in specific diameters which might or might not be compatible with the system. Secondly and mainly due there shock absorbing property as these balls rebound when they come in contact with the ground which might lead to the collapse of the whole system . These all drawbacks can be overcome with the help of a custom made in which the core in of aluminum coated with polyutherane coating which combines the properties of the sports ball with none of the above stated shortcomings and also has a high load bearing capacity[(11)].

III. MATHEMATICAL MODELING

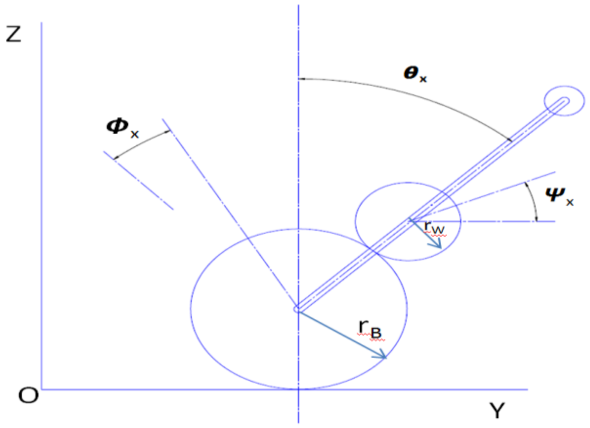
The two dimensional model for the ballbot about YZ plane is described. Only two plane are considered for generating the equation of motion i.e. vertical plane YZ/XZ plane and horizontal plane XY plane. As the calculations for two vertical YZ and XZ plane will be same therefore modeling for only one vertical plane will be taken into account and same values will be assumed for other vertical plane. Therefore here equations will be found for YZ and XY planes only.

A. Assumptions

The process of designing a ball balancing robot is an intricate process due to which a few assumptions are made in the system so as to simplify the system. The following are the assumptions made [(5) (14) (21)]

- There are two perpendicular planes considered but the mathematical modeling for only one plane is done as the two planes are identical and have the same equations of motion.
- Any distortions in the system is neglected as the system is divided into two mains parts namely the ball and the body further omni wheels are attached to the body which drive the system.

- Friction is considered to be negligible.
- Slip for this model is not taken into consideration for both the ball and the floor and the ball and the Omni wheels.
- The value of static friction is sufficiently high so as to prevent sliding of the ball and also to avoid the motion of rotation of the ball about its axis.
- The ball makes constant contact with the ground and with no shoot or hindrances in the path of movement.
- Ground considered for the movement of this robot is considered to be flat implying that the ball has zero height or zero potential energy
- Omni wheel is considered to be a single part making single point of contact with the surface of the ball



2D Model of the Ball Balancing Robot

Where
r_B Radius of the ball
r_W Radius of virtual actuating wheel
I_B Moment of inertia of the ball
I_W Moment of inertia of the virtual actuating wheel in the YZ-/XZ-plane
I_{Wxy} Moment of inertia of the virtual actuating wheel in the XY-plane
I_A Moment of inertia of the body of the robot in the YZ-/XZ-plane
I_{Axy} Moment of inertia of the body of the robot in the XY-plane
l Distance between COM of the ball and COM of the body of the robot

Where
φ_x And φ_y indicate the orientation of the ball,
θ_x,θ_y and θ_z indicate the orientation of the body and
ψ_x,ψ_y and ψ_z indicate the orientation of the virtual actuating wheels.

B. Mathematical model

All coordinates can be expressed as functions of minimal coordinates

$$X_B = \varphi_x . r_B$$

$$X_W = \varphi_x . r_B + \sin\theta_x . (r_B + r_W)$$

$$X_A = \varphi_x . r_B + \sin\theta_x . l$$

The relation of actuating wheel expressed as function of minimal coordinates yields

$$\dot{\psi}_x = \frac{r_B}{r_W} . (\dot{\varphi}_x - \dot{\theta}_x) - \dot{\theta}_x$$
$$\dot{\psi}_y = \frac{r_B}{r_W} . (\dot{\varphi}_y - \dot{\theta}_y) - \dot{\theta}_y$$
$$\dot{\psi}_z = \frac{r_B}{r_W} . (\dot{\varphi}_z - \dot{\theta}_z) . sin\alpha$$

Lagrangian equation for the YZ/XZ Plane is given by summation of kinetic energies for ball, body and Omni wheels and subtracting the summation of potential energies.

$$L(\varphi_x, \theta_x, \dot{\varphi}_x, \dot{\theta}_x) = T_{B,yz} + T_{A,yz} + T_{W,yz} - V_{B,yz} - V_{A,yz} - V_{W,yz}$$

Further to which as the Lagrangian for the YZ/XZ plane is used to find the equation of motions by using Euler Lagrangian equation, which is given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_x}\right) - \frac{\partial L}{\partial \varphi_x} = \tau_{\varphi_x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_x}\right) - \frac{\partial L}{\partial \theta_x} = \tau_{\theta_x}$$

The equations of motion are derived from the above Euler Lagrange further to which result is obtain by combining the equations in the form below

$$M(q_{yz})\ddot{q}_{yz} + C(q_{yz}, \dot{q}_{yz})\dot{q}_{yz} + G(q_{yz}) = \tau_{ext}$$

$$M_{yz} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

IV. State Space

State Space for YZ/XZ Plane

$M_{11}.\ddot{\varphi}_x + M_{12}.\ddot{\theta}_x + C_{12}.\dot{\theta}_x = \tau_{x1}$ (1)

$M_{21}.\ddot{\varphi}_x + M_{22}.\ddot{\theta}_x + G_2.\theta_x = \tau_{x2}$ (2)

$V_1 = \dot{\varphi}_x$, $\dot{V}_1 = \ddot{\varphi}_x$ (a)

$\dot{V}_1 = (\tau_{x1} - M_{12}.\dot{V}_2 - C_{12}.V_2)/M_{11}$ (b)

$V_2 = \dot{\theta}_x$ (c)

$\dot{V}_2 = (\tau_{x2} - M_{21}.\dot{V}_1 - G_2.\theta_x)/M_{22}$ (d)

$$\dot{X} = Ax + Bu$$

$$\begin{bmatrix} \dot{\varphi}_x \\ \dot{V}_1 \\ \dot{\theta}_x \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{M_{12}.G_2}{M_{22}.M_{11} - M_{12}.M_{21}} & -\frac{M_{22}.C_{12}}{M_{22}.M_{11} - M_{12}.M_{21}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{M_{11}.G_2}{M_{21}.M_{12} - M_{11}.M_{22}} & -\frac{M_{21}.C_{12}}{M_{21}.M_{12} - M_{11}.M_{22}} \end{bmatrix} \begin{bmatrix} \varphi_x \\ V_1 \\ \theta_x \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tau_{x1} \\ 0 \\ \tau_{x2} \end{bmatrix} [u]$$

$$y = Cx + Du$$

$$\begin{bmatrix} \varphi_x \\ \theta_x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_x \\ V_1 \\ \theta_x \\ V_2 \end{bmatrix}$$

State Space for XY Plane

$M_{xy}.\ddot{\theta}_z = \tau_{xy}$ (1)

$V_1 = \dot{\theta}_z$,$\dot{V}_1 = \ddot{\theta}_z$ (a)

$\dot{V}_1 = \frac{-\tau_{xy}}{I_{Wxy}.\frac{r_B^2}{r_W^2} + m_W(r_B + r_W)^2 + I_{Axy}}$ (b)

$$\dot{X} = Ax + Bu$$

$$\begin{bmatrix} \dot{\theta}_z \\ \dot{V}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_z \\ V_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I_{Wxy}.\frac{r_B^2}{r_W^2} + m_W(r_B + r_W)^2 + I_{Axy}} \end{bmatrix} [u]$$

$$y = Cx + Du$$

$$[\theta_z] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_z \\ V_1 \end{bmatrix}$$

V. Conclusions and Future Work

Attainment of the Equations of Motion: Equations of Motion of a generalised ballbot system have been derived in linearised form. Several assumptions were required during the derivation, and as such the equations do not give a perfect model of the system. However, the derived equations are believed to provide a fairly accurate model, as simulation results were similar to actual results.

Development of a Controller program to Stabilise a Ballbot: A controller program has been developed in which successfully balances the Ballbot. Additionally, the controller is able to maintain balance when subjected to small disturbances, and also provides command tracking capabilities.

Design of the ball balancing robot is considered and also various parameters utilised were considered.
A 3D model was designed consisting the various parts imparting dimensions and material and collaborating them into an assembly
For future work 3D modeling of the robot has to be done for getting more accurate results by considering three planes and prototype according to the solved equations using different mechanical and electronic components has to be made. Stabilization of the model utilizing state space equation of the system using a controller can be measured.

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